第三章　　序列效用分析法

題型：消費者行為公理

1.是非題

(1) 一人代表不同效用水準之兩條無異曲線只能有一個交點；

(2) 若個人的受補償需求曲線較正常需求曲線為陡，則此物品必為劣等財。

(3) 政府對自然獨占廠商可採取邊際成本定價之管制方式。

(4) 在兩期的跨期選擇模型中，若此人在第一期有正儲蓄，則利率上升時，替代效果與所得效果對儲蓄的影響方向相同。

(5)當一消費者以全部的所得去消費 X、Y 兩種物品，此兩種物品可均為劣等財。【97 中央經研所】

解：(1)錯誤；兩條無異曲線不可以相交，否則會違反消費者行為公理，遞移性與單調性會產生矛盾。

(2)錯誤；如果價格上漲，受補償需求曲線比普通需求曲線更平坦；如果價格下跌，受補償需求曲線比普通需求曲線更陡。

(3)錯誤；自然獨佔理想管制法為「平均成本定價法」。

(4)錯誤；此人為儲蓄者，SE 與 IE 變動方向相反，當實質利率上漲時，SE 為本期消費量減少，本期儲蓄量增加，而所得效果為隨著利息收入提高，實質所得增加，本期消費量增加，本期儲蓄量會減少。

(5)錯誤，社會上不可能所有財貨皆為劣等財，至少有一個正常財。

凹向原點

2.偏好排序(Preference Orderings)

(1)張三對（4 個蘋果，6 根香蕉）的喜好程度與（6 個蘋果，4 根香蕉）的喜好程度相同，但對（5 個蘋果，5 根香蕉）的喜好程度遠不如（4 個蘋果，6 根香蕉）。由上面之陳述，我們能判斷張三對蘋果與香蕉之偏好排序滿足偏好多樣性（或嚴格凸性(strict convexity)）的特性嗎？請說明你的答案。令(x，y)代表（香蕉，蘋果）的數量，符號 A～B 表示對 A 和 B 同等地喜歡；符號 $A \succ B$ 表示對 A 的喜歡甚於對 B 的喜歡。張三對下列各組合的關係如下：

$(9,5) \sim (7,7) \sim (5,9)$　　　$(9,6) \succ (7,7) \succ (8,5)$

(2)列出(9,6)與(5,9)的偏好排序。 (3)列出(5,9)與(8,5)的偏好排序。

(4)張三的偏好排序，滿足 MRS 遞減的特性嗎？為什麼？【97 政大國貿所】

解：(1)令$(X_1, Y_1) = (4, 6)$，$(X_2, Y_2) = (6, 4)$

由題目可知 $(4, 6) \sim (6, 4)$

則$\left(\frac{1}{2}X_1 + \frac{1}{2}X_2, \frac{1}{2}Y_1 + \frac{1}{2}Y_2\right) = (5, 5)$

但$(4, 6) \succ (5, 5)$　　∴ 表示消費者之偏好序列不滿足偏好的多樣性，即不滿足凸性公

理。

(2)$\because$ (9 , 6) $\succ$ (7 , 7) ~ (5 , 9)　　根據遞移性可知 (9, 6) $\succ$ (5, 9)

(3)$\because$ (5, 9)~(7, 7) $\succ$ (8, 5)　　根據遞移性可知 (5, 9) $\succ$ (8, 5)

(4)$\because$ (9 , 5) ~ (7, 7) ~ (5, 9)　$\therefore$ 無異曲線為一直線，$\frac{dMRS}{dX}=0$ $\therefore$ 不具 MRS 遞減之特性

3.令(x, y)代表（香蕉，蘋果）的數量。符號 A~B 表示對 A 和 B 同等地喜歡；符號 A $\succ$ B 表示對 A 的喜歡甚於對 B 的喜歡。張三對下列各組合的關係如下：(9, 5) ~ (7, 7) ~ (5, 9) (9, 6) $\succ$ (7, 7) $\succ$ (8, 5).

(1) 張三比較喜歡(9, 6) 或(5, 9)？

(2) 張三比較喜歡(5, 9) 或(8, 5)？

(3) 張三的偏好排序，滿足 MRS 遞減的特性嗎？為什麼？【97 政大國貿所】

解：(1)根據偏好的遞移性 $\because (9,6)\succ(7,7)\sim(5,9)$ $\therefore (9,6)\succ(5,9)$

(2)根據偏好的遞移性 $\because \begin{cases}(7,7)\sim(5,9)\\(7,7)\succ(8,5)\end{cases}$ $\therefore (5,9)\succ(8,5)$

(3) 由(9,5) ~ (7,7) ~ (5,9)可知：每減少 2 單位香蕉(X)的消費量，必須增加 2 單位蘋果(Y)的消費量，亦即，MRS = 1，$\frac{dMRS}{dX}=0 \Rightarrow$ MRS 不會遞減，此人的無異曲線為線性函數，香蕉與蘋果對此人而言為完全替代品。

4.假若消費者喜歡 Y 財貨，但是不喜歡 X 財貨。請畫出此消費者之無異曲線(indifference curve)，並說明此無異曲線的特性。(10%)【98 靜宜企研所】

解：只要 X 財貨為厭惡品，Y 財貨為喜好品，無異曲線為「正斜率」。

5.(20%)周公的效用函數為 $U(x,y)=x+2y$，其中 $x\geq 0$，$y\geq 0$。請用數學說明或是說明此效用函數背後所代表周公對 x 和 y 財貨的偏好，是否滿足下列特性（僅使用文字或是圖型說明者，不予計分）：

$MRS_{xy}=\frac{MU_x}{MU_y}=\frac{1}{2}$

(1)完整性(completeness) (2)遞移性(transitivity) (3)嚴格凸性(strict convexity)(4)越多越好(more is better)【98 政大經研所】

解：

(1)完整性(completeness)：假設有兩個組合 $(x_0,y_0)=(3,2)$ 與 $(x_1,y_1)=(2,3)$，則效用函數 $U(x_0,y_0)=7$；$U(x_1,y_1)=8$，則我們可以確定 $U(x_1,y_1)\succ U(x_0,y_0)$ 成立。

(2)遞移性(transitivity)：假設存在另一個商品組合 $(x_2,y_2)=(3,3)$，則效用函數 $U(x_2,y_2)=9$，因為 $U(x_2,y_2)\succ U(x_0,y_0)$，並且 $U(x_2,y_2)\succ U(x_1,y_1)$，保證 $U(x_2,y_2)\succ U(x_1,y_1)\succ U(x_0,y_0)$ 成立。

(3)嚴格凸性(strict convexity)：不成立

由於 $MRS_{XY}=\frac{1}{2}$，$\frac{dMRS_{XY}}{dx}=0$，無異曲線為負斜率的直線，不滿足 MRS_{XY} 遞減假設，因此

此效用函數不滿足嚴格凸性公理。

(4)單調性(Monotonic)：隨著財貨數量愈多，效用愈高，假設存在另一個商品組合 $(x_3,y_3)=(4,4)$，則效用函數 $U(x_3,y_3)=12$，因為 $x_3>x_2$，$y_3>y_2$，則 $U(x_3,y_3)=12>U(x_2,y_2)=9$，此效用函數單調性會成立。

ordinal!!

6.Assume my preference over fruits are ordinal. I prefer orange (O) to banana (B) to apple (A). Let $u(j), j=O,B,A$ be the respective utilities i derive from eating one unit of fruit j. Write down three sets of utilities which represent my ordinal preference over these three kinds of fruits. (10 分)【98 中正國經所】

解：$u(O,B,A)=3O+2B+A$；$u(O,B,A)=O^3+B^2+A$

$$u(O,B,A)=O+B^{\frac{1}{2}}A^{\frac{1}{3}}$$

7.若消費者 A 與 B 的效用函數分別為 $U_A=X_A^2+Y_A^2$ 及 $U_B=X_BY_B$，則下列敘述何者為正確？ (A)消費者 A 符合無異曲線凸向原點之假設。(B)消費者 B 符合無異曲線凸向原點之假設。 (C)消費者 A 符合無異曲線不能相交之假設。 (D)消費者 B 符合無異曲線不能相交之假設。 (E)消費者 A 符合邊際效用遞減之假設。【96 北大經研所】

解：<u>(B)</u>

❶ $U_A=X_A^2+Y_A^2$ $\quad MRS_{XY}^A=\dfrac{MU_X^A}{MU_Y^A}=\dfrac{2X_A}{2Y_A}=\dfrac{X_A}{Y_A}$

$\dfrac{dMRS_{XY}^A}{dX_A}=\dfrac{1}{Y_A}>0$ ⇒「不符合」邊際替代率遞減（凸向原點）之假設

❷ $U_B=X_BY_B$ $\quad MRS_{XY}^B=\dfrac{MU_X^B}{MU_Y^B}=\dfrac{Y_B}{X_B}$

$\dfrac{dMRS_{XY}^B}{dX_B}=-\dfrac{2Y_B}{X_B^2}<0$ ⇒「符合」邊際替代率遞減（凸向原點）之假設

故消費者 A 不符合無異曲線凸向原點之假設；消費者 B 符合無異曲線凸向原點之假設。

題型：偏好種類

8.Let x and y be the quantities of two goods consumers consume. In the following table, the utility functions of several consumers are listed:

Consumer	Utility function
A	3xy
B	$2\sqrt{xy}$
C	2x + y
D	x^2+y^2+2xy
E	Min{x, y}

$MRS_{xy}=\frac{y}{x}$

A, B

(1)Which consumers have the same preferences (demand functions)? Why?

