毛慶生老師總體經濟學筆記

注意事項

1. 本筆記中若有錯誤請自行修正。
2. 前 3 章錯誤可能比較多，請注意。
3. 因作者習慣，所以並沒有將老師上課每一個字記下，若不去上毛慶生老師的課，將有很多地方不懂，而且老師上課非常精彩，推薦大家去修。
4. 毛慶生老師有自己編寫教材，請有上課的人一定要看，另外本筆記中有些圖引用老師教材(懶得畫了)。
5. 若想要將此份筆記改得更完美，可以來信索取 word 檔與圖形，信箱為 colinjet2003@gmail.com，並附上 50 字以上 100 字以下理由(體諒我做了那麼久這要求應該不過份)。

電機系 B96 級

林佟鴻

Chapter 1－Introduction

- Two types of economics
  - Positive economics 實然
  - Normative economics 應然

  Positive：How “**does**” the real world work?

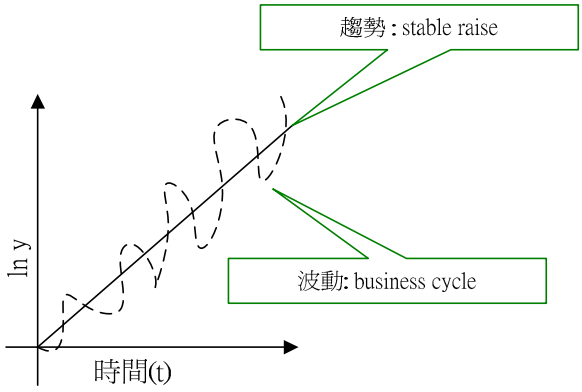
  Normative：How “**should**” the real world work?

- The scope of macroeconomics

  The performance of an economy **as a whole**

  衡量兩種量
  - Quantities ex：出口、進口、消費、投資、外匯
  - Prices ex：物價水準、貨幣價值

- 無論哪一學派都要研究的 market economics 有以下 5 特點
  1. 成長理論：stable raise of output and income
  2. 景氣循環理論：business cycle(短期波動)
  3. 貨幣理論：平均物價水準穩定成長
  4. 政府重要性增加：increase role of government policy in fiscal and monetary
  5. 全球化(經濟開放)：openness of economy

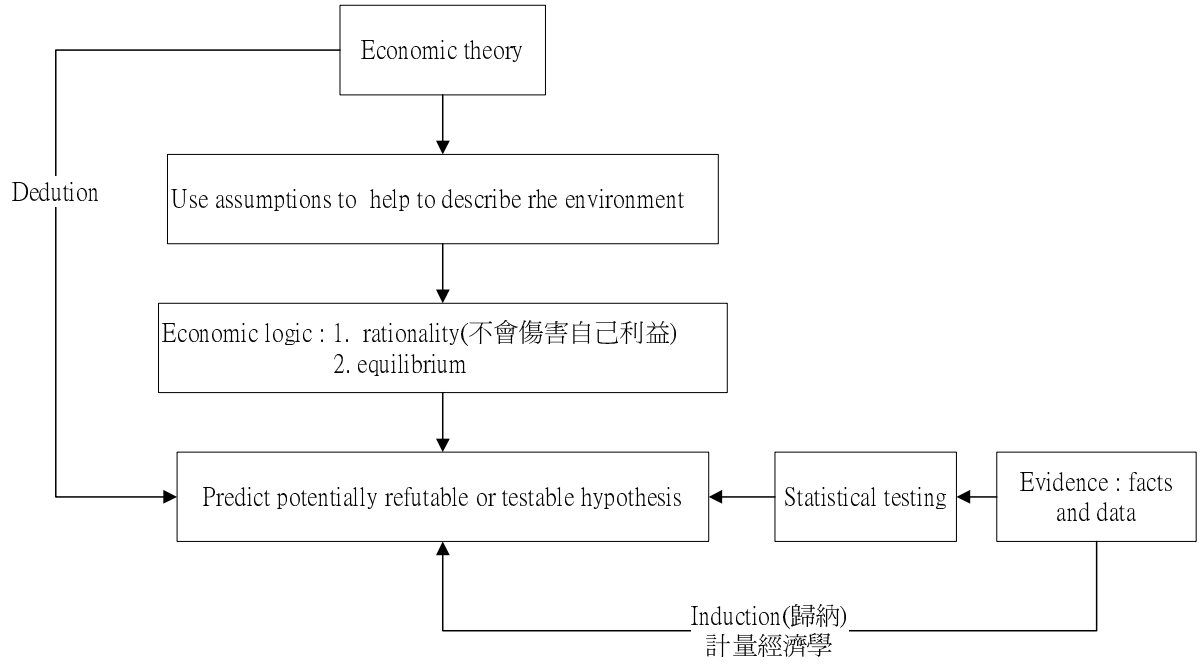


- Something about economy

  Observation equivalence：結果相同但各學派解釋不同

  經濟模型：Abstraction and simplification NOT realistic representation

- Methodology in general



- Anatomy 包含四要素
  1. Technology & feasibility (技術&可行性) 客觀條件
  2. Preference & objectives (偏好&目標) 主觀意願
  3. behavior implication ex：demand and supply function

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  4. equilibrium outcome ex：market consequence

  總經

- The types of variables depend on the model
  Exogenous variables(外生變數)：不在 model 中，無法選擇及控制
  Endogenous variables(內生變數)：The variables that can be determined in the model

Chapter 2－Measurement : Quantities & Prices

(Ⅰ) Quantities：GDP & GNP

- GDP(Gross domestic product 國內生產毛額)

  Definition：The nominal (dollar) **market value** of all final goods and services that an economy produces **in a given period** within its bounder (在國界內)

  Cf：GNP(Gross nation product 國民生產毛額)

  Definition：The nominal (dollar) **market value** of all final goods and services that an economy produces **in a given period** by its residents / citizens (由此國籍的人所生產)

  Final output：

  $$\begin{array}{l} \textit{Total value of production} \\ - \quad \textit{Intermediate input} \\ \hline \textit{Value added to production} \end{array}$$

  Ex：一台電腦 total value=1 萬元，但其中 CPU(1000 元)是和別人買的，並非自己生產的，此 1000 元就是 intermediate inputs，所以只有 9000 元是自己產的 final output(value added to production)。

  ※ 定義中的 final goods and services 要使用 value added to production，若使用 total value 會重複計算到 intermediate inputs(因此 intermediate inputs 在其他企業中是 value added to production，所以已經算入一次了。

  ※ 因爲只計算 market value，所以不經由市場的活動不被計入(ex: underground economy 黑市「使智利的 GDP 少算 70%的原因」、housewife service)

- GNP

Factor income from abroad：contribution to foreign production by domestic factors.

Factor payment from abroad：contribution to domestic production by foreign factors.

$$\begin{array}{r} \text{Factor income from abroad} \\ -\quad \text{Factor payment from abroad} \\ \hline \text{Net Factor Income From Abroad}\left(\text{ NFIA }\right) \end{array}$$

Net factor payment (NFP) = NFIA

$$\boxed{\text{GNP} = \text{GDP} + \text{NFIA}}$$

- Pitfalls of GDP as a measurement of welfare

①. Ignore income distribution
②. Exclude non-market transactions
③. Ignore the value of leisure
④. Ignore environmental quality ⇒行政院主計處編 Green GDP 來修正
⑤. Ignore product quality
⑥. Ignore wealth implications

- 補充

Arthum Okun 提出：Misery Index(痛苦指數)=失業率+通貨膨脹率

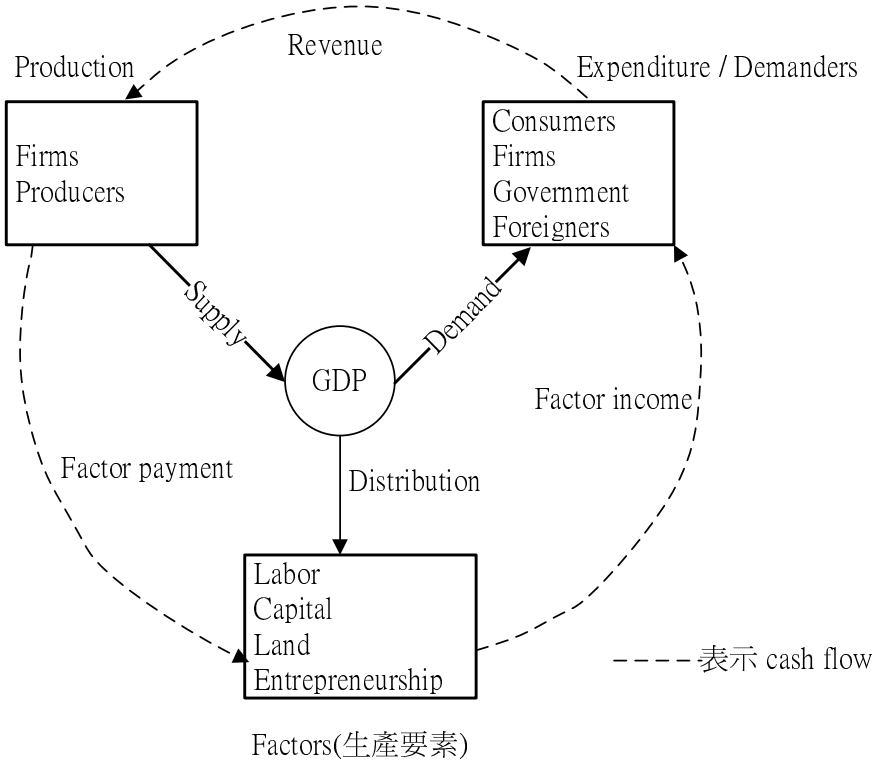
Stagnation(經濟不景氣)+inflation = stagflation(1970 年代)
但通常經濟不景氣，就會少消費，因此較不易通貨膨脹，1970 年代出現原因爲：凱因斯學派的對策。

凱因斯學派面對 stagnation 之對策：

a. Cut the tax (讓大家錢變多)
b. Monetary easing(貨幣寬鬆)
c. 增加消費⇒inflation

(II) Measurement of GDP

- Use System of National Accounts (SNA93,1993 版本)，以前用 SNA68
  Principal：How to produce / consume / distribute 由以上可推出 3 種方法