(2)Which consumers consider the two goods as perfect substitutes? → MRS_{xy} ⇒ constant

(3)Which consumers consider the two goods as perfect complements?【95 海洋應經所】

解：

$A: MRS_{XY} = \dfrac{MU_X}{MU_Y} = \dfrac{Y}{X}$　　$B: MRS_{XY} = \dfrac{MU_X}{MU_Y} = \dfrac{Y}{X}$　　$C: MRS_{XY} = \dfrac{2}{1} = 2$.......完全替代偏好

$D: MRS_{XY} = \dfrac{MU_X}{MU_Y} = \dfrac{2X+2Y}{2X+2Y} = 1$.......完全替代偏好

(1)效用函數經過「單調遞增轉換」後，MRS_{XY}相同，無異曲線形狀相同，表示消費者有相同偏好結構，因此 A 與 B 效用函數皆為齊序偏好效用函數，具有相同偏好。

(2)(C)、(D)　　　　(3)(E)

9.Which of the following utility functions defined on $x_1 > 0$ and $x_2 > 0$ are (is) not homothetic「齊序偏好」？(A) $U(x_1, x_2) = \frac{1}{3}\ln x_1 + \frac{2}{3}\ln x_2$　(B) $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$

(C) $U(x_1, x_2) = x_1^{\frac{1}{3}} + x_2^{\frac{2}{3}}$　(D) $U(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} + x_1^{\frac{1}{2}} x_2$ (E) $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} + x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$。【95 台大經研所】

解：(C)不是 Homothetic utility function

(A) $MRS_{12} = \dfrac{MU_1}{MU_2} = \dfrac{\frac{1}{3x_1}}{\frac{2}{3x_2}} = \dfrac{1}{2}\left(\dfrac{x_2}{x_1}\right)$　　(B) $MRS_{12} = \dfrac{MU_1}{MU_2} = \dfrac{\frac{1}{3} x_1^{\frac{-2}{3}} x_2^{\frac{2}{3}}}{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{\frac{-1}{3}}} = \dfrac{1}{2}\left(\dfrac{x_2}{x_1}\right)$

(C) $MRS_{12} = \dfrac{MU_1}{MU_2} = \dfrac{\frac{1}{3} x_1^{\frac{-2}{3}}}{\frac{2}{3} x_2^{\frac{-1}{3}}}$

(D)令 $V = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} \Rightarrow U = V + V^2 \Rightarrow \dfrac{\partial U}{\partial V} = 1 + 2V > 0$，表示 U 為 V 的單調遞增轉換函數，因 V 為 Homothetic preference，則 U 效用函數亦為 Homothetic utility function。

P) 10.(1)下列敘述何者為正確？ (A)當消費者面對兩種財貨，其中 X 財貨會愈多愈好，而財貨 Y 是愈少愈好，則其無異曲線的斜率為負斜率。 (B)對一位消費者而言，任何兩條無異曲線間有無限多條的無異曲線 (C)對一位消費者而言，兩條無異曲線的距離差異為其邊際替代率。(D)當消費者面對兩種完全互補性財貨時，其無異曲線的斜率為負斜率。(E)自原點射出之射線與任何一條準線性偏好的無異曲線相切之切點的邊際替代率值，皆為相等之常數。(2 分)

(2)若消費者效用函數為 $U = XY + X$，預算函數為 $I = P_X \cdot X + P_Y \cdot Y$，則： (A)X 財貨為季芬財 (B)X 財貨為劣等財 (C)X 財貨為正常財 (D)X 財貨為自由財 (E)資料不完整，無法判斷。(2 分)【99 台北公行所】

解： (1)(B)；(A)若 X 財為喜好品，Y 財為厭惡品，則無異曲線為正斜率。(C)無異曲線斜率值為 MRS_{XY}，並非兩條無異曲線的垂直間距。(E)Cobb-Douglas utility function 為 Homothetic preference 效用函數，每一條無異曲線和通過原點的射線相交的切線斜率值會相等。

(2)(C)；$MRS_{XY}=\frac{MU_X}{MU_Y}=\frac{P_X}{P_Y}\Rightarrow\frac{Y+1}{X}=\frac{P_X}{P_Y}\Rightarrow Y=\left(\frac{P_X}{P_Y}X-1\right)$代回預算線：

$P_X X+P_Y\left(\frac{P_X}{P_Y}X-1\right)=I$，可解出普通需求函數：$X^*=\frac{I+P_Y}{2P_X}$；$Y^*=\frac{I-P_Y}{2P_Y}$，

◆比較靜態分析可知：$\frac{\partial X^*}{\partial I}=\frac{1}{2P_X}>0$；$\frac{\partial Y^*}{\partial I}=\frac{1}{2P_Y}>0$，表示 X 財與 Y 財皆為正常財。

11.設有兩個產品。王五的所得是 m，且其效用函數是 U（x_1, x_2）＝$2x_1^3x_2^6$。

(1)請問王五的無異曲線是否會凸向原點？（需證明）

(2)令產品 1 與 2 的單價是 p_1 與 p_2，請求出 x_1 與 x_2 的需求函數。

(3)王五的偏好是否是一個 homothetic 偏好？（需證明）

(4)設另一個人的效用函數是 V(x_1, x_2) = $\log x_1 + 2\log x_2$，請問此人的偏好與王五的偏好有何關係？【96 中央產經所】

$V=\frac{1}{3}\log(X_1X_2^2)^3=\frac{1}{3}\log\left(\frac{U}{2}\right)^{\frac{1}{3}}$ ∴有相同

解：

(1)將效用函數進行全微分：$0=6X_1^2X_2^2dX_1+12X_1^3X_2^5dX_2$

$MRS_{XY}=\frac{dX_2}{dX_1}=-\frac{MU_X}{MU_Y}=\frac{-6X_1^2X_2^6}{12X_1^3X_2^5}=\frac{-X_2}{2X_1}\langle 0$【表示無異曲線呈現負斜率】

$$\frac{d|MRS_{12}|}{dX_1}=\frac{d\left(\frac{X_2}{2X_1}\right)}{dX_1}=\frac{(2X_1)\left(\frac{dX_2}{dX_1}\right)-X_2(2)}{(2X_1)^2}=\frac{2X_1\left(\frac{-X_2}{2X_1}\right)-X_2(2)}{4X_1^2}$$

$\therefore\frac{d|MRS_{12}|}{dX_1}=\frac{-3X_2}{4X_1^2}\langle 0$ ....符合邊際替代率遞減，無異曲線凸向原點

(2) $MaxU=2X_1^3X_2^6 \quad st \quad P_1X_1+P_2X_2=m$

2 $X_1^M=\frac{m}{3P_1} \qquad X_2^M=\frac{2m}{3P_2}$

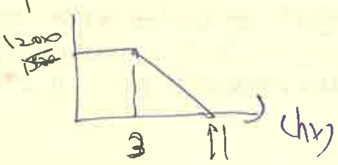
(3)$|MRS_{XY}|=\left|\frac{dX_2}{dX_1}\right|=\left|-\frac{MU_X}{MU_Y}\right|=\left|\frac{-6X_1^2X_2^6}{12X_1^3X_2^5}\right|=\frac{X_2}{2X_1}$.......只要 MRS_{XY} 可以化簡成 $f\left(\frac{X_2}{X_1}\right)$ 之比例函數，表示消費者為齊序偏好效用函數，ICC 是一條過原點直線。

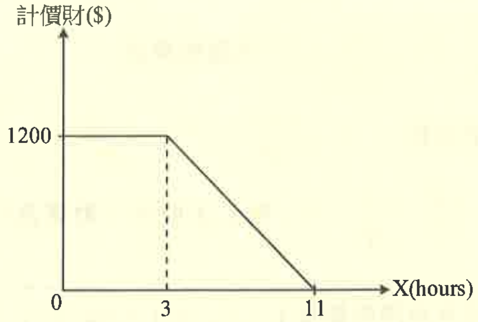
(4)$V=\log X_1X_2^2=\log\left(X_1^3X_2^6\right)^{\frac{1}{3}}=\frac{1}{3}\log\frac{U}{2} \qquad \frac{dV}{dU}=\frac{1}{U}>0$

因此 V 為 U 之單調遞增轉換函數，有相同偏好結構；表示消費者為齊序偏好效用函數。

12.布希有現金 1500 元，想帶布萊爾去「伊拉克 KTV」唱歌，若「伊拉克 KTV」收費方式為「包廂費 300 元，免費歡唱三個小時後若想續唱則每小時收費 150 元」，請依此繪出布希所面對的預算限制線。【中山企研所】

$\boxed{\frac{1200}{150}=8\ hr}$





13.A person can buy 9 units of X and 10 units of Y with her total income. Or she can buy 3 units of X and 12 units of Y with her total income. The price of X is $8. What is her total income?【96 中興財金所】

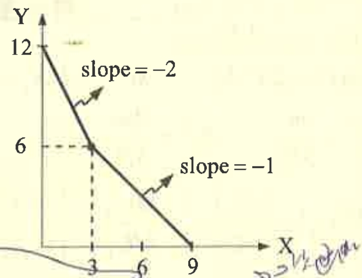
解: $P_X X + P_Y Y = M$

$\begin{cases} 8\times 9 + P_Y \times 10 = M \\ 8\times 3 + P_Y \times 12 = M \end{cases}$ → 解聯立，得 M = 312

14.王先生消費 X、Y 兩種財貨，其效用函數為 U(X, Y) = XY，又其所得為 12 元。財貨 Y 的單位價格為 1 元。銷售財貨 X 的廠商進行促銷。令 Q 為某消費者購買的財貨 X 數量。若 $Q \le 3$，則每單位 X 價格為 2 元。若 $Q > 3$，則前 3 單位仍為每單位 2 元，但超過 3 單位的部份則每單位 1 元。又兩種財貨均可無限細分，故購買及消費時不一定要購買或消費整數單位。

(1)請求出王先生的預算限制線，並將預算限制線畫在(X, Y)平面上。

(2)求出王先生的最適消費。【台大財金所】



解：(a) $\boxed{\begin{cases} 2X + Y = 12 & X \le 3 \\ 3(2) + (X-3) + Y = 12 & X > 3 \end{cases}}$

$\begin{cases} 2X + Y = 12 & X \le 3 \\ X + Y = 9 & X > 3 \end{cases}$

(b)若 $X \le 3$ $\boxed{\begin{array}{l} Max \quad U = XY \\ s.t \quad 2X + Y = 12 \end{array}}$ $\Rightarrow$ $X^* = \frac{12}{2\times 2} = 3$, $Y^* = \frac{12}{2\times 1} = 6$ $\Rightarrow U^* = 18$

若 $X > 3$ $\boxed{\begin{array}{l} Max \quad U = XY \\ s.t \quad X + Y = 9 \end{array}}$ $\Rightarrow$ $X^* = \frac{9}{2\times 1} = 4.5$, $Y^* = 4.5$ $\Rightarrow U^* = 20.25$

∴ 最適消費 $X^* = 4.5$，$Y^* = 4.5$

思考國高中題有相同的式子

15.With some services, e.g., checking accounts, phone services, or pay TV, a consumer is offered a choice of two or more payment plans. One can either pay a high entry fee and get a low price per unit of service or pay a low entry fee and a high price per unit of service. Suppose you have an income of $100. There are two plans. Plan A has an entry fee of $20 with a price of $2 per unit. Plan B has an entry fee of $40 with a price of $1 per unit of using the service. Let x be expenditure on other good and y be consumption of the service.

(1) Write down the budget equation that you would have after you paid the entry fee for each of the two plans.

(2) If your utility function is xy, how much y would you choose in each case?