由上圖可以看出 Measure GDP 的三種方法：

(1) Measure by production(所有 Production 相加)
(2) Measure by expenditure(所有 Expenditure 相加)
(3) Measure by factors(所有 Factor 相加)

(III) Measure by supply

$$\begin{array}{l} \text{Total value of production (sales revenue)} \\ - \quad \text{Intermediate inputs (cost of goods sold)} \\ \hline \text{Value-added to production (gross profits)} \end{array}$$

GDP = sum of value-added by domestic firms

GNP = GDP + NFIA

(IV) Measure by expenditure

$$GDP = C + I + G + (X - M)$$

- Parameters

  C：Consumer (expenditures on consumption)
  I：Firms (investment expenditures)
  G：Government (government expenditures)
  X：Export
  M：Input
  C+I+G：Domestic absorption (demand)
  X-M：可寫成 NX⇒Net Export

- C：Private consumption expenditure (60% of GDP)

  種類(by types)：
  (1) Non durables：food
  (2) Services：education、entertainment、utility
  (3) Durables：cars (因房子超耐久所以算在投資中)

- I：Gross domestic investment(國內投資毛額) or Gross capital formation(資本形成毛額)(20%)

  1. 分類
     By ownership：
     a. I^P：Private investment (11~12%)
     b. I^g：Public investment (8~9%) { government capital / Public enterprise (公營企業)

     By types：
     a. Nonresidential physical investment (廠房、機器、設備)
     b. Residential physical investment (房子)
     c. "Change" in inventories (存貨) 要當年的才可計入 GDP，以前的不算，此項在算 GDP 時最容易出錯。
     d. Corporate software expenditure
     e. Mining expenditure

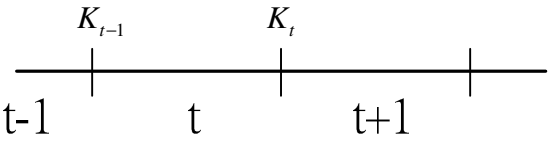
d，e 兩項在 SNA68 中算成是中間投入，SNA93 才計入

2. GDP & NDP & Dep

Investment：Expenditure on “capital” goods (for producing) by producers

Gross investment = Net investment + Depreciation Investment (Dep)

Let K_{t-1} = initial capital stock at time t



$$K_t = K_{t-1} + \underbrace{I_t}_{\text{投資流量}} - \underbrace{\delta K_{t-1}}_{\text{折舊}} \quad 0 \le \delta \le 1$$

$$\underbrace{I_t}_{\text{毛投資}} = \underbrace{\left(K_t - K_{t-1}\right)}_{\text{net investment}} + \underbrace{\delta K_{t-1}}_{\text{depreciation}}$$

同理 GDP - Dep = NDP

GNP - Dep = NNP

- G：Government consumption expenditure (purchase)經常帳

(13~15%)

1、 Examples：national defense、public education、law & enforcement、foreign affairs

2、 Government spending (outlays)

消費性支出

a. G

b. TR (transfer payments)⇒not include in GDP because nothing produced or consumed by it

c. Net interest payment on public debts 公債 ⇒not include in GDP

非消費性支出

a. I^g (Public investment)

3、 TR

a. Unemployment compensations (Insurance)
b. Retirement compensations (National pension or Social security)
c. National health program
d. Other subsides (Disaster reliefs)

- NX：Foreign demand / Net export

NX (net export) = X (export) – M (input)
∴ GDP = C + I + G + NX
GDP + NFIA = GNP
經常帳餘額(Current account balance) = NX + NFIA

(V) Measure GDP (GNP) by distribution (factor income)

- National income (NI)

(1) Labor component：wage 約佔 GDP 60%
(2) Capital component：Interest (rental 租令所得、dividend 股利)
(3) Land：rent (地租)
(4) After sales tax profit (企業能力)

※ Dividend 中借錢給銀行所賺的利息不可計入，一定要賺國外的錢才算，因國內的有人賺必有人賠，正負相抵。

NI = Wage + Interest + Rent + After sales tax profit

- NI V.S GNP

1. Difference
①. Depreciation(Dep) ⇒Not in NI but in GNP
②. Indirect taxes (Value-added tax VAT 可轉價的間接稅) ⇒Not in NI but in GNP
③. Government product subsidy⇒In NI but NOT in GNP
②-③ = Net indirect tax

GNP = NI + Dep + Net indirect tax
= NI + ① + ② - ③

- Income and Saving

1. Private income (y) 民間所得
   不勞而獲：TR (transfer income from government)
   INT (公債利息)

$$y = (GDP + NFIA) + TR + INT$$

※Personal income(個人所得) = y- undistributed business earning(有賺但廠商沒分給你)

2. Private disposable income (y^d)

$$y^d = y - T(\text{ total tax 勞而不獲})$$

3. Private saving (S^p)

$$y^d = C \text{ (consumption)} + S^p$$

$$S^p = y^d - C = y - T - C$$

$$=(GDP+NFIA+TR+INT)-T-C$$

4. Government deficit (D) & Government savings(S^g)

$$D = \text{政府支出} - \text{政府收入(Tax)}$$

$$= (G+I^g+TR+INT)-T$$

$$S^g = \text{政府收入(Tax)} - \text{消費性支出}$$

$$=T - (G+TR+INT)$$

$$D+S^g = I^g \Rightarrow S^g = (-D) + I^g \quad (-D \text{ 表預算盈餘})$$

5. National saving (S)

(1) $S = \boxed{S^p + S^g}$

=[GDP+NFIA+TR+INT-T-C]+[T-(G+TR+INT)]

=$\boxed{\text{GDP+NFIA-(C+G)}}$

=GNP-(C+G)

(2) S= GDP+NFIA-(C+G)

=(C+G+I+NX) +NFIA-(C+G)

=$\boxed{\text{I + NX+NFIA}}$

= I + CA

NX：國際間 goods & services 結果

NFIA：國際間 factors 移動結果

CA：current account balance = NX+NFIA (國外效果)

I：國內投資效果

6. National wealth(W)

$$W_t = W_{t-1} + S_t$$

- S＞0⇒National wealthy ↑
- S＜0⇒National wealthy ↓

S＞0⇒
- I＞0：Domestic assent (capital) ↑
- CA＞0：
  - NX ↑
  - NFIA ↑

(VI) Real GDP & Nominal GDP

- Conventional (Basic) approach

Nominal GDP：名目上的 GDP，以當時價格來衡量，但若通貨膨脹使價格上升，GDP 亦會上升，此上升並不代表經濟變好。

Assume N goods, no intermediate inputs

P_{it} = price of good i at time t　　i=1,2,3,…N

q_{it} = quantity of good i at time t　　i=1,2,3,…N

1. Nominal GDP at time t

$$y_t = \sum_{i=1}^{N} p_{it} q_{it}$$

GDP at ***current*** dollar

2. Real GDP at time t (base year = 0　以第 0 期的價格來衡量)

$$\overline{y}_t^0 = \sum_{i=1}^{N} p_{i0} q_{it}$$

將價格波動排除

GDP at ***constant*** dollar

3. Example－2 goods economy

p_1^2 = price of good 1 base on the time 2

	time	q_1	p_1	p_1q_1	$p_1^1q_1$	$p_1^2q_1$
good 1	1	100	\$1	100	1×100	3×100
	2	50	\$3	150	1×50	3×50

	time	q_2	p_2	p_2q_2	$p_2^1q_2$	$p_2^2q_2$
good 2	1	50	\$4	200	4×50	6×50
	2	100	\$6	600	4×100	6×100

Nominal GDP

$y_1 = p_1q_1 + p_2q_2 = 100 + 200 = 300$

$y_2 = p_1q_1 + p_2q_2 = 150 + 600 = 750$

Growth = 2.5 (價格上升影響極大)

Real GDP (base year = 1)

$\overline{y}_1^1 = p_1^1q_1 + p_2^1q_2 = 100 + 200 = 300$

$\overline{y}_2^1 = p_1^1q_1 + p_2^1q_2 = 50 + 400 = 450$

Growth =1.5

Real GDP (base year = 2)

$\overline{y}_1^2 = p_1^2q_1 + p_2^2q_2 = 300 + 300 = 600$

$\overline{y}_2^2 = p_1^2q_1 + p_2^2q_2 = 150 + 600 = 750$

Growth =1.25

缺點：Base year 不同 GDP 成長程度不同，是因爲相對價格改變所早成的結果，第一期 $\frac{p_1}{p_2}=\frac{1}{4}$，第二期 $\frac{p_1}{p_2}=\frac{1}{2}$，在第二期 good1 變得相對較貴。未解決此問題，大約每 5 年 Base year 會換一次，以減輕相對價格改變的效果

優點：將價格波動效果排除

- Chain-weighting (chain-indexed) real GDP

Rolling base year

1. Simplified example (base year=1)

t	q_1	p_1	$\overline{p}_1$ (上下兩期價格取平均)	$\overline{p}_1 q_1$
1	100	\$1	\$2	2×100
2	50	\$3		2×50 4×50
3	100	\$5	\$4	4×100

t	q_2	p_2	$\overline{p}_2$	$\overline{p}_2 q_2$	Total	chain index
1	50	\$4	\$5	5×50	450	$\frac{450}{450}=1$
2	100	\$6		5×100 6×100	600 800	$\frac{600}{450}\cdot\frac{450}{450}=\frac{4}{3}$
3	100	\$6	\$6	6×100	1000	$\frac{1000}{800}\cdot\frac{600}{450}=\frac{5}{3}$

Base price：用前後兩期取平均

由前面 Basic 做法知 Nominal GDP at time = 1) $y_1=300$

Chain-indexed real GDP：

$$\overline{y}_1^c=300\times 1=300 \qquad \overline{y}_2^c=300\times\frac{4}{3}=400$$

$$\overline{y}_3^c=300\times\frac{5}{3}=500$$

2. Actual procedure (US National income & Product income)

①. Compute Fisher quantity index for each time t

$$F_t = \sqrt{\left(\frac{\sum_{i=1}^{N} p_{t-1}^i q_t^i}{\sum_{i=1}^{N} p_{t-1}^i q_{t-1}^i}\right)\left(\frac{\sum_{i=1}^{N} p_t^i q_t^i}{\sum_{i=1}^{N} p_t^i q_{t-1}^i}\right)}$$