(3) Which plan would you prefer?【97 成大經研所】

解：(1)First plan: A low entry fee and a high price per unit of service

Budget equation: $x+2y=100-20=80$

Second plan: A high entry fee and a low price per unit of service

Budget equation: $x+y=100-40=60$

(2) First plan: $\text{Max } U=xy \quad \text{s.t. } x+2y=80 \quad MRS_{xy}=\frac{y}{x}=\frac{P_x}{P_y}=\frac{1}{2}\Rightarrow 2y=x \quad x^*=40, y^*=20$

Second plan: $\text{Max } U=xy \quad \text{st. } x+y=60 \quad MRS_{xy}=\frac{y}{x}=\frac{P_x}{P_y}=\frac{1}{1}\Rightarrow y=x \quad x^{**}=30, y^{**}=30$

(3)由於 $U_{first}=x^*y^*=800<U_{second}=x^{**}y^{**}=900$，因此會選擇第一個 plan

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16.已知效用函數為：$U=100X^{\frac{1}{4}}Y^3$

(1)求 X、Y 的邊際效用？($MU_X=?\ \ MU_Y=?$)(8 分)

(2)求 X、Y 的邊際效用是否會遞減？請證明之。(8 分)

(3)求 X、Y 的邊際替代率？($MRS_{XY}=?$)(8 分)【98 高應大人資所】

解： (1) $MU_X=\frac{\partial U}{\partial X}=25X^{\frac{-3}{4}}Y^3>0$ ，$MU_Y=U_Y=300X^{\frac{1}{4}}Y^2>0$

(2) $U_{XX}=\frac{\partial MU_X}{\partial X}=\frac{-75}{4}X^{\frac{-7}{4}}Y^3<0$ ，$U_{YY}=\frac{\partial U_Y}{\partial Y}=600X^{\frac{1}{4}}Y>0$

表示 X 財符合邊際效用遞減法則；而 Y 財呈現邊際效用遞增現象。

(3) $MRS_{XY}=\frac{dY}{dX}=\frac{-MU_X}{MU_Y}=\frac{-25X^{\frac{-3}{4}}Y^3}{300X^{\frac{1}{4}}Y^2}=\frac{-Y}{12X}<0$ ，表示無異曲線為負斜率。

17.Derive the demand curves for x_1, x_2 assuming utility function 輕鬆的基本題型

(1) $u=x_1 x_2$ (9 分)

(2) $u=x_1^{\alpha_1}x_2^{\alpha_2}$ (9 分)

What are the price and income elasticities for x_1 and x_2 ?【99 中央產經所】

解：

(1)Cobb-Douglas utility function，可解出效用極大化普通需求函數：

$x_1=\frac{M}{2P_1}$ ； $x_2=\frac{M}{2P_2}$

❶ x_1 的需求彈性：

$$\varepsilon_{x_1}^d=\frac{-dx_1}{dP_1}\frac{P_1}{x_1}=\frac{M}{2P_1^2}\times\frac{P_1}{\frac{M}{2P_1}}=1$$

❶ x_2 的需求彈性：

$$\varepsilon_{x_2}^d=\frac{-dx_2}{dP_2}\frac{P_2}{x_2}=\frac{M}{2P_2^2}\times\frac{P_2}{\frac{M}{2P_2}}=1$$

❷ x_1 的所得彈性：

$$\varepsilon_{x_1}^M = \frac{dx_1}{dM}\frac{M}{x_1} = \frac{1}{2P_1} \times \frac{M}{\frac{M}{2P_1}} = 1$$

❷ x_2 的所得彈性：

$$\varepsilon_{x_2}^M = \frac{dx_2}{dM}\frac{M}{x_2} = \frac{1}{2P_2} \times \frac{M}{\frac{M}{2P_2}} = 1$$

(2)Cobb-Douglas utility function：$x_1^M = \frac{\alpha_1}{(\alpha_1+\alpha_2)} \cdot \frac{M}{P_1}$ ；$x_2^M = \frac{\alpha_2}{(\alpha_1+\alpha_2)} \cdot \frac{M}{P_2}$

將 x_1 普通需求函數取自然對數：

$$\ln x_1 = \ln\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right) + \ln M - \ln P_1$$

$$d\ln x_1 = d\ln M - d\ln P_1$$

❶ x_1 的需求彈性 $\varepsilon_{x_1}^d = \frac{-\partial \ln x_1}{\partial \ln P_1} = 1$

❷ x_1 的所得彈性 $\varepsilon_{x_1}^M = \frac{\partial \ln x_1}{\partial \ln M} = 1$

將 x_2 普通需求函數取自然對數：

$$\ln x_2 = \ln\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right) + \ln M - \ln P_2$$

$$d\ln x_2 = d\ln M - d\ln P_2$$

❶ x_2 的需求彈性 $\varepsilon_{x_2}^d = \frac{-\partial \ln x_2}{\partial \ln P_2} = 1$

❷ x_2 的所得彈性 $\varepsilon_{x_2}^M = \frac{\partial \ln x_2}{\partial \ln M} = 1$

18.某甲的所得為 46，其消費 X 與 Y 兩種財貨，而其效用函數為 U(xy) = xy，其中 x 與 y 分別為某甲對財貨 X 與 Y 的消費量。財貨 Y 之單位價格為 4。財貨 X 之單位價格隨購買量而有不同：購買前 3 單位之財貨 X 時之價格為 6；若購買數量超過 3 單位，則超過 3 單位的部份每單位價格為 4。某甲對財貨 X 的最適消費量為 ______。當消費最適的財貨組合時，某甲的效用水準為 ______。【97 台大財金所】

解：$\begin{cases} 6X+4Y=46 & X\le 3 \\ 18+4(X-3)+4Y=46 & X>3 \end{cases}$ $\begin{cases} 6X+4Y=4 & X\le 3 \\ 4X+4Y=4 & X>3 \end{cases}$

若 X ≤ 3 $\boxed{\begin{array}{ll} Max & U=XY \\ & 3X+2Y=23 \\ s.t & 6X+4Y=46 \end{array}}$ $\Rightarrow X^* = \frac{46}{2\times 6} = 3.83(\times)$ 矛盾

若 X > 3 $\boxed{\begin{array}{ll} Max & U=XY \\ s.t & 4X+4Y=40 \end{array}}$ $\Rightarrow X^* = \frac{40}{2\times 4} = 5$ $Y^* = 5$ $\Rightarrow U^* = 25$

∴ 最適消費 $X^* = 5$ ，$Y^* = 5$

19.Preference and Demand

(1)When we know the representative consumer with fixed proportion of expenditure on good X and good Y, please derive the possible utility form to describe the preference of the representative consumer. (5 分)

(2)When we know the representative consumer with fixed quantities of consumption on good X and good Y, please derive the possible utility form to describe the preference of the representative consumer. (5 分)【99 逢甲財稅；國貿所】

解：(1)財貨支出占所得比例固定不變，表示消費者偏好型態為「Cobb-Douglas utility function」，證明如下：$U = X^{\alpha}Y^{1-\alpha}$ ，

效用極大化普通需求函數為 $X^M = \frac{\alpha M}{P_X}$ ；$Y^M = \frac{(1-\alpha)M}{P_Y}$

$$\begin{cases} X\text{ 財支出占所得比例:}\dfrac{P_X\cdot X}{M}=\alpha \\ Y\text{ 財支出占所得比例:}\dfrac{P_Y\cdot Y}{M}=(1-\alpha \end{cases}$$

(2)消費者偏好型態為「完全互補」效用函數，假設 α 個 X 財必須搭配 β 個 Y 財才有一單位的效用，則效用函數表達為 $U=\min\left(\dfrac{X}{\alpha};\dfrac{Y}{\beta}\right)$，則效用極大化的普通需求函數為

$X^M=\dfrac{\alpha M}{\alpha P_X+\beta P_Y}$ ； $Y^M=\dfrac{\beta M}{\alpha P_X+\beta P_Y}$ 。

20.(1)A consumer's utility function is $U=xy$, where x and y denote the amounts of two goods. Please specify the consumer's Marshallian demand curve for good x if the consumer's income equals 200. (5%)

(2)Based on the demand curve from (1), find the price elasticity of demand for x when its price equals 5. 【99 清大經研所】 (5%)

解：(1)Cobb-Douglas utility function：

x 財普通需求函數為 $x^M=\dfrac{200}{2P_x}=\dfrac{100}{P_x}$ ， $y^M=\dfrac{200}{2P_y}=\dfrac{100}{P_y}$

(2)當 $P_x=5$ 時，x 財需求量 $x^*=20$，則 x 財需求彈性為 $\varepsilon^d=\dfrac{-dx}{dP_x}\dfrac{P_x}{x}=\dfrac{100}{P_x^2}\times\dfrac{P_x}{\dfrac{100}{P_x}}=1$

21.Given the budget constraint $4X+Y=10$ for buying two goods X and Y, find the optimal choice on X this individual if the utility function is $\min(X,Y)$? ________

【99 元智財金所】

解： $\begin{cases} Max\ U=\min(X,Y) \\ s.t.\quad 4X+Y=10 \end{cases} \Rightarrow X^*=Y^*=2$ ， $U^*=\min(X,Y)=2$

22.小王決定利用下班補習英文(X)及電腦(Y)，假設英文課程每小時 400 元，電腦課程每小時 600 元，假設其一個月的進修預算 12000 元，其效用函數為：$U=X^{\frac{1}{2}}Y^{\frac{1}{2}}$，試問：

(1)小王最適課程進修時數為何？(10 分)

(2)如果小王一個月最多只能撥出的進修時間只有 23 小時，請問其最適課程進修時數為何？(10 分)【98 嘉大應經所】

解：

(1) $X^*=\dfrac{12000}{2\,(400)}=15$ ， $Y^*=\dfrac{12000}{2\,(600)}=10$

(2)效用極大化決策：

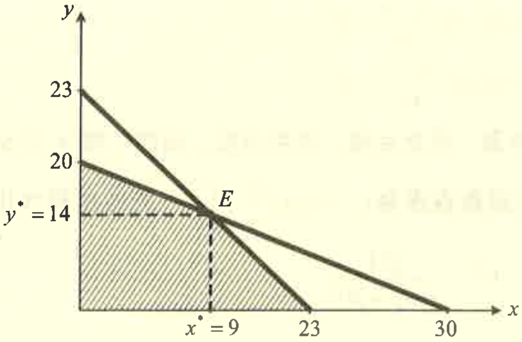
Max $U=X^{\frac{1}{2}}Y^{\frac{1}{2}}$

s.t. $400X+600Y=12{,}000$

$X+Y=23$

同時滿足時間限制條件與所得預算限制式，因此效用極大化需求量發生在二條限制條件相交 E 點：

$X^*=9$ ， $Y^*=14$

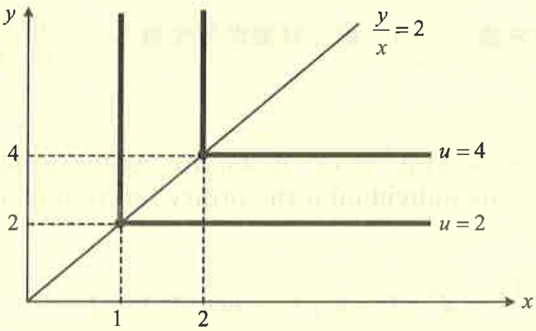


23.Suppose a consumer has preferences over two goods that can be represented by the utility function $U=\min\{2X, Y\}$

(1)Describe the shape of the indifference curves. (4%)

(2)Describe the special properties of the $MRS_{X,Y}$. (4%)【99 成大財金所】

解： (1)消費者的偏好型態為「完全互補」效用函數，表示 1 個 X 財必須搭配 2 個 Y 財才有 2 單位的效用，此時無異曲線是以 $2X=Y \Rightarrow \dfrac{Y}{X}=2$ 為角點的直角形。



(2)因為無異曲線為直角形，在 $Y>2X$ 區域中，無異曲線為垂直線，其 $MRS_{XY}=\infty$，在 $Y<2X$ 區域中，無異曲線為水平線，其 $MRS_{XY}=0$，但是角點上的 MRS_{XY} 則無法定義。

24.A person's utility function is of the form $U(X,Y)=5XY$. The prices of good X and Y are $P_X=4$ and $P_Y=2$, respectively. The person's income, M is $=1200$. (15%)

(1)What quantities of X and Y should the consumer purchase to maximize his utility?

(2)Determine the person's income offer curve (IOC).

(3)Derive the Engel curve for X and Y, respectively.【99 淡江財金所、國貿所】

解：效用極大化決策：$\begin{cases} Max\, U=5XY \\ s.t \quad M=P_X X+P_Y Y \end{cases} \Rightarrow$ 普通需求函數 $X^M=\dfrac{M}{2P_X}$ ； $Y^M=\dfrac{M}{2P_Y}$

(1)當 $P_X=4$ ， $P_Y=2$ ， $M=1200$ 時

$X^*=\dfrac{1200}{2\,(4)}=150$ ， $Y^*=\dfrac{1200}{2\,(2)}=300$ ， $U^*=225{,}000$

(2)ICC 方程式：滿足消費者均衡條件 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}$

$\dfrac{Y}{X}=\dfrac{4}{2}\Rightarrow Y=2X$，ICC 過原點直線。

(3) ❶Engel Curve for X 財

$X=\dfrac{M}{2\,(4)}\Rightarrow M=8X$

❷Engel Curve for Y 財

$Y=\dfrac{M}{2\,(2)}\Rightarrow M=4Y$

Engel Curve 過原點直線，保證 $\varepsilon_M^X=1$；$\varepsilon_M^Y=1$

25.Suppose an individual utility function has the form of $U=X^{0.4}Y^{0.6}$, where X and Y are the commodity consumed and M is this individual income. P_X, P_Y denote the market prices of these two commodities. Please derive the Marshallian and Hicksian demand function for X and Y. (15%)【99 中興企研所】

解：

Marshallian demand function：

$$\begin{cases} Max\,U=X^{0.4}Y^{0.6} \\ s.t.\quad M=P_XX+P_YY \end{cases}$$

$$\mathrm{L}=X^{0.4}Y^{0.6}+\lambda\,(M-P_XX-P_YY)$$

F.O.C：

$$\frac{\partial \mathrm{L}}{\partial X}=0\Rightarrow 0.4X^{-0.6}Y^{0.6}+\lambda\,(-P_X)=0$$

$$\frac{\partial \mathrm{L}}{\partial Y}=0\Rightarrow 0.6X^{0.4}Y^{-0.4}+\lambda\,(-P_Y)=0$$

$$\frac{\partial \mathrm{L}}{\partial \lambda}=0\Rightarrow M-P_XX-P_YY=0$$

$\dfrac{0.4Y}{0.6X}=\dfrac{P_X}{P_Y}\Rightarrow Y=\dfrac{3P_X}{2P_Y}X$ 代回預算式

$X^M=\dfrac{2M}{5P_X}$；$Y^M=\dfrac{3M}{5P_Y}$

Hicksian demand function： 跳了 ⇒ 後面的單元

$$\begin{cases} Min\,E=P_XX+P_YY \\ s.t.\quad U=X^{0.4}Y^{0.6} \end{cases}$$

支出極小化均衡條件 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}$

$Y=\dfrac{3P_X}{2P_Y}X$ 代回效用函數：

$$U=X^{0.4}\left(\frac{3P_X}{2P_Y}X\right)^{0.6}\Rightarrow X^H=\left(\frac{2P_Y}{3P_X}\right)^{0.6}U$$

$$Y^H=\left(\frac{3P_X}{2P_Y}\right)\cdot\left(\frac{2P_Y}{3P_X}\right)^{0.6}U$$

可得 $Y^H=\left(\dfrac{3P_X}{2P_Y}\right)^{0.4}U$

26.小明對橘子(X_1)與蘋果(X_2)的效用函數為 $U(X_1,X_2)=4X_1^{3/2}+X_2$，若他原來消費 25 顆橘子與 18 顆蘋果，現在其要增加 11 顆橘子消費，在維持效用不變之下，則小明最多要放棄多少顆蘋果？(10 分)【98 台大森林所】

解：原來組合：$X_1=25$，$X_2=18$，$U_0=38$

新組合：$38=4\,(36)^{\frac{1}{2}}+X_2'\Rightarrow X_2'=14$，$\Delta X_2=-4$

小明要放棄 4 個蘋果，才可以維持原效用水準不變。

27.假定效用函數與預算限制分別為：$TU=2X(Y+1)$，$2X+Y=11$；試以 Lagrangian 乘數法求取消費者均衡狀態下之 X、Y 值與總效用。【99 高應大人資所】

解：

$$\mathrm{L}=2X(Y+1)+\lambda\,(11-2X-Y)$$

F.O.C：

$\dfrac{\partial L}{\partial X}=0 \Rightarrow 2(Y+1)+\lambda(-2)=0 \ldots$ ❶

$\dfrac{\partial L}{\partial Y}=0 \Rightarrow 2X+\lambda(-1)=0 \ldots$ ❷

$\dfrac{\partial L}{\partial \lambda}=0 \Rightarrow 11-2X-Y=0 \ldots$ ❸

$\dfrac{2(Y+1)}{2X}=2 \Rightarrow Y+1=2X$ 代入 ❸ 式

$\therefore Y^*=5$ ， $X^*=3$ ， $U^*=36$

S.O.C

$$|\overline{H_2}|=\begin{vmatrix} 0 & 2 & -2 \\ 2 & 0 & -1 \\ -2 & -1 & 0 \end{vmatrix}=8>0$$

滿足效用極大化充分條件。

$X:Y=4:3$

28.效用函數 $U(X,Y)=\min\{3X,4Y\}$ ，所得 $I=20$ ，X，Y 的價格為 P_X ， P_Y。

(1)試繪出效用函數的圖形。X，Y 兩物品之間為何種關係？

(2)假設 $P_Y=1$ ，請導出 X 的需求函數。【99 高應大企研所】

解： (1) $U=\min(3X,4Y)$ ，表示 4 單位 X 財，搭配 3 單位 Y 財，才有 12 單位效用，消費者的偏好為「完全互補」效用函數，無異曲線是以 $3X=4Y$ 為角點的直角形。

(2) $\begin{cases} Max\, U=\min(3X,4Y) \\ s.t \quad 20=P_X X+Y \end{cases} \Rightarrow 3X=4Y$ 代入預算線可得：

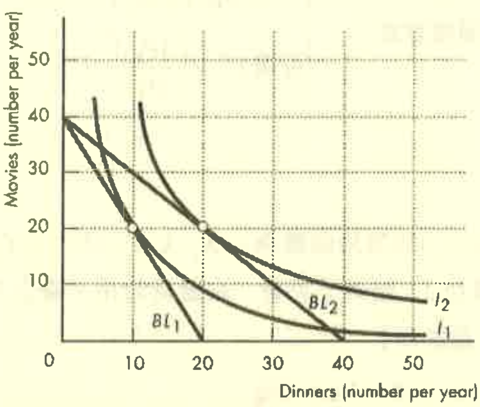
$X^M=\dfrac{20}{P_X+\dfrac{3}{4}}=\dfrac{80}{4P_X+3}$ ； $Y^M=\dfrac{60}{4P_X+3}$

easy!

要用自己的方法算

$X^M=4^{倍}\left(\dfrac{20}{4\times P_X+3\cdot 1}\right)=\dfrac{80}{4P_X+3}$

$Y^M=3^{倍}\cdot\dfrac{20}{(4P_X+3)}=\dfrac{60}{4P_X+3}$



29.George has a \$600 annual entertainment budget that he uses to buy trips to the movies and dinners at local restaurants. The figure above shows indifference curves and budget lines for these two goods. The price of a movie is \$15. (10 分)

(1) Along budget line BL_1, what is the price of a dinner?

(2)What combination of dinners and movies will George select along budget line BL_1?

(3) Budget line BL_2 represents a change in the price of dinners from that along BL_1. What is the new price of dinners along this budget line?

(4)What combination of dinners and movies will George select along budget line BL_2?

(5) Use the information in this problem to give two points on George's demand curve for dinners.【99 嘉大管研所】

解：

(1) $P_X X + 15Y = 600 \Rightarrow \dfrac{600}{P_X^0} = 20 \Rightarrow P_X^0 = 30$

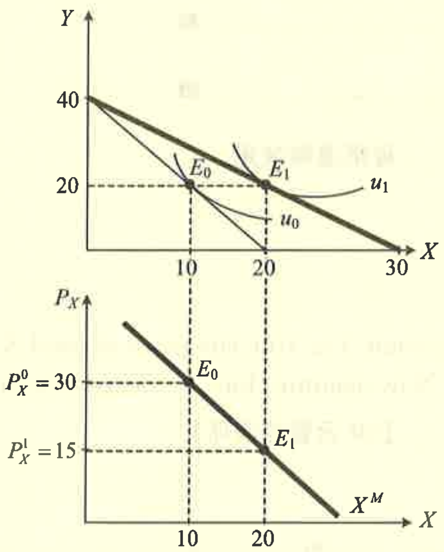
(2) $X_0^* = 10$ ，$Y_0^* = 20$

(3) $\dfrac{600}{P_X'} = 40 \Rightarrow P_X' = 15$

(4) $X_1^* = 20$ ，$Y_1^* = 20$

(5)利用($P_X^0 = 30, X_0 = 10$)與($P_X' = 15, X_1 = 20$)

可推導 X 財普通需求曲線：



30.某消費者的所得為 100，效用函數為 U = XY，X 與 Y 的價格同為 10；

(1) 試求消費者的消費組合與最大效用；

(2) 如果 X 的價格從 10 漲為 20，求新的消費組合與最大效用。【97 淡江企研所】

解：消費最適化問題：$\begin{cases} \max U = XY \\ s.t. \quad P_X X + P_Y Y = M \end{cases}$