Growth of real GDP (base year=t-1)　　Growth of real GDP (base year=t)

上是非線性，所以兩邊同取 log

※$1+z=\sqrt{(1+x)(1+y)}$

$$\ln(1+z)=\frac{1}{2}\ln(1+x)+\frac{1}{2}\ln(1+y)$$

$\because \ln(1+x)\approx x$ for x sufficient

(在經濟學中，x 常代表利率或成長率，所以夠小

$$\Rightarrow z\approx\frac{1}{2}(x+y)$$

②. Chained index (I_t)

令 $I_0=1$

$I_1=I_0F_1=F_1$

$I_2=I_1F_2=F_1F_2$

$I_t=I_{t-1}F_t=F_1F_2\cdots F_t$

③. Chained-weighting real GDP (base year= 0)

$$\overline{y}_t^c = y_0 \bullet I_t$$

y_0 ：nominal GDP at t=0

缺點：GDP≠C+I+G+NX 但差不多

④. Example

t	y_t^1	y_t^2	Chain-index	chained real GDP
1	300	600	$I_1 = 1$	300
2	450	750	$I_2 = F_2 = 1.37$	$300 \times 1.37 = 410.6$
Gross Factor	1.5	1.25		

$$F_2 = \sqrt{(1.5)(1.25)} = 1.37$$

(VII) Price index

- 用處

  Weighted average price (index) of a basket of goods and services at a given point of time

- GDP deflator (GDP 平減指數)

$$P_t = 100 \frac{No\min al\ GDP_t}{\mathrm{Re}al\ GDP_t} = 100 \frac{\sum_{i=1}^{N} p_{it} q_{it}}{\sum_{i=1}^{N} p_{i0} q_{it}} \quad \text{(以第 0 期爲基準)}$$

$$= 100 \sum_{i=1}^{N} \left[\frac{p_{it} q_{it}}{\sum_{i=1}^{N} p_{i0} q_{it}} \right] \quad \because \frac{d+e+f}{a+b+c} = \frac{d}{a+b+c} + \frac{e}{a+b+c} + \frac{f}{a+b+c}$$

$$= 100 \sum_{i=1}^{N} \frac{p_{it}}{p_{i0}} \left[\frac{p_{i0} q_{it}}{\sum_{i=1}^{N} p_{i0} q_{it}} \right] \quad \text{令} \left[\frac{p_{i0} q_{it}}{\sum_{i=1}^{N} p_{i0} q_{it}} \right] = w_{it} \quad \text{Paasche index}$$

$$= 100 \sum_{i=1}^{N} \frac{p_{it}}{p_{i0}} w_{it} \qquad ※ \sum_{i=1}^{N} w_{it} = 1$$

w_{it} =Expenditure share of good i at time t (表第 i 種商品在 t 的比重)

P_t 表示現在的 goods 在過去值多少

GDP deflator 一般會低估痛苦指數，因爲忽略了調整的過程，ex：當一個商品價錢上升，一般人消費量會下降，導致該物的 w_{it} 下降，使該物較不重要，但調整過程是痛苦的，卻被忽略。

- CPI (consumer price index) or WPI(whole sale (producer) price index) 消費者物價指數

$$P_t = 100\frac{\sum_{i=1}^{N} p_{it}q_{i0}}{\sum_{i=1}^{N} p_{i0}q_{i0}} = 100\sum_{i=1}^{N}\left(\frac{p_{it}}{p_{i0}}\right)\left(\frac{p_{i0}q_{i0}}{\sum_{i=1}^{N} p_{i0}q_{i0}}\right) \quad 令 \left(\frac{p_{i0}q_{i0}}{\sum_{i=1}^{N} p_{i0}q_{i0}}\right) = \tilde{w}_i$$

$$= 100\sum_{i=1}^{N}\left(\frac{p_{it}}{p_{i0}}\right)\tilde{w}_i$$

※ $\tilde{w}_i$ 和時間 t 無關，代表 Expenditure share of good i at time t=0， Laspeyres index

P_t 表示過去的 goods 拿到今天會值多少

CPI or WPI 一般會高估痛苦指數

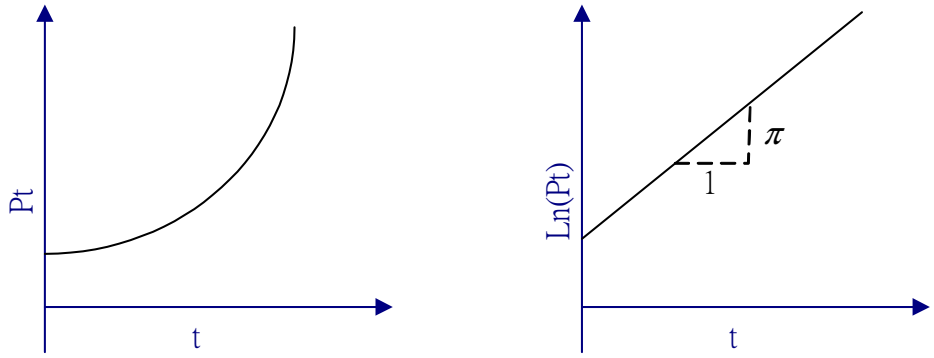
- Chain-weighting price index (Fisher price index)

$$P_t = \frac{\text{Nominal GDP at t}}{\text{Chain}-\text{indexed GDP at t}} = \sqrt{(\text{Paasche index})(\text{Laspeyres index})}$$

- Inflation rate

$$\boxed{\pi_t = \frac{p_{t+1} - p_t}{p_t}} = \frac{p_{t+1}}{p_t} - 1 \Rightarrow 1 + \pi_t = \frac{p_{t+1}}{p_t} \quad \text{nonlinear}$$

$$\ln(1+\pi_t) = \ln p_{t+1} - \ln p_t \quad \Rightarrow \pi_t \approx \ln p_{t+1} - \ln p_t \quad \text{linear}$$



Chapter 3－Measuring Business Cycle

(Ⅰ) Basic concept

- Definition of business cycle

  Ups and downs of real activities (real GDP) along the long term trade

- Transform of time series

  令 y_t ：GDP、利率…　　g：成長率

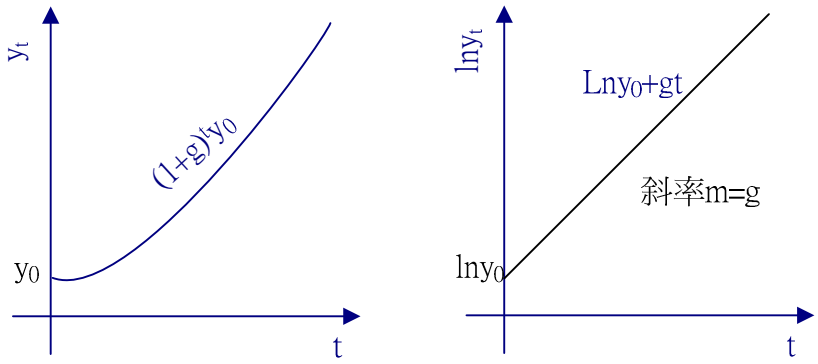
  $y_t = (1+g)\,y_{t-1} = (1+g)^2\,y_{t-2} = \cdots = (1+g)^t\,y_0$　for discrete time

  因爲式子爲非線性，所以兩邊同取 ln

  $$\ln y_t = \ln(1+g) + \ln y_{t-1}$$

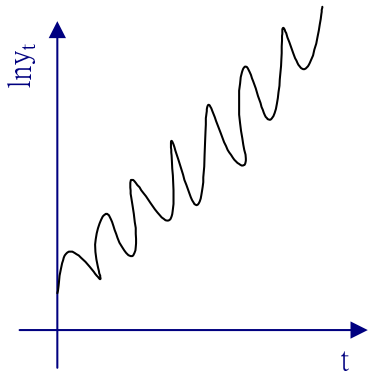
  $$= \ln y_0 + t\ln(1+g) \qquad \text{當 g 極小，} \ln(1+g) \approx g$$

  $$= \ln y_0 + gt$$



(II) Decomposition (filtering) of time series unobservable consequences

- Unobservable consequences



實際數據不可能是漂亮的直線，而是包括成長率和波動，那要如何 filter 出想要知道的成長率 g(也就是線的斜率)？
解一：迴歸線

Unobservable consequences：

①. Trend component (growth、long-term、secular)
②. Cyclical component (business cycle、short-term)
③. Seasonal component (特殊節日)
④. Irregular component

如何衡量 Cyclical component(business cycle)？ HP Filter (下面)

- Decomposition

Hodnck Prescott (HP) filter

y_t^T ：Trend component of y_t

y_t^C ：Cyclical component of y_t

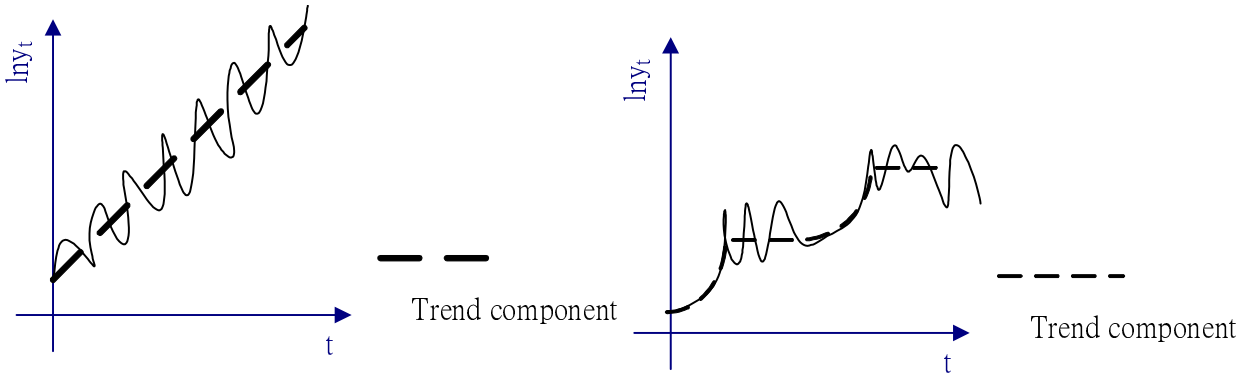
$$y_t^C = \frac{y_t - y_t^T}{y_t^T} = \frac{y_t}{y_t^T} - 1 \Rightarrow 1 + y_t^C = \frac{y_t}{y_t^T} \Rightarrow \ln\left(1 + y_t^C\right) = \ln\left(\frac{y_t}{y_t^T}\right) = \ln y_t - \ln y_t^T$$

$$\boxed{y_t^C \approx \ln y_t - \ln y_t^T}$$

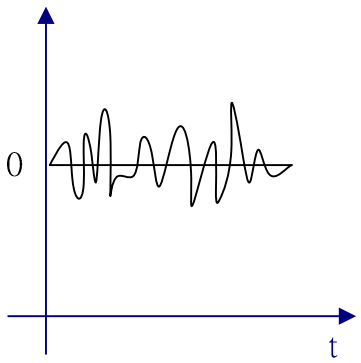
Cyclical component = 該季觀察值取 ln - Trend component 取 ln