消費者均衡的必要條件：$\begin{cases} MRS = \dfrac{P_X}{P_Y} \Rightarrow P_X X = P_Y Y \\ P_X X + P_Y Y = M \end{cases}$；一般需求函數：$\begin{cases} X^* = \dfrac{M}{2P_X} \\ Y^* = \dfrac{M}{2P_Y} \end{cases}$

(1) $P_X = P_Y = 10, M = 100 \Rightarrow \begin{cases} X^* = \dfrac{100}{20} = 5 \\ Y^* = \dfrac{100}{20} = 5 \end{cases}$　(2) $P_X = 20, P_Y = 10, M = 100 \Rightarrow \begin{cases} X^* = \dfrac{100}{40} = 2.5 \\ Y^* = \dfrac{100}{20} = 5 \end{cases}$

31.Assuming that a frequently used utility function is the Cobb-Douglas utility function, which can be represented in the following form : $U(X, Y) = a\ln(X) + (1-a)\ln(Y)$, where ln signifies the logarithm to the base $e(=2.71828)$, and subject to the constraint that all income is spent on the two goods : $P_X X + P_Y Y = I$, and in the meantime, let $a = 0.8$, $P_X = \$2$, $P_Y = \$1$, and $I = \$100$ To find the demand functions for X and Y, we first write the Lagrangian : $\Phi = a\ln(X) + (1-a)\ln(Y) - \lambda(P_X X + P_Y Y - I)$. Please answer the following questions :

(1) What does that mean by Lagrange Multiplier? (15%)

(2) $X = ?$ (5%) (3) $Y = ?$ (5%)【99 北大國企所】

解：$U = 0.8\ln X + 0.2\ln Y$

$L = 0.8\ln X + 0.2\ln Y + \lambda(100 - 2X - Y)$

F.O.C：$\frac{\partial L}{\partial X} = 0 \Rightarrow \frac{0.8}{X} + \lambda(-2) = 0$ ……… ❶

$\frac{\partial L}{\partial Y} = 0 \Rightarrow \frac{0.2}{Y} + \lambda(-1) = 0$ ……… ❷

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow 100 - 2X - Y = 0$ ……… ❸

聯立求解可得：$X^* = 40$ ，$Y^* = 20$ ，$\lambda = \frac{\partial U}{\partial M} = MU_m =$ **貨幣邊際效用**

$\lambda^* = \frac{0.2}{Y} = \frac{0.2}{20} = 0.01$

32.Roger's utility function is $U(x, y) = xy^2$, his income 1 is 100, the price of good X, P_x, is 10 initially, and the price of good Y, P_y, is 10. Now assume that P_x decreases to 5. What is the Slutsky substitution effect? __(3)__.【99 元智企研所】

解：普通需求函數：$X^M = \frac{I}{3P_x}$ ；$Y^M = \frac{2I}{3P_y}$

原均衡：$X_0 = \frac{100}{3(10)} = \frac{10}{3}$ ；$Y_0 = \frac{200}{3(10)} = \frac{20}{3}$ ，**新均衡**：$X_2 = \frac{20}{3}$ ；$Y_2 = \frac{20}{3}$

$CD = (P_x^0 - P_x')X_0 = (10-5)\frac{10}{3} = \frac{50}{3}$ ，$I' = 100 - \frac{50}{3} = \frac{250}{3}$

$X_1 = \frac{\frac{250}{3}}{3(5)} = \frac{50}{9}$ ；$Y_1 = \frac{2\left(\frac{250}{3}\right)}{3(10)} = \frac{50}{9}$

$SE_S : \begin{cases} X : \frac{50}{9} - \frac{10}{3} = \frac{20}{9} \\ Y : \frac{50}{9} - \frac{20}{3} = \frac{-10}{9} \end{cases}$ $IE_S : \begin{cases} X : \frac{20}{3} - \frac{50}{9} = \frac{10}{9} \\ Y : \frac{20}{3} - \frac{50}{9} = \frac{10}{9} \end{cases}$

33.Suppose that one consumer has the following utility function: $u(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$ where x and y are two goods. Denote by p_x and p_y the prices of x and y, respectively.

(1)Suppose that the price of x is 10, the price of y is 10 and the consumer's income is 100. What are the optimal consumption bundle, denoted by $(x^*, y^*) =$ ______ and the

consumer's maximum utility level achieved $u^* =$ ______ ?
(2)Find the optimal consumption bundle $(x^*, y^*) =$ ______ when the price of x drops to 5.
(3)Find the compensated bundle $(x_H^*, y_H) =$ ______ and the compensated income $(M_H) =$ ______ . Write down the compensated budget line $(BL_H) =$ ______ .
(4)Find the changes in demand of x due to the substitution effect and income effect $(SE, IE) =$ ______ . (28 分)【99 成大交管/電管所】

解：

(1)效用極大化決策 $\begin{cases} Max\, u = x^{\frac{1}{2}} y^{\frac{1}{2}} \\ s.t \quad 10x + 10y = 100 \end{cases} \Rightarrow x^* = 5$ ， $y^* = 5$ ， $u^* = 5$

(2) $p_x' = 5$ ， $x' = \dfrac{100}{2(5)} = 10$ ， $y' = \dfrac{100}{2(10)} = 5$

(3)支出極小化決策 $\begin{cases} Min\ E = 5x + 10y \\ s.t \quad u_0 = x^{\frac{1}{2}} y^{\frac{1}{2}} = 5 \end{cases} \Rightarrow x^H = \sqrt{\dfrac{p_y}{p_x}} u$ ， $y^H = \sqrt{\dfrac{p_x}{p_y}} u$

$$x^H = \sqrt{\frac{10}{5}} 5 = 5\sqrt{2} \text{ ， } y^H = \sqrt{\frac{5}{10}} 5 = \frac{5\sqrt{2}}{2} \text{ ， } E = 5(5\sqrt{2}) + 10\left(\frac{5\sqrt{2}}{2}\right) = 50\sqrt{2}$$

受補償預算限制式為 $BL_H = 50\sqrt{2} = 5x^H + 10y^H$

(4)以 x 財需求量變動量來表達 SE 與 IE：

$$SE = x^H - x^* = 5\sqrt{2} - 5 \text{ ， } IE = x' - x^H = 10 - 5\sqrt{2}$$

34.【是非題】During recessions when workers lose their jobs and experience reduced incomes, sales of durable goods (goods with a life expectancy of 3 years or more) decline. Apparently, durables are inferior goods.【97 中央企研所】

解：錯誤。假設其他條件不變下，隨著所得下降耐久財消費量卻減少，表示耐久財為正常財。

35.效用函數 $U(x, y) = \min\{x, y\}$ ，所得 $I = 20$ ，x，y 的價格為 P_x、 P_y。(20%)

(1) 試繪出效用函數的圖形。

(2) x、y 兩物品之間為何種關係？

(3) 給定 $P_y = 5$ ，請導出 x 的需求函數。

(4) 在最適點上，邊際替代率等於物品相對價格的條件 $(MRS = P_x / P_y)$ 是否會成立？為什麼？

【99 高雄應用科大企研所】

解：

(1)(2)消費者偏好為「完全互補」效用函數，一單位 x 財必須搭配一單位 y 財才有一單位效用，無異曲線是以 $x = y$ 為角點的直角形。

(3) $\begin{cases} Max\, U = \min(x, y) \\ s.t \quad 20 = P_x x + 5y \end{cases} \Rightarrow x^M = \dfrac{20}{5 + P_x}$ ； $y^M = \dfrac{20}{5 + P_x}$

(4)消費者均衡解發生在無異曲線直角點之處，此時 MRS_{xy} 無法定義，因此消費者均衡解將不

會滿足 $MRS_{xy}=\frac{P_x}{P_y}$ 條件，消費者均衡解會滿足 $\begin{cases} x=y \\ I=P_x x+P_y y \end{cases}$ 條件。

36.Assuming that a frequently used utility function is the Cobb-Douglas utility function, which can be represented in the following form: U(X, Y) = a log(X) + (1 - a)log(Y), subject to the constraint that all income is spent on the two goods: $P_X X + P_Y Y$ = I, and in the meantime, let a = 0.5, P_x = \$2, P_Y = \$4, and I = \$200。To find the demand functions for X and Y, we first write the Lagrange in:

φ = a log(X) + (1 - a)log(Y) - λ(P_xX + P_y - 1). Please answer the following questions:

(1)What does that mean by Lagrange Multiplier?

(2)X = ? (3)Y = ? 【96 北大國企所】

解：

(1) Lagrange Multiplier 即為貨幣的邊際效用；

$$\begin{cases} Max\,U(X,Y) \\ s.t. P_X\cdot X+P_Y\cdot Y=M \end{cases} \Rightarrow L=U(A,B)+\lambda\left(M-P_X\cdot X-P_Y\cdot Y\right)$$

由一階條件可得：$MRS=\frac{P_X}{P_Y}\Rightarrow\frac{MU_X}{P_X}=\frac{MU_Y}{P_Y}=\lambda$

由邊際效用均等法則可知：$MRS=\frac{P_X}{P_Y}\Rightarrow\frac{MU_X}{P_X}=\frac{MU_Y}{P_Y}=\lambda=MUm$

(2)、(3)

由消費者均衡條件：$\begin{cases} MRS=\frac{P_X}{P_Y} \\ P_X X+P_Y Y=M \end{cases}\Rightarrow\begin{cases} \frac{\frac{0.5}{X}}{\frac{0.5}{Y}}=\frac{2}{4}\Rightarrow X=2Y \\ 2X+4Y=200 \end{cases}\Rightarrow X^*=50, Y^*=25$

37.Eric receives utility from days spent traveling on vacation domestically. (D) and days spent traveling in a foreign country (F) as given by the utility $U(D,F)=DF$. The price of a day spent traveling domestically is \$160 and in a foreign country \$200. Eric's annual budget for traveling is \$8,000. (20%)

(1) Find Eric's utility maximizing choice of days traveling domestically and in a foreign country. Find also his utility level from consuming that bundle. (5%)

(2) Suppose that the price of domestic traveling increases to \$250 per day. Calling his budget for traveling x, (suppose by now that it is unknown) find the demand for D and F under the new prices as a function of x. (5%)

(3) Find the income necessary to make Eric reach the same utility level as before the price change. (4%)

(4) Compute the quantities demanded with the new prices and the income you found in section c. Compute also the quantities demanded with the new prices and the original income. Using your answers tell us what is the total change in quantity of D due to the price increase in P_D that the consumer experiences and what part of that change is due to income or substitution effects. (6%)【98 雲科大財金所、工管所、運籌所】