Determine Trend component

ex：



通常 y_t^C 會在 0 附近波動



(III) Characterization

- 特性

If $\{x_t\}_{t=1}^T$ ，$\{y_t\}_{t=1}^T$ 都是 Cyclical components

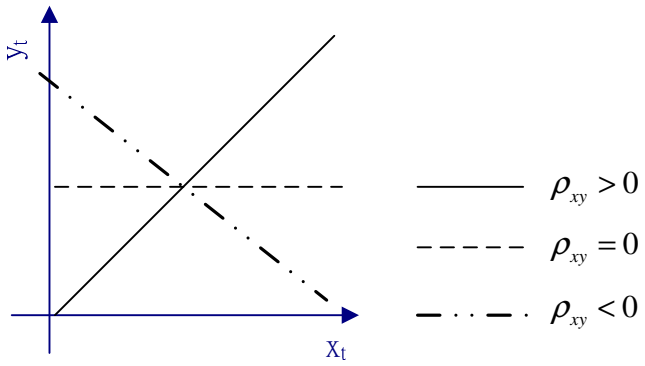
1. Volatility

Standard deviation： $\sigma_x = \sqrt{\dfrac{\sum_{t=1}^{T}(x_t - \bar{x})^2}{T}}$

2. Co-movement

Covariance： $\sigma_{xy} = \dfrac{\sum_{t=1}^{T}(x_t - \bar{x})(y_t - \bar{y})}{T}$

標準化：correlation coefficient： $\rho_{xy}=\frac{\sigma_{xy}}{\sigma_x\sigma_y}$　$-1\le\rho_{xy}\le 1$



同期之下：

$\rho_{xy}>0$ ：procyclical (正相關)

$\rho_{xy}=0$ ：acyclical (不相關)

$\rho_{xy}<0$ ：counter cyclical (負相關)

3. Persistence

(1) Auto covariance of order 1 (只和上一期作比較)：

$$\sigma_x^{\,2}(1)=\frac{\sum_{t=2}^{T}(x_t-\bar{x})(x_{t-1}-\bar{x})}{T-1}$$

(2) Auto covariance of arbitrary order：

$$\sigma_x^{\,2}(\tau)=\frac{\sum_{t=\tau+1}^{T}(x_t-\bar{x})(x_{t-\tau}-\bar{x})}{T-\tau}\qquad \tau=0\Rightarrow\sigma_x^2(0)=\sigma_x^2$$

上式表示：if τ=1~20 皆很大，表示此周期維持了 20 季

(3) 標準化： $AR(\tau)=\frac{\sigma_x^2(\tau)}{\sigma_x^2}$

$|AR(\tau)|\le 1$ 因爲自己和自己最像，所以$\sigma_x^2(0)=\sigma_x^2$ 最大

4. 補充

$$\sigma_{xy}(\tau)=\frac{\sum_{t=\tau+1}^{T}(x_t-\bar{x})(y_{t-\tau}-\bar{y})}{T-\tau}$$

$$\sigma_{xy}(\tau)\neq\sigma_{xy}(-\tau)$$

$\sigma_{xy}(\tau)$：本季和τ季之前的關係。

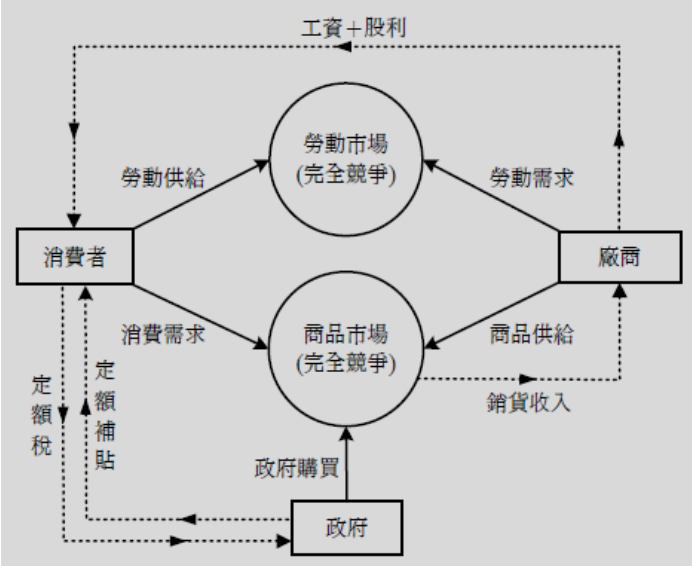
$\sigma_{xy}(-\tau)$：本季和τ季之後的關係。

5. Volatility of GDP
   (1) 消費波動＜所得波動＜投資波動
   (2) 台灣和美國比政府波動較高
   (3) 台灣 GDP 和進出口成正相關(景氣好就出超，景氣不好就入超)，此現象只有少數國家才有
   (4) GDP 和實質利率成負相關(景氣好利率低)，美國的相關性爲 0

Chapter 4－Steady Choice of Firm

(I) The model economy

●



(摘自毛老師的書)

Model 中所含：

(1) 3 agents：
- Consumers (identical, same preference)
- Firms (identical)
- Government

(2) 2 competitive markets：
- Goods market
- Labor market

Assumes：

①. 因假設靜態模型，所有人都只活一期，所以 consumer no saving。(一期可以自行定義，例如：一天、一年、80 年)

②. 因只活一期，所以對廠商而言 no investment。

③. 所有的稅 T=net of transfer lump-sum tax (表示每個人交一樣多)

④. Government income 全來自稅收(T)

(II)生產函數

- 型式

$$\boxed{y = AF(k,n)}$$

A：Product shock (外生衝擊)，表示任何可能對生產要素產生影響的外生因素，ex：晴天、雨天。A 也可以稱爲 total factor productivity (TFP)，因爲 A 不僅可以影響產量 y，也可以影響投入要素(k,n)，ex：晴天使工人快樂，所以生產力上升。

k：Capital stock(外生決定)，因只活一期，所以可想成定值。

n：Labor input

y：output

- Assume on $F(k,n)$

(1) $F(k,0) = F(0,k) = 0$

(2) Both k, n are productive

$$\begin{cases} k\rightarrow\ ,\ \ n\uparrow\ \ \Rightarrow y\uparrow \\ k\uparrow\ \ ,\ \ n\rightarrow\ \Rightarrow y\uparrow \end{cases}$$

表示成數學式子就是 MPL(marginal product of labor)

$$\begin{cases} MPL = \lim\limits_{\Delta n\to 0}\dfrac{\Delta y}{\Delta n} = AF_n(k,n) \geq 0 \ \ \Rightarrow\ \ k\rightarrow, n\uparrow \Rightarrow y\uparrow \\ MPK = \lim\limits_{\Delta k\to 0}\dfrac{\Delta y}{\Delta k} = AF_k(k,n) \geq 0 \ \ \Rightarrow\ \ k\uparrow, n\rightarrow \Rightarrow y\uparrow \end{cases}$$

$$\begin{cases} F_n(k,n) \text{ 可以想成 } F(k,n) \text{ 對 n 一階偏微分 或 n change 對 } F(k,n) \text{ 的影響} \\ F_k(k,n) \text{ 可以想成 } F(k,n) \text{ 對 k 一階偏微分 或 k change 對 } F(k,n) \text{ 的影響} \end{cases}$$

(3) MPK and MPL 遞減

$$\begin{cases} k\rightarrow\ ,\ \ n\uparrow\ \ \Rightarrow MPL\downarrow \\ k\uparrow\ \ ,\ \ n\rightarrow\ \Rightarrow MPK\downarrow \end{cases}$$

$$\begin{cases} \lim\limits_{\Delta n\to 0}\dfrac{\Delta MPL}{\Delta n} = AF_{nn}(k,n) \leq 0 \\ \lim\limits_{\Delta k\to 0}\dfrac{\Delta MPK}{\Delta k} = AF_{kk}(k,n) \leq 0 \end{cases}$$

$F_{nn}(k,n)$：n 改變對 $F_n(k,n)$ 的影響 or $F_n(k,n)$ 對 n 一階偏微。

(4) k & n are complementary in the sense that

$$\begin{cases} k\rightarrow\ ,\ \ n\uparrow\ \Rightarrow \text{MPK}\uparrow \text{(有 100 台電鬧和一個人，若多 10 人，每台電腦使用量上升)} \Rightarrow F_{kn}(k,n)\geq 0 \\ k\uparrow\ ,\ \ n\rightarrow\ \Rightarrow \text{MPL}\uparrow \text{(有 20 人用一台影印機，若多一台機器，每個人生產力上升)} \Rightarrow F_{nk}(k,n)\geq 0 \end{cases}$$