解：首先處理消費者最適化問題的一般式

$$\begin{cases}\max U = DF \\ s.t.\ P_D D + P_F F = B\ (budget)\end{cases}$$

利用 *Lagrange* 法可求解一般需求函數為：$D^M = \dfrac{B}{2P_D}$ ；$F^M = \dfrac{B}{2P_F}$

將一般需求函數代入目標函數可得間接效用函數：$V = U(D^M, F^M) = \dfrac{B^2}{4P_D P_F}$

利用對偶理論可求得支出函數為：$E = 2\sqrt{P_D P_F U}$

利用 *Shephard lemma* 可得受補償需求函數：$D^H = \dfrac{\partial E}{\partial P_D} = \sqrt{\dfrac{P_F}{P_D}U}$ ； $F^H = \dfrac{\partial E}{\partial P_F} = \sqrt{\dfrac{P_D}{P_F}U}$

(1) 經濟體系原始狀況：$P_D = 160$，$P_F = 200$，$B = 8{,}000$

$$D_0 = \frac{B}{2P_D} = \frac{8{,}000}{2\times 160} = 25 \qquad F_0 = \frac{B}{2P_F} = \frac{8{,}000}{2\times 200} = 20 \qquad U_0 = 25\times 20 = 500$$

(2) 依題意：$P_D = 250$，$P_F = 200$，$B = X \Rightarrow D^M = \dfrac{B}{2P_D} = \dfrac{X}{500}$，$F^M = \dfrac{B}{2P_F} = \dfrac{X}{400}$

(3) 依題意：$P_D = 250$，$P_F = 200$，$U_0 = 500$

$$E = 2\sqrt{P_D P_F U_0} = 2\sqrt{250\times 200\times 500} = 10{,}000$$

$$CV = 10{,}000 - 8{,}000 = 2{,}000$$

(4) 1. 維持原效用之下：$P_D = 250$，$P_F = 200$，$U_0 = 500$

$$D^H = \sqrt{\frac{200}{250}\times 500} = 20,\quad F^H = \sqrt{\frac{250}{200}\times 500} = 25$$

2. 依題意：$P_D = 250$，$P_F = 200$，$B = 8{,}000$

$$D_2 = \frac{B}{2P_D} = \frac{8{,}000}{2\times 250} = 16$$

$$F_2 = \frac{B}{2P_F} = \frac{8{,}000}{2\times 200} = 20$$

$$\text{D 財貨的}\begin{cases}SE = D^H - D_0 = 20 - 25 = -5 \\ IE = D_2 - D^H = 16 - 20 = -4 \\ PE = -5 + (-4) = -9 \\ PE = D_2 - D_0 = 16 - 25 = -9\end{cases}$$

38.If a consumer has preferences between two goods, x and y. His preferences over the two goods can be presented by the Cobb-Douglas utility function u(x, y) = $rx^a y^b$. where r, a, b are all constants. Assume the price of x is Px , the price of y is Py , and his income is I. (1)If there are 10 consumers in the market with the same preference described above, find the market demand curve

(2)Drive the Engel curve. 【96 中山企研所】

解：

$$(1)\begin{cases}\max U = rX^aY^b \\ s.t.\ \ P_X X + P_Y Y = I\end{cases} \Rightarrow L = rX^aY^b + \lambda\left(I - P_X X - P_Y Y\right)$$

由消費者最適化的必要條件：$MRS=\dfrac{P_X}{P_Y}$

$$\begin{cases}\dfrac{raX^{a-1}Y^b}{rbX^aY^{b-1}}=\dfrac{P_X}{P_Y}\\ P_XX+P_YY=I\end{cases}\Rightarrow\begin{cases}bP_XX=aP_YY\\ P_XX+P_YY=I\end{cases}$$

聯立求解上式可得代表性消費者的「一般需求函數」$\Rightarrow\begin{cases}X^*=\left(\dfrac{a}{a+b}\right)\dfrac{I}{P_X}\\ Y^*=\left(\dfrac{b}{a+b}\right)\dfrac{I}{P_Y}\end{cases}$

市場需求函數：$\begin{cases}X_{市}=10X^*=\left(\dfrac{10a}{a+b}\right)\dfrac{I}{P_X}\\ Y_{市}=10Y^*=\left(\dfrac{10b}{a+b}\right)\dfrac{I}{P_Y}\end{cases}$

(2)由一般需求函數可求得 Engel curve $\begin{cases}I=P_XX\left(\dfrac{a+b}{a}\right)\\ I=P_YY\left(\dfrac{a+b}{b}\right)\end{cases}$

39.設市場只有 X 及 Y 二物品，效用函數（utility function）為 $U(X,Y)=2X^{0.4}Y^{0.6}$，Y 之價格為 Py＝1，所得為 I＝600，

(1)X 之價格為 Px＝2，問效用最大時的 X 及 Y 消費量為何？

(2)X 的需求函數為何？

(3)現行公司 X 產品定價為 3 元，你認為公司降價可以增加收入嗎？證明之。【96 暨南財金所】

解：普通需求函數： $X^*=\dfrac{2M}{5P_X}\quad Y^*=\dfrac{3M}{5P_Y}$

(1)**當** $P_X=2,P_Y=1,M=600,X^*=\dfrac{2\times600}{5\times2}=120,Y^*=\dfrac{3\times600}{5\times1}=360$

(2)**普通需求函數：** $X^*=\dfrac{2M}{5P_X}$

(3) $X^*=\dfrac{2M}{5P_X}=\dfrac{240}{P_X}\quad P_X^0=3,X_0=80,TR_0=240;P_X^1=2,X_1=120,TR_1=240$

因為 X 財需求彈性等於一，普通需求曲線為雙曲線，消費者對 X 財貨支出金額固定不變，因此即使公司降價，總收益仍維持不變。

40.給定一個消費者的邊際效用如下： $MU_x=0.5\dfrac{Y^{0.5}}{X^{0.5}}\qquad MU_Y=0.5\dfrac{X^{0.5}}{Y^{0.5}}$

(1)消費者的 MRS_{yx} 是什麼？當這個消費者同時消費兩倍的 X 與 Y 的數量，會發生什麼改變？三倍的話呢？增加或減少任何的一個共同的分數 t 呢？

(2)導出消費者對於 X 與 Y 的需求函數。

(3)這個消費者會將她總所得的什麼樣的比率花費在 X 面？她將總所得的什麼樣的比率花費在 Y 上面呢？ (4)這個消費者對於 X 的需求的價格彈性是什麼？

(5)假設相關價格保持不變，證明當所得變動之時，對於 Y 對 X 的消費比率仍然固定不變。【95 台大商研所】

解：

(1)MRS：邊際替代率；假設消費者只消費 X、Y 兩財，在維持相同的滿足程度之下，每增加一單位 X 的消費量，所願意放棄的 Y 財數量；數學定義為：$MRS = \dfrac{MU_X}{MU_Y}$；

$MRS = \dfrac{MU_X}{MU_Y} = \dfrac{Y}{X} \Rightarrow MRS(tX, tY) = \dfrac{tY}{tX} = \dfrac{Y}{X}, \forall t > 0$；消費量以相同比例變動時，MRS 不變：效用函數為齊序函數；

(2)消費者最適化的必要條件：$\begin{cases} MRS = \dfrac{Y}{X} = \dfrac{P_X}{P_Y} \\ P_X X + P_Y Y = M \end{cases} \Rightarrow \begin{cases} P_X X = P_Y Y \\ P_X X + P_Y Y = M \end{cases} \Rightarrow \begin{cases} X = \dfrac{M}{2P_X} \\ Y = \dfrac{M}{2P_Y} \end{cases}$

(3) 由需求函數：$\begin{cases} X = \dfrac{M}{2P_X} \\ Y = \dfrac{M}{2P_Y} \end{cases} \Rightarrow \dfrac{P_X X}{M} = \dfrac{P_Y Y}{M} = 0.5$

(4) 由需求函數：$X = \dfrac{M}{2P_X} \Rightarrow \ln X = \ln M - \ln 2 - \ln P_X \Rightarrow E_d = -\dfrac{\partial \ln X}{\partial \ln P_X} = 1$

(5) $\dfrac{Y}{X} = \dfrac{\dfrac{M}{2P_Y}}{\dfrac{M}{2P_X}} = \dfrac{P_X}{P_Y} \Rightarrow \dfrac{\partial\left(\dfrac{Y}{X}\right)}{\partial M} = 0 \Rightarrow$ 當所得變動之時，對於 Y 對 X 的消費比率仍然固定不變。

41.The utility function for a consumer are given by: $U = \sqrt{XY}$

(1)What is the MRSYX for this consumer? MRSYX is the marginal rate of substitution of Y for one unit X. What happens to the MRSYX as the amount of X and Y this consumer has simultaneously doubles? Triples?

(2)What share of her total income does this consumer spend on X? What share does she spend on Y?

(3)Derive the consumer's demand functions for X and Y.

(4)What is the price elasticity of demand for X for this consumer? 【95 中原企研所】

上次小考的題目考過

解：

(1)MRS：邊際替代率；假設消費者只消費 X、Y 兩財，在維持相同的滿足程度之下，每增加一單位 X 的消費量，所願意放棄的 Y 財數量；數學定義為：$MRS = \dfrac{MU_X}{MU_Y}$；

$MRS = \dfrac{MU_X}{MU_Y} = \dfrac{Y}{X} \Rightarrow MRS(tX, tY) = \dfrac{tY}{tX} = \dfrac{Y}{X}$；消費量以相同比例變動時，MRS 不變；

(2)、(3)

$Max \quad U = \sqrt{X^2 + Y^2}$

$s.t \qquad M = P_X X + P_Y Y$

由消費者最適化的必要條件可聯立求解一般需求函數：

$$\begin{cases} MRS = \dfrac{Y}{X} = \dfrac{P_X}{P_Y} \\ P_X X + P_Y Y = M \end{cases} \Rightarrow \begin{cases} P_X X = P_Y Y \\ P_X X + P_Y Y = M \end{cases} \Rightarrow \begin{cases} X = \dfrac{M}{2P_X} \\ Y = \dfrac{M}{2P_Y} \end{cases}$$

由需求函數可知，兩種財貨的支出佔所得比重均為 1/2： $\begin{cases} X = \dfrac{M}{2P_X} \\ Y = \dfrac{M}{2P_Y} \end{cases} \Rightarrow \dfrac{P_X X}{M} = \dfrac{P_Y Y}{M} = 0.5$

(4) 由需求函數： $X = \dfrac{M}{2P_X} \Rightarrow \ln X = \ln M - \ln 2 - \ln P_X \Rightarrow E_d = -\dfrac{\partial \ln X}{\partial \ln P_X} = 1$

42.假設消費者對商品 X 與 Y 的消費決策如下所示：

Max　　　U = f(X, Y) = X^2Y

Subject to　300 = 10X + 20Y

試求：(1)X 商品的恩格爾曲線為________；(2)X 商品的需求曲線為需求價格彈性=1 的雙曲線。【96 高科大風管所】

解：$\begin{cases} \max U = X^2 Y \\ s.t.\ \ P_X X + P_Y Y = M \end{cases} \Rightarrow L = X^2Y + \lambda(M - P_X X - P_Y Y)$

由消費者最適化的必要條件$\left(MRS = \dfrac{P_X}{P_Y}\right)$與預算線可得「一般需求函數」$\Rightarrow \begin{cases} X^* = \left(\dfrac{2}{3}\right)\dfrac{M}{P_X} \\ Y^* = \left(\dfrac{1}{3}\right)\dfrac{M}{P_Y} \end{cases}$

(1)　$Engel\ \ curve: M = \dfrac{3}{2}P_X X = 15X \Rightarrow Engel\ \ curve$為一過原點的直線

(2)　$X^* = \left(\dfrac{2}{3}\right)\dfrac{M}{P_X} = \left(\dfrac{2}{3}\right)\dfrac{300}{P_X} = \dfrac{200}{P_X} \Rightarrow P_X X = 200$

43.有一消費者之效用函數為 $U = \ln X + Y$，令 Pi 為 i 財貨之價格，i = X or Y；其所得限制為 I。

(1)試推導此消費者對 Y 財貨之需求線。

(2)請問 X、Y 兩財為互補財貨替代財？【94 中原企管所】

解：

$\frac{1}{X} = \frac{P_X}{P_Y}$

$P_X X = P_Y$

$Y = \frac{M - P_Y}{P_Y}$

$$\begin{cases} \max U = \ln X + Y \\ s.t.\ \ P_X X + P_Y Y = I \end{cases} \Rightarrow \ell = \ln X + Y + \lambda\left(I - P_X X - P_Y Y\right)$$

$$f.o.c \begin{cases} \dfrac{\partial \ell}{\partial X} = 0 \Rightarrow \dfrac{1}{X} = \lambda P_X \\ \dfrac{\partial \ell}{\partial Y} = 0 \Rightarrow 1 = \lambda P_Y \\ \dfrac{\partial \ell}{\partial \lambda} = 0 \Rightarrow P_X X + P_Y Y = I \end{cases} \Rightarrow \begin{cases} X^* = \dfrac{P_Y}{P_X} \\ Y^* = \dfrac{I - P_X X^*}{P_Y} = \dfrac{I}{P_Y} - 1 \end{cases}$$

$$\frac{\partial X^*}{\partial P_Y} = \frac{1}{P_X} > 0 \Rightarrow \text{兩財為替代品}$$

$$\frac{\partial Y^*}{\partial P_X} = 0 \Rightarrow \text{兩財為獨立品}$$

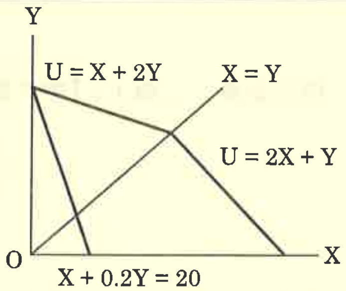
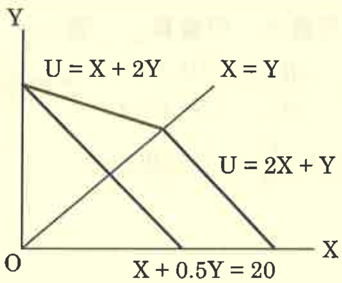
題型：拗折形無異曲線

44.小球是一隻聰明、活潑的黃金鼠。牠的主人是經濟學家，因此小求深知在預算限制下，追求效用最大的道理。小球只消費兩種東西：葵花子（X）與起司（Y），由此得到的效用為 U(X,Y) = Max(X+2Y,2X+Y)。主人每天給小球一個固定的總額度 M，讓牠可以在葵花子與起司的消費上，自由選擇。一單位葵花子要用到 1 單位的額度，1 單位起司則要用掉 P 單位的額度。

(1)如果 M = 20，P = 0.2，聰明的小球的最適消費組合是什麼？

(2)如果 M = 20，P = 0.5，聰明的小球的最適消費組合是什麼？【淡江企研、財金所】

解：$\text{if} \begin{cases} X > Y,\ X+2Y < 2X+Y \Rightarrow U = 2X+7,\ MRS_{XY} = 2 \\ X < Y,\ X+2Y > 2X+Y \Rightarrow U = X+2Y,\ MRS_{XY} = 1/2 \end{cases}$

(1) $\begin{cases} \text{Max } U = \max(X+2Y, 2X+Y) \\ \text{s.t.}\ \ X+0.2Y = 20 \end{cases}$ $\begin{cases} MRS_{XY}^{X>Y} = 2 < 5 = P_X/P_Y \\ MRS_{XY}^{X<Y} = 0.5 < 5 = P_X/P_Y \end{cases}$ $\Rightarrow X^* = 0,\ \ Y^* = \dfrac{20}{0.2} = 100,$ $U^* = X^* + 2Y^* = 200$	(2) $\begin{cases} \text{Max } U = \max(X+2Y,\ 2X+Y) \\ \text{s.t.}\ \ X+0.5Y = 20 \end{cases}$ $\begin{cases} MRS_{XY}^{X>Y} = 2 = 2 = P_X/P_Y \\ MRS_{XY}^{X<Y} = 0.5 < 2 = P_X/P_Y \end{cases}$ $\Rightarrow X^* = 0,\ \ Y^* = \dfrac{20}{0.5} = 40,$ $U^* = X^* + 2Y^* = 80$
 	

45.當甲的 $MRS=\begin{cases}1, X\geq Y\\ 10, X<Y\end{cases}$，則當所得 I = 100、價格為 Px = 2、Py = 6 時，他會買多少 X？多少 Y？請說明你的理由。 【元智企研所】

解： $Given\ MRS=\begin{cases}1, X\geq Y\\ 10, X<Y\end{cases} > \frac{P_X}{P_Y}=\frac{1}{3} \Rightarrow Y^*=0, X^*=\frac{100}{2}=50$

46.小安準備把 250 元花在 X 和 Y 上，$P_X=2$，$P_Y=3$。

(1)若她的效用函數是 $U(X, Y)=(X+Y)^2$，則最適消費組合是？

(2)若她的效用函數是 U(X, Y) = min(X + Y, Y)，則最適消費組合是？ 【95 淡江企研所】

解：

(1)完全替代效用函數，效用極大化均衡解為角隅解：

$$\begin{cases}\max U=(X+Y)^2\\ s.t.\ \ P_X X+P_Y Y=M\end{cases} \Rightarrow MRS=1>\frac{P_X}{P_Y}=\frac{2}{3} \Rightarrow \begin{cases}X^*=\frac{250}{2}=125\\ Y^*=0\end{cases}$$

(2) U(X, Y) = min(X + Y, Y)，無異曲線呈現拗折狀，效用極大化均衡解為角隅解：分兩種情況討論

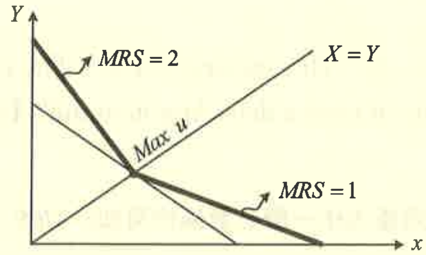
CASE 1： $X+Y>Y\Rightarrow X>0$	CASE 2： $X+Y<Y\Rightarrow X<0$(不合理)
$\begin{cases}\max U=Y\\ s.t.\ \ P_X X+P_Y Y=M\end{cases}$	$\begin{cases}\max U=X+Y\\ s.t.\ \ P_X X+P_Y Y=M\end{cases}$
$X^*=0$ $Y^*=U^*=\frac{250}{3}$	$MRS=1>\frac{P_X}{P_Y}=\frac{2}{3}\Rightarrow\begin{cases}X^{**}=\frac{250}{2}=125>0\\ Y^{**}=0\end{cases}$ (不合 X < 0 條件)
均衡解為 CASE 1： $\begin{cases}X^*=0\\ Y^*=U^*=\frac{250}{3}\end{cases}$	

47.若物品 X 和 Y 的市場價格為 $P_X=3$ 和 $P_Y=2$，

當某甲的偏好為：$\frac{MU_X}{MU_Y}=\begin{cases}1 \text{ if } X\geq Y\\ 2 \text{ if } X<Y\end{cases}$，其中 X 和 Y 表示甲的消費量，若甲的所得為 100 元，則 Y 為追求效用最大，甲會買____個 X。

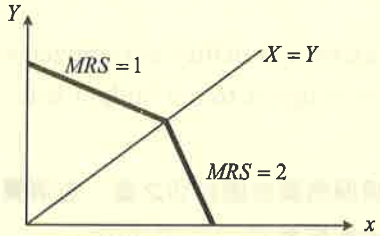
若乙的偏好為：$\frac{MU_X}{MU_Y}=\begin{cases}2 \text{ if } X\geq Y\\ 1 \text{ if } X<Y\end{cases}$，乙的所得亦為 100 元，則乙會買____個 X。【中央財金所】

解： (1)$|MRS|=1<\frac{P_X}{P_Y}=\frac{3}{2}<|MRS|=2$



均衡解 $\begin{cases} X = Y \\ 3X + 2Y = 100 \end{cases}$， $X^* = 20$， $Y^* = 20$

(2)



效用極大化均衡解必為 Corner Solution

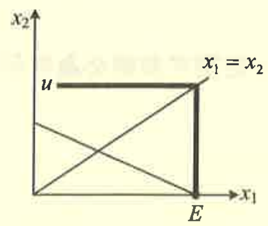
$$\begin{cases} X = \dfrac{100}{3}; Y = 0 \\ X = 0,\ Y = \dfrac{100}{2} = 50 \end{cases}$$

由於題目未給定效用函數型式，無法得知哪一組角隅才是效用極大化均衡解。

48.Suppose that Mary's utility function is $U(X_1, X_2) = \max\{X_1, X_2\}$. X_1 and X_2 are the consumption of the good 1 and 2. P_1 and P_2 are the prices of the good 1 and 2. Please analyze Mary's optimal choice. 【98 高雄大學金融管理】

解： $U = \max(X_1, X_2)$，效用極大化均衡解有三種情況：

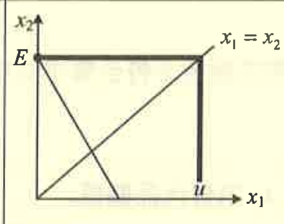
❶ 若 $P_1 < P_2 \Rightarrow \dfrac{P_1}{P_2} < 1$