$F_{nk}(k,n)$ ：k 改變對 $F_n(k,n)$ 的影響 or $F_n(k,n)$ 對 k 一階偏微。

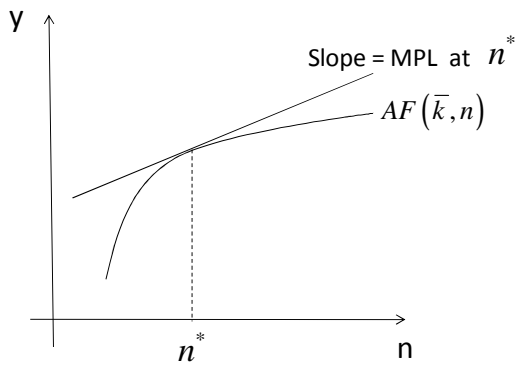
(5) Inada condition(非必要，但可以化簡模型

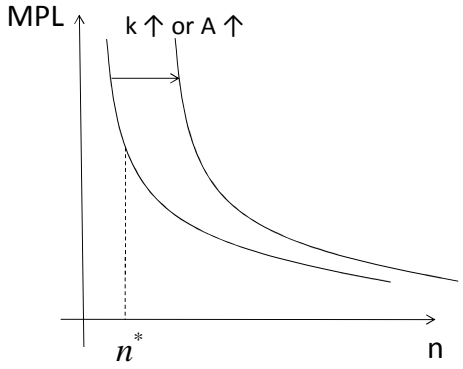
$$\begin{cases} k\rightarrow\infty\Rightarrow \text{MPK}\rightarrow 0 \\ n\rightarrow\infty\Rightarrow \text{MPL}\rightarrow 0 \end{cases} \qquad \begin{cases} k\rightarrow 0\Rightarrow \text{MPK}\rightarrow\infty \\ n\rightarrow 0\Rightarrow \text{MPL}\rightarrow\infty \end{cases}$$

(6) $F(k,n)$ is CRTS (constant return to scale)

$F(k,n)$ is CRTS if $\forall a>0 \quad ay=AF(ak,an)$

符合以上假設圖形 ex：





- Example－Cobb Dauglas function

$$y=Ak^{\alpha}n^{1-\alpha}\quad(0\leq\alpha\leq 1)$$

可自行驗證符合上面 6 個假設

- Elasticity

(1) Output elasticity of capital

$$\lim_{\Delta k\to 0}\frac{\Delta y/y}{\Delta k/k}=\frac{dy/y}{dk/k}=\frac{k}{y}\frac{dy}{dk}=\boxed{\frac{d\ln y}{d\ln k}}$$

意即：k 變化 1%時，y 變化多少%

or

$$\frac{k}{y}\frac{dy}{dk}=\frac{k\bullet MPK}{y}=\frac{\text{Capital income}}{\text{Total income}}=\text{Capital share}$$

Capital share：所有收入中有百分之多少是由 capital 賺到

(2) Example

$$y=Ak^{\alpha}n^{1-\alpha}$$

$$\begin{cases}\text{Capital share}=\dfrac{k}{y}\dfrac{dy}{dk}=\dfrac{k}{y}A\alpha k^{\alpha-1}n^{1-\alpha}=\dfrac{\alpha Ak^{\alpha}n^{1-\alpha}}{Ak^{\alpha}n^{1-\alpha}}=\alpha \quad \text{or} \quad \dfrac{d\ln y}{d\ln k}=\alpha \\ \text{Labor share}=\dfrac{n}{y}\dfrac{dy}{dn}=1-\alpha \quad \text{or} \quad \dfrac{d\ln y}{d\ln n}=1-\alpha\end{cases}$$

(III)Objective：Profit (dividend)

- 假設所有生產都是爲了 Dividend= Profit

$$\boxed{d=AF\left(k,n\right)-wn}$$

d： dividend

w： real wage rate 單位：$\left(\dfrac{\text{多少單位產品}}{1\text{單位}labor}\right)$

外生：A,k,w

此式可看成：收入-支出(單位爲產品而非錢)

(IV)Decision Problem

- 目標

  Given $\{\mathrm{A}, k, w\}$ choose n such that

  $$\max_{\{n\}} d = AF(k,n) - wn$$

- FOC (first order condition)

  $\Delta n = 1 \Rightarrow$ marginal cost ：w　　d↓

  marginal gain ： $AF_n(k,n)$=MPL　　d↑

  MPL＞w⇒d↑ as $n^d \uparrow \Rightarrow n^d \uparrow$

  MPL＜w⇒d↓ as $n^d \uparrow \Rightarrow n^d \downarrow$

  Optimal requires：

  $$\boxed{MPL = AF_n(k,n) = w}$$

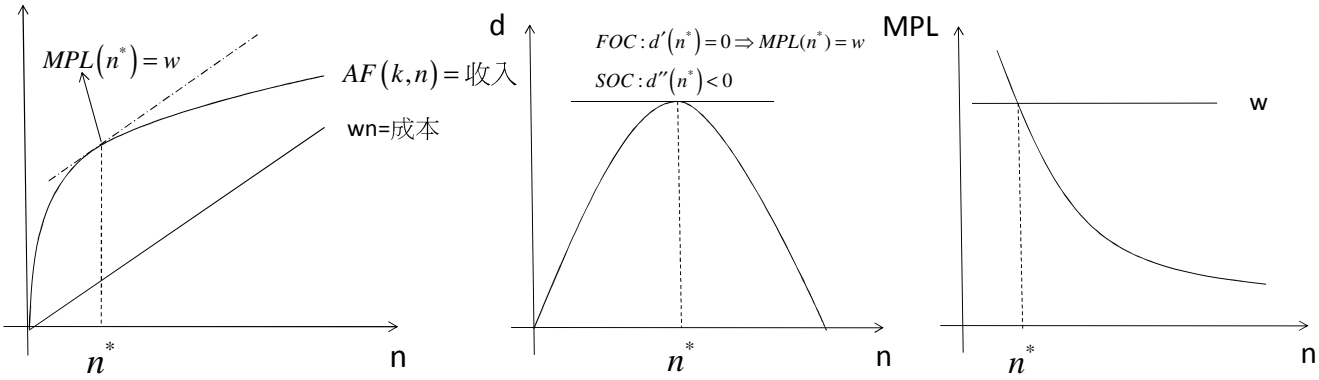
- Second order condition

  $\mathrm{AF}_{nn}(k,n^d) < 0$

  但其實在 6 個假設中已經有 $\mathrm{AF}_{nn}(k,n^d) < 0$，所以其實不需要管 Second order condition

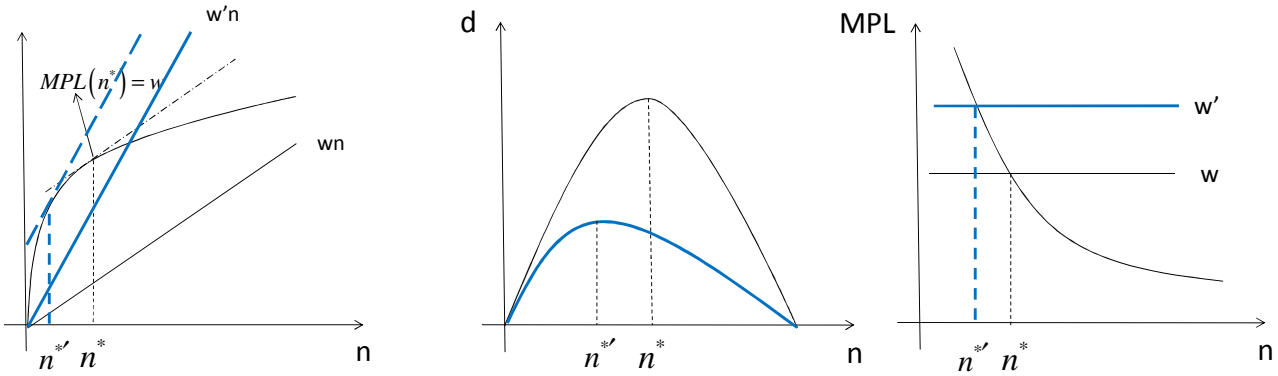
- 分析用圖形

  幾乎所有問題都可以用以下三種圖形來分析。

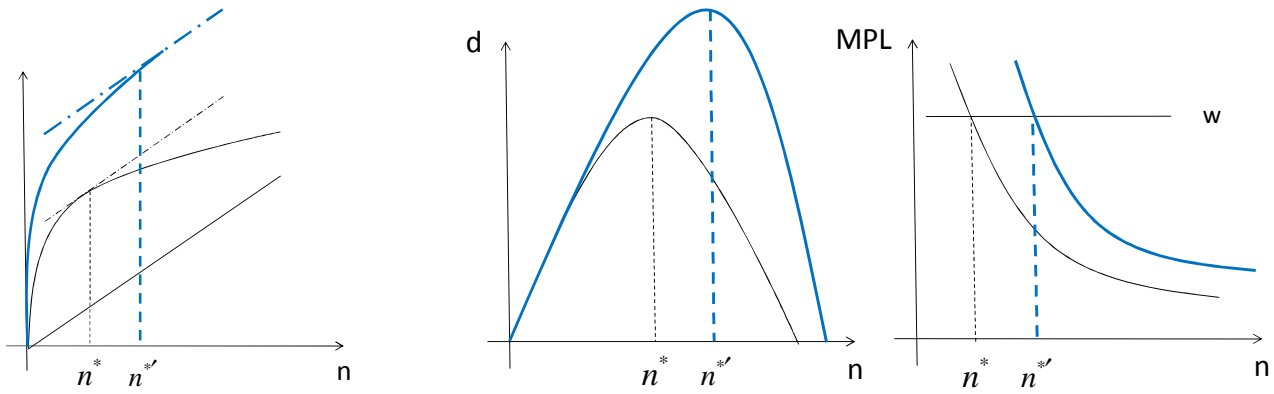


- Shocks

  (1) w↑，A→，k→ ⇒ labor demand(n^d)↓



  (2) w→，A↑ or k↑ ⇒ labor demand(n^d)↑



(3) Summary

由 FOC： $AF_n(k,n)=w$ 可以找出 n^d

$$\Rightarrow n^d = n^d\left(\underset{(-)}{w},\underset{(+)}{A},\underset{(+)}{k}\right)$$

$$\Rightarrow \text{goods supply}\quad y^s = AF\left(k,n^d\right) = y^s\left(\underset{(-)}{w},\underset{(+)}{A},\underset{(+)}{k}\right)$$

A： { 直接效果： $AF_n(k,n)$ 中的 A
　　{ 間接效果：使 n^d ↑

k： { 直接效果： $AF_n(k,n)$ 中的 k
　　{ 間接效果：使 n^d ↑

※上方中(＋)表示正相關，ex：A↑ $\Rightarrow n^d$ ↑

※上方中(－)表示負相關，ex：w↑ $\Rightarrow n^d$ ↓

- Example

(1) $y = Ak^{\alpha}n^{1-\alpha}$

MPL=$(1-\alpha)Ak^{\alpha}n^{-\alpha} = w$

$$\Rightarrow n^d = \left[\frac{(1-\alpha)A}{w}\right]^{\frac{1}{\alpha}}k$$

$$\ln n^d = const + \frac{1}{\alpha}\ln A - \frac{1}{\alpha}\ln w + \ln k$$

取 ln 優點爲 elasticity 容易「看」出

Ex：wage elasticity of $n^d = \dfrac{d\ln n^d}{d\ln w} = -\dfrac{1}{\alpha} \approx -3$ in Taiwan

表示 w 上升 1%， n^d 下降 3%

(2) 能源危機

$$\max_{\{n,x\}} d = F\{k,n,x\} - (wn+qx)$$

x：投入 oil

q：price of oil

Chapter 5－Static Choice of Consumers

(Ⅰ) The Representative Consumer

- 式子

(1) Time constrain

$$l+n=H=1$$

l：leisure time
n：labor time
H：time endowment

(2) Budget constrain

$$c=(d-T)+wn=a+wn$$

c：consumption
d：dividend income
T：lump-sum tax
a=(d-T)：非勞動所得
wn：勞動所得

(3) 轉換

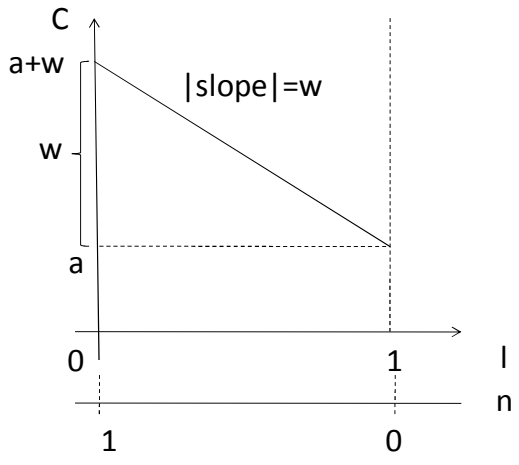
將$n=1-l$ 代入 budget constrain

$$c+wl=a+w=x$$

x=a+w：full income 所有時間都工作可得

w：單位爲$\left(\frac{\text{產品}}{\text{時間}}\right)$可以想成休閒的機會成本

- 圖形



(II) Preference

- Utility function

  $U = u(c,l)$ ：ordinal (只看相對，不看絕對)

- Assumption

  (1) “more” is prefer to “less”

  $$\begin{cases} c\uparrow\ ,\ l\rightarrow\ \Rightarrow u\uparrow \quad \text{ie}\ \ u_c(C,l)>0 \\ c\rightarrow,\ \ l\ \uparrow\Rightarrow u\uparrow \quad \text{ie}\ \ u_l(C,l)>0 \end{cases}$$

  (2) Diminishing MU of C & l

  $$\begin{cases} c\uparrow\ ,\ l\rightarrow\ \Rightarrow u_c\downarrow \quad \text{ie}\ \ u_{cc}(C,l)<0 \\ c\rightarrow,\ \ l\ \uparrow\Rightarrow u_l\downarrow \quad \text{ie}\ \ u_{ll}(C,l)<0 \end{cases}$$

  (1)&(2)表示邊際效用遞減但恆正

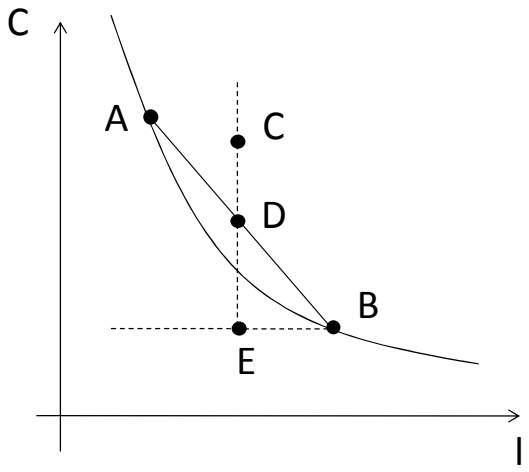
  (3) $u_{cl}(c,l)\begin{matrix}>\\=\\<\end{matrix}0$ 皆可(沒有限制)

  (4) Prefer “diversity”

  (5) c & l are normal goods

- Indifference Set

(1) $\{(c,l)|u(c,l)=cons\tan t\}$



①. $u_C > u_A = u_B > u_D$ ， $u_E > u_A = u_B$

②. Marginal rate of substitution (邊際替代率 MRS) of l for

c=$-\left.\frac{\Delta c}{\Delta l}\right|_{u=u^*} = -\left.\frac{dc}{dl}\right|_{u=u^*} = MRS_{l,c}$ ↓ as l ↑ (c ↓)⇒邊際效用遞減

$MRS_{l,c}$=增加一單位 l，願意放棄的 c 量

=一單位 l 相當於多少 c

(2) Measure MRS

$u = u(c,l)$

$du = u_c(c,l)dc + u_l(c,l)dl$ 衡量 MRS 時 du=0

$-\left.\frac{dc}{dl}\right|_{u=u^*} = MRS_{l,c} = \frac{u_l(c,l)}{u_c(c,l)}$ will ↓ as l ↑

(III) Decision Problem

- 目標

Given $\{a = d - T, w\}$

$\max_{\{c,l,n\}} u(c,l)$ such that

$\begin{cases} l + n = 1 \\ c = a + wn \end{cases}$ or $c + wl = a + w$

- FOC

$\Delta n = 1 \Rightarrow \Delta l = -1$

$\begin{cases} \text{Marginal cost of working：} u_l(c,l) \\ \text{Marginal gain of working：} u_c(c,l) \cdot w \end{cases}$

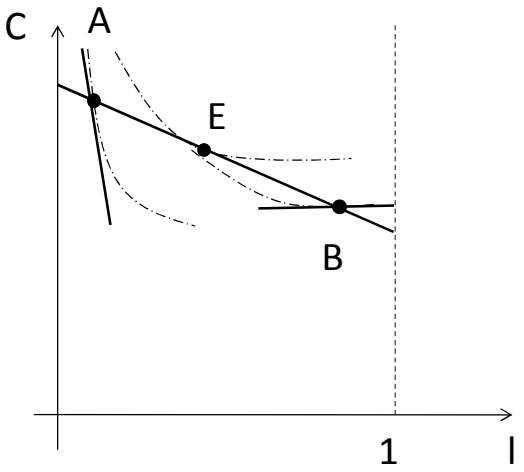
Optional requires

$$u_l(c,l) = u_c(c,l) \cdot w \Rightarrow MRS_{l,c} = \frac{u_l(c,l)}{u_c(c,l)} = w$$

- 求解

$\begin{cases} l + n = 1 \\ c = a + wn \quad \Rightarrow \quad \text{get} \quad \begin{cases} c^d = c^d(w,a) \\ n^s = n^s(w,a) \end{cases} \\ \dfrac{u_l(c,l)}{u_c(c,l)} = w \end{cases}$

- 圖形



E：MRS=w⇒平衡

A：MRS>w⇒少工作

B：MRS<w⇒多工作

（想法）：以 B 爲例，若多工作 1 小時，cost=MRS(B)，但市場給的爲 w，比 cost 還多，所以多工作。

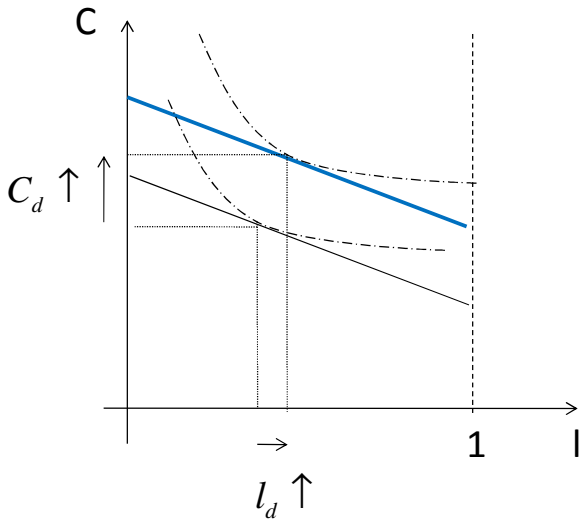
- Predictions

外生變數$\{a,w\}$改變所造成的影響

(1) a↑ w→ (d↑ or T↓)$\Rightarrow x=a+w\uparrow$

∵c , l are normal goods

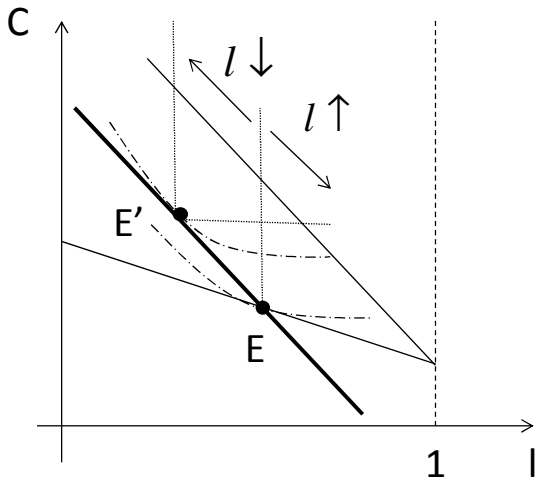
∴所得效果：$c^d\uparrow, l^d\uparrow, n^s\downarrow$



(2) a→ w↑

$$\begin{cases} \text{w}\uparrow \Rightarrow \text{休閒的機會成本}\uparrow \Rightarrow \text{替代效果：} c^d \uparrow, l^d \downarrow, n^s \uparrow \\ \text{a+w}\uparrow \Rightarrow \text{所得效果：} c^d \uparrow, l^d \uparrow, n^s \downarrow \end{cases}$$

（註）：通常所得效果較小∵ w↑ firm cost↑ ∴可以給的 dividend (d) ↓又 a=d-T.∵a↓ ⇒ a + w 改變較少



Income compensation 後替代效果使平衡點由 E 移到 E’ ，所得效果使平衡點由 E’ 移到東北的區域內。

(3) Summary

$$c^d = c^d \left(\underset{\substack{(+)\\ IE+SE}}{w}, \underset{\substack{(+)\\ IE}}{a} \right)$$

$$n^s = n^s \left(\underset{(?)}{w}, \underset{\substack{(-)\\ IE}}{a} \right)$$

IE：Income effect

SE：Substitution effect

?：不確定 ┌ SE>IE (+)
　　　　└ SE<IE (-)

- Application

(1) Wage income tax

Max u(c,l) st $\begin{cases} l+n=1 \\ c=a+(1-\tau)wn \end{cases}$

τ ：tax rate

$(1-\tau)w$ ：after tax wage rate

FOC：$u_l(c,l)=(1-\tau)wu_c(c,l)$

$$\text{MRS}=\frac{u_l}{u_c}=(1-\tau)w$$

$\tau\uparrow$ ，$(1-\tau)w\downarrow$ $\begin{cases} IE: c^d\downarrow, l^s\downarrow \\ SE: c^d\downarrow, l^s\uparrow \end{cases}$

Ex：$u(c,l)=\ln c+\ln l$

FOC：$\frac{c}{l}=(1-\tau)w \Rightarrow \begin{cases} c^d=\frac{1}{2}\left[a+(1-\tau)w\right] \\ n^s=\frac{1}{2}\left[1-\frac{a}{(1-\tau)w}\right] \end{cases}$

a=0

a>0 有錢人：$\tau\uparrow \Rightarrow n^s\downarrow$ SE＞IE

a<0 ：$\tau\uparrow \Rightarrow n^s\uparrow$ SE＜IE

(2) Consumption voucher 消費卷

v：voucher

c+v = a+wn+v $\Rightarrow c = a + wn$ unchanged

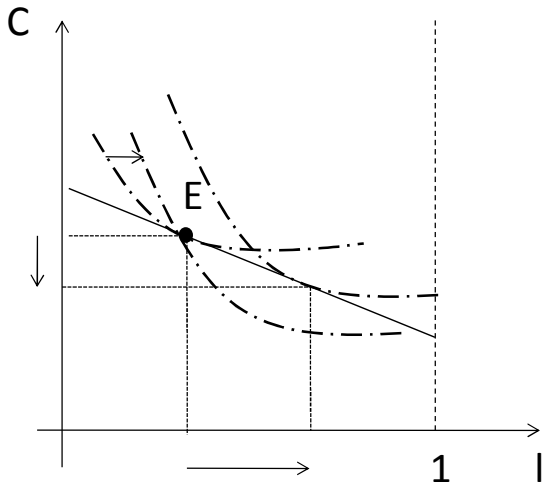
c：budget consumption

v：voucher consumption

改變偏好：$\max u(c+v)+s(l)$ st c+wl=a+w

$$MRS_{l,c} = \frac{s'(l)}{u'(c+v)} = w$$

∵ v↑使$u'(c+v)$ ↓ $MRS_{l,c}$ ↑



個人消費c^d ↓，但總消費$c^d + v$↑

Ex：$u(c,l) = \ln(c+v) + \ln l$

$$c^d = \frac{1}{2}(a+w-v) \qquad n^s = \frac{1}{2}\left(1 - \frac{a+v}{w}\right)$$

總消費$c^d + v = \frac{1}{2}(a+w+v)$

表示發 3600 的消費卷會替代掉$\frac{1}{2}v = 1800$的個人消費，此爲本來不論有沒有發消費卷都一定會花的錢，總消費其實只有上升$\frac{1}{2}v = 1800$，並沒有到 3600 元。

Chapter 6－General Equilibrium in Static Model

(I) Definition of equilibrium

- Definition of equilibrium

  “ex ante”(事前) demand = “ex ante” supply
  “ex post”(事後) demand = “ex post” supply

- Definition of competitive equilibrium (CE)

  A CE is a set of quantities (allocations) {c, n, y, T} and the real wage rate w, such that given {A, k} and policy (G)

(II) Competitive equilibrium (CE)

- Equations

  Given $\{A, k, G\}$ find real wage rate w^* and quantities $\{c^*, n^*, y^*, T\}$

  (1) Facing w^* consumer optimize

  $$\begin{cases} MRS_{l,c} = w^* \quad : \dfrac{u_l(l^*, c^*)}{u_c(l^*, c^*)} = w^* & \text{———①} \\ \text{Budget constrain}: c^* = (d^* - T) + w^* n^* = a^* + w^* n^* & \text{———②} \end{cases}$$

  (2) Facing w^* firms optimize

  $$\begin{cases} MPL = w^* \ : \ AF_n(k, n^*) = w^* & \text{———③} \\ \text{Profit} \quad : \ d = AF(k, n^*) - w^* n^* \end{cases}$$

  (3) Government budget constrain is fulfilled
  $G = T$ (G：政府花費) ———④

  (4) w^* adjusts so that all markets clear

  $$\begin{cases} \text{Goods market}: C^d(w^*, a^*) + G = y^s(w^*, A, k) & \text{———⑤} \\ \text{Labor market}: n^d(w^*, A, k) = n^s(w^*, a) = n^* & \text{———⑥} \end{cases}$$

※ 在 A ,k ,G 都不變的情況下若上方 ① ~ ⑥式都成立，則達到全面均衡狀態。

※ 對個別消費者或生產者而言 w 爲外生變數，但對整個市場而言，w 爲內生變數。

※ 有 6 調方程式，但只有 5 個未知數，表示 6 條方程式並非互相獨立，以下將介紹 Walra's law 可以推出式子⑤→⑥

- Walra's law

$$\begin{cases} c^d = (d - T) + wn^s \\ G = T \\ d = y^s - wn^d \end{cases}$$

(將 d 和 T 代入)

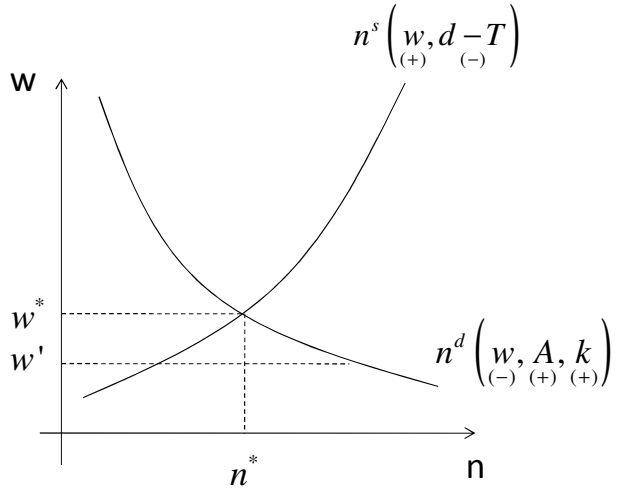
$$\underbrace{\left[\left(c^d + G\right) - y^s\right]}_{\text{Excess demand in goods market}} + \underbrace{w\left[n^d - n^s\right]}_{\text{Excess demand in labor market}} = 0$$

上式無論 w 是多少都成立，當均衡時的 w^* 使前後兩項皆=0。且由上式可知若存在一個 w^* 使其中一市場達到均衡，則另一市場會自動達到均衡。

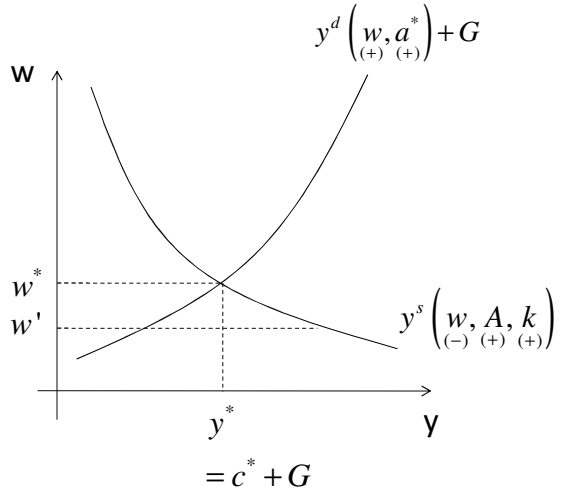
(III) 均衡圖形分析

- Demand supply Representation

Labor market



Good market

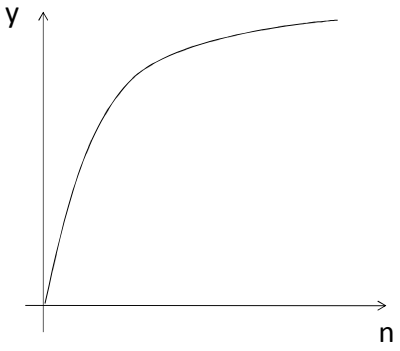


(1) 左右兩張圖的 w^* 必在同一大小。

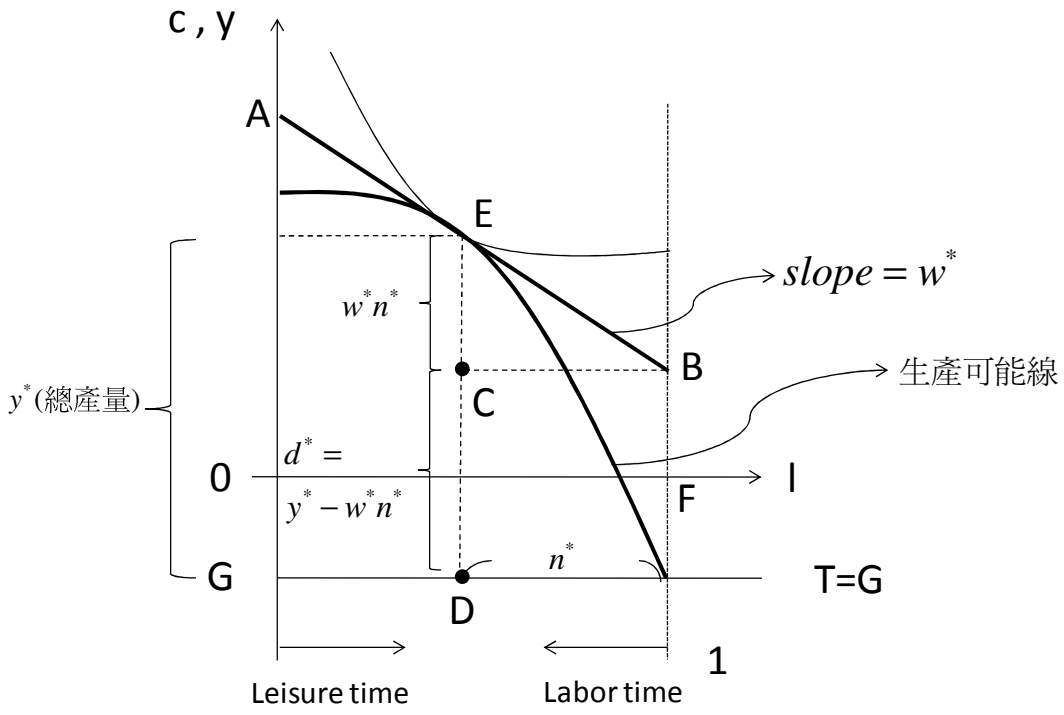
(2) If $w' < w^*$，labor market 會有 excess demand，則根據 Walra's law，good market 必有 excess supply。

- Explicit Representation

將 6 條方程式畫在同一張圖上：
首先將廠商生產可能線畫在 l-c 圖上，



因爲接下來所畫的圖橫軸爲 leisure 所以上圖的生產可能線需左右鏡射，且因有政府課稅，所以圖形下移 G 單位，就可繪出下圖。

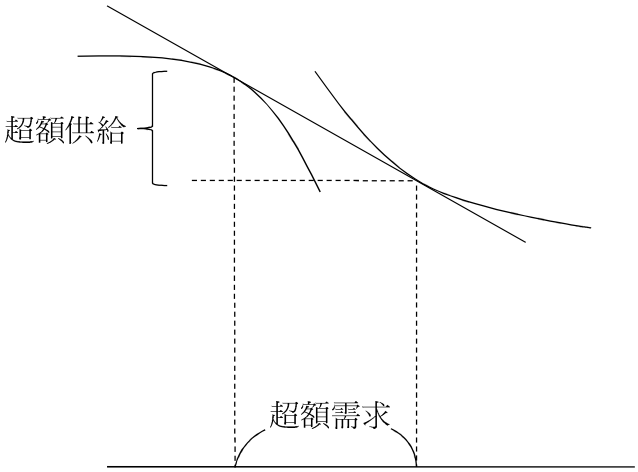


$\overline{ABF}$ ：消費者選擇(budget line)

$\overline{BF} = a^* = d^* - G = d^* - T$

$MRS(E) = MPL(E) = w^*(E)$

If $w' \neq w^*$ 圖形會變成下方，所以會調整至 $w' = w^*$ 才達到均衡。



- Robinson Crusoe Representation

(1) 此爲 Pareto Optimal (PO)與之前的 CE 有一些不同。

(2) 想成只有一個人”Crusoe”，沒有市場，所以沒有價格。而此人的選擇爲：

Given $\{A,k,G\}$ $\max u(c,l)$

st $l+n=1$ & $c = \underset{Crusoe生產的}{AF(k,n)} - \underset{被偷走或壞掉}{G}$

FOC： $\Delta n = 1$ $\Delta c = MPL = AF_n(k,n)$

MC of working： $u_l(c,l)$

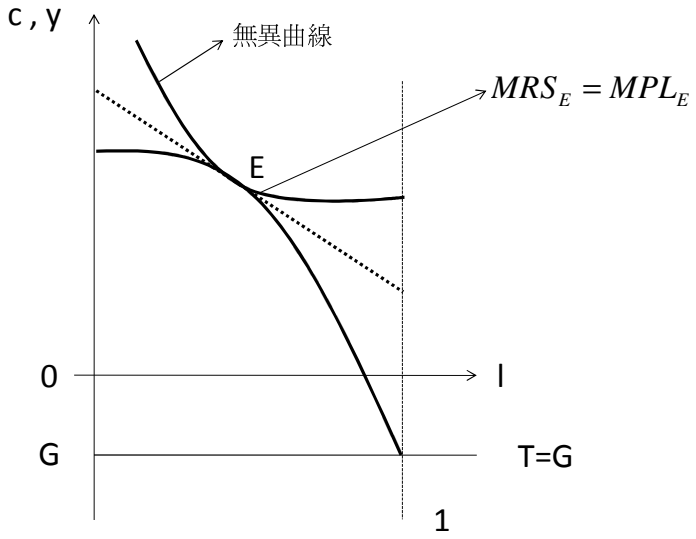
MG from working： $u_c(c,l) \bullet MPL = u_c(c,l) \bullet AF_n(k,n)$

$\therefore MRS_{l,c} = \dfrac{u_l(l,c)}{u_c(l,c)} = AF_n(k,n) = MPL$(以上都沒有價格 w)

PO requires：

$$\begin{cases} MRS_{l,c} = \dfrac{u_l(l,c)}{u_c(l,c)} = AF_n(k,n) = MPL \\ c = AF(k,n) - G = y - G \\ l + n = 1 \end{cases}$$

get$\{c^*, n^*, y^*, l^*\}$



對照前面的圖，可以畫出虛線 MPL=w，此 w 稱爲影子價格(shadow price)

(3) 比較

CE requires：

$$\begin{cases} \text{Consumer 效用極大：} MRS_{l,c} = \dfrac{u_l(l,c)}{u_c(l,c)} = w \\ \text{Firms 利潤極大：} AF_n(k,n) = MPL = w \\ \text{商品市場均衡：} c = y - G \ \text{(Walra's law 使另一市場均衡)} \\ l + n = 1 \end{cases}$$

get$\{c^*, n^*, y^*, l^*, w^*\}$

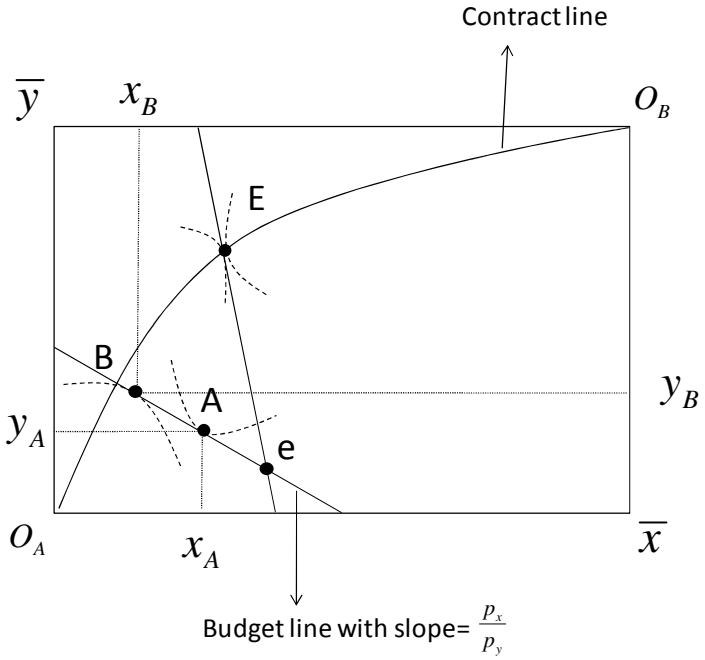
(IV) Welfare theorem

- Definition

  The first welfare theorem：CE⇒PO
  The second welfare theorem：PO⇒CE

- CE⇒PO

  Suppose 2 goods 2 agents endowment economy
  Goods：x , y
  Endowment (fix supply)： $\overline{x}, \overline{y}$
  Agents：A ,B
  Edgeworth box



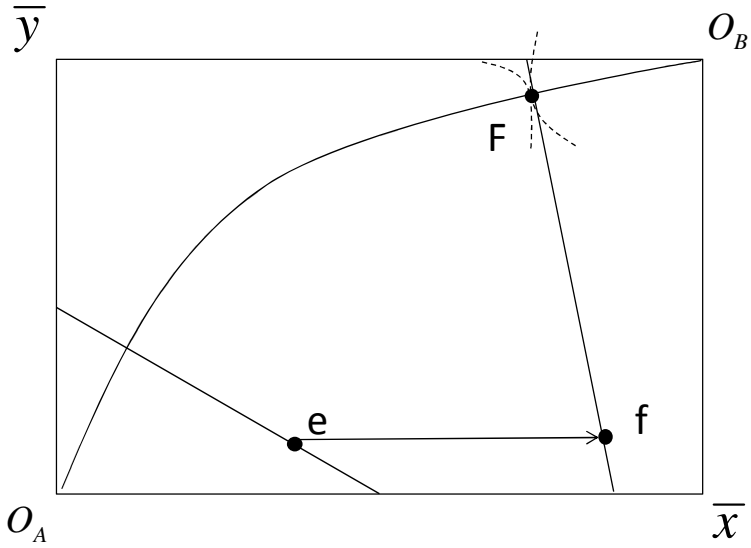
  Initial endowment at point e , under the price p_x, p_y

$$\begin{cases} x_A + x_B > \overline{x} & \text{：Excess demand (ED)} \\ y_A + y_B < \overline{y} & \text{：Excess supply (ES)} \end{cases}$$

$$\Rightarrow p_x \uparrow, p_y \downarrow, \frac{p_x}{p_y} \uparrow$$

  新預算限制線爲過 e 點，斜率較大的線
  ∴若 CE(E 點)⇒必在 contract line 上(PO)

- PO⇒CE



如何由 e→F，必須先到過 F 點的線上任一點，由革命、共產重分配等方法，可以使 e→f(initial allocation adjusted)，再由市場定理調整至 F。

- Source of market inefficiency

(1) Externality 外部性
(2) Incomplete or asymmetric information
(3) Distortion tax (非 lump-sum tax)
(4) Monopoly power