效用極大化均衡解：

$X_1^* = \dfrac{M}{P_1}$ ； $X_2^* = 0$

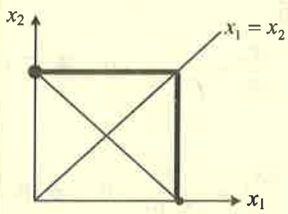
❷ 若 $P_1 > P_2 \Rightarrow \dfrac{P_1}{P_2} > 1$



效用極大化均衡解：

$X_1^* = 0$ ； $X_2^* = \dfrac{M}{P_2}$

❸ 若 $\dfrac{P_1}{P_2} = 1$



效用極大化均衡解：

$(X_1^*; X_2^*) = \left(\dfrac{M}{P_1}; 0\right)$ 與 $\left(0, \dfrac{M}{P_2}\right)$

題型：消費者均衡解計算

49.Ann's utility function is $U(x,y)=x+47y-3y^2$. Her income is \$107. The price of x is \$1, and the price of y is \$23. How many units of good x does Ann demand?【96 中興財金所】

解：$MRS_{xy}=\dfrac{MU_x}{MU_y}=\dfrac{1}{47-6y}$　　根據效用極大化一階必要條件可知: $MRS_{xy}=P_x/P_y$

$\Rightarrow \dfrac{1}{47-6y}=\dfrac{1}{23}\Rightarrow 47-6y=23$ ，$y^*=4$

50.Ture/False Questions. Briefly explain your answer.A utility maximizer will always choose a bundle at which his indifferent curve is tangent to his budget line. (5 分)【98 交大財金所】

解：錯誤；效用極大化均衡解不一定發生在預算線與無異曲線相切之處，若消費者呈現凹性偏好或是無異曲線為負斜率的直線，則效用最大均衡解為 Corner Solution。

51.Examine the properties of the demand functions of a consumer with the following utility function: $U(X)=f(X_1)+X_2$; $(f'>0, f''<0)$.(10 分)【98 中山經研所】

解：　$L=f(X_1)+X_2+\lambda(M-P_1X_1-P_2X_2)$

F.O.C　$\dfrac{\partial L}{\partial X_1}=0\Rightarrow f'(X_1)+\lambda(-P_1)=0$ ....(1)

$\dfrac{\partial L}{\partial X_2}=0\Rightarrow 1+\lambda(-P_2)=0$ ....(2)

由 $\dfrac{(1)}{(2)}$ 可得消費者均衡條件：$f'(X_1)=\dfrac{P_1}{P_2}$ ，將均衡條件全微分：

$$f''dX_1=\frac{P_2\cdot dP_1-P_1\cdot dP_2}{P_2^2}=\frac{dP_1}{P_2}-\frac{P_1}{P_2^2}dP_2$$

◆比較靜態分析：

❶ 令 $dP_2=0\Rightarrow\dfrac{\partial X_1}{\partial P_1}=\dfrac{1}{f''\cdot P_2}<0$，表示 X_1 的需求函數必符合需求法則，X_1 的需求曲線必為負斜率。

❷ 令 $dP_1=0\Rightarrow\dfrac{\partial X_1}{\partial P_2}=\dfrac{-P_1}{f''\cdot P_2^2}>0$，表示 X_1 與 X_2 為替代品關係。

❸ 令 $dP_1=0$ ，$dP_2=0\Rightarrow\dfrac{\partial X_1}{\partial M}=0$ ，表示 X_1 為中性財。　說明

52.(True or false with interpretation) Diminishing marginal rate of substitution implies diminishing marginal utility. [Hint: Considering the utility function, $U(X,Y)=\sqrt{XY}$, and its monotonic transformations to demonstrate your argument.]. (10 分)【98 中山經研所】

解：False，**邊際替代率遞減不一定保證財貨邊際效用遞減。**

(1)以效用函數 $U=\sqrt{XY}$ 說明：

$$MU_X=\frac{1}{2}X^{\frac{-1}{2}}Y^{\frac{1}{2}}\text{，並且}\frac{dMU_X}{dX}=\frac{-1}{4}X^{\frac{-3}{2}}Y^{\frac{1}{2}}<0$$

$$MU_Y=\frac{1}{2}X^{\frac{1}{2}}Y^{\frac{-1}{2}}\text{，並且}\frac{dMU_Y}{dY}=\frac{-1}{4}X^{\frac{1}{2}}Y^{\frac{-3}{2}}<0$$

$$MRS_{XY}=\frac{MU_X}{MU_Y}=\frac{Y}{X}\text{，}\frac{d|MRS_{XY}|}{dX}=\frac{\left(\frac{dY}{dX}\right)X-Y}{X^2}=\frac{\left(\frac{-Y}{X}\right)X-Y}{X^2}=\frac{-2Y}{X^2}<0$$

，表示此效用函數存在 MRS_{XY} 遞減，且邊際效用亦遞減特性。

(2) 將 $U=\sqrt{XY}$ 經過單調遞增轉換為 $V=U^4=X^2Y^2$

$$MU_X=2XY^2\text{，}\frac{dMU_X}{dX}=2Y^2>0\text{；}MU_Y=2X^2Y\text{，}\frac{dMU_Y}{dY}=2X^2>0$$

$$MRS_{XY}=\frac{MU_X}{MU_Y}=\frac{Y}{X}\text{，}\frac{dMRS_{XY}}{dX}=\frac{-2Y}{X^2}<0$$

，表示效用函數經過單調遞增轉換後，邊際效用可能遞增；固定不變或是遞減，但是仍滿足 MRS_{XY} 遞減特性，因此 MRS_{XY} 遞減的效用函數不一定隱含邊際效用遞減。

53.If a consumer has a utility function $u(x_1,x_2)=\ln x_1+\ln x_2$, where x_1 and x_2 are good 1 and good 2 respectively. Well, the market is offering a rate of exchange to the consumer of $-P_1/P_2$, where P_1 and P_2 are the prices of good 1 and good 2 respectively. Please explain why the consumer will choose the consumption where the Marginal Rate of Substitution is equal to this rate of exchange. What fraction of her income will she spend on good 2. (10 分) 【98 暨南經研所】

解：利用消費者均衡條件求解可得普通需求函數：$x_1=\frac{I}{2P_1}$ ，$x_2=\frac{I}{2P_2}$ ；兩個財貨支出占所得比例各為 $\frac{1}{2}$。

54.【是非題】根據一個消費者的偏好型態所定義的效用函數，若不符合邊際替代率遞減(diminishing marginal rate of substitution)的情況，那麼此消費者效用最大化的最適消費決策，有可能現在所得的邊際效用(marginal utility of income)為零的情況下。(5 分)【96 台北財政所】

解：錯誤；假設消費者效用函數為完全替代偏好，無異曲線為負斜率的直線，MRS_{XY} 固定不變，不滿足 MRS_{XY} 遞減，此時效用最大均衡解為 Corner Solution＜假設全部所得買 X 財，必須滿足均衡條件：$\frac{MU_x}{P_x}=MU_m$ ，只要 MU_x 為正數，保證所得邊際效用亦為正數，不可能出現 $MU_m=0$ 情況。

55.Sarah has $300 to allocate between opera tickets and movie tickets. The price of each opera tickets is $60, and the price of each movie ticket is $6. Her marginal rate of substitution (MRS) of opera tickets for movie tickets equal 5, regardless of what market basket she choose. (1)How many opera and movie tickets will she purchase? (10 分)(2)If the price of each opera ticket is \$30 and the price of each movie ticket is \$6, how many opera and movie tickets will she purchase? (10 分)【96 高雄經管所】

解：

(1) $MRS_{XY}=5<\dfrac{P_X}{P_Y}=10$，因此效用最大均衡解為 $X^*=0$，$Y^*=50$

(2) $MRS_{XY}=\dfrac{P_X}{P_Y}=5$，此時預算限制式上任一組合皆可能是效用最大均衡解。

56.【是非題】Suppose Michelle buys juice and only juice. Michelle would never consider buying any other good. Juice is always a Giffen good for Michelle. (5 分)【96 中央企研所】

解：錯誤；Michelle 只買 juice，此為 Corner Solution，並不表示 juice 為季芬財。

57.Suppose you has an utility function given by: $\ln(x_1)+x_2$. The budget constraint is given by: $I\geq P_1X_1+P_2X_2$, and $P_i>>0$.

(1)Verify that the Marshallian demand functions for x_1 and x_2 are homogeneous of degree 0, and derive the expressions for price and income elasticity of x_1. (10 分)

(2)Verify the weighted average of the income elasticities is unity. Are x_1 and x_2 necessities or luxuries? Why? (10 分)

(3)Suppose you initially consume the consumption bundle (1, 6), you now face the market price, $P=(\$2,\$10)$. How much income would you need to attain the same utility that you enjoyed at your initial consumption bundle? Round your answer to 2 decimal points. (5 分)【97 東華經研所】

$P_X X_1 + P_Y X_2 = M$

解：準線性偏好效用函數

$\dfrac{M-P_2}{P_2}=X_2$

(1)

$$\mathrm{L}=\ln x_1+x_2+\lambda(I-P_1x_1-P_2x_2)$$

$$\text{F.O.C}\ \frac{\partial \mathrm{L}}{\partial x_1}=0\Rightarrow \frac{1}{x_1}+\lambda(-P_1)=0\ \ldots❶$$

$$\frac{\partial \mathrm{L}}{\partial x_2}=0\Rightarrow 1+\lambda(-P_2)=0\ \ldots❷$$

$$\frac{\partial \mathrm{L}}{\partial I}=0\Rightarrow I-P_1x_1-P_2x_2=0\ \ldots❸$$

可得 x_1 的普通需求函數 $x_1=\dfrac{P_2}{P_1}$，代入預算限制式可得 $x_2=\dfrac{I}{P_2}-1$

Marshallian demand function 為零階齊次函數：

$$f(\lambda P_1,\lambda P_2)=\frac{\lambda P_2}{\lambda P_1}=\frac{P_2}{P_1}=x_1$$

$$f(\lambda P_2,\lambda I)=\frac{\lambda I}{\lambda P_2}-1=\frac{I}{P_2}-1=x_2$$

(2) x_1 的所得彈性 $=0$；x_2 的所得彈性 $\varepsilon_I=\dfrac{\partial x_2}{\partial I}\dfrac{I}{x_2}=\dfrac{1}{P_2}\cdot\dfrac{I}{\frac{I}{P_2}-1}=\dfrac{I}{I-P_2}$ 大於 1，表示 x_1 為中性財，x_2 為奢侈品。將所得彈性乘上財貨支出占所得比重加總可得：

$\varepsilon_I^{x_1}\alpha_1 + \varepsilon_I^{x_2}\alpha_2 = 0 + \left(\frac{I}{I-P_2}\right)\left(\frac{I-P_2}{I}\right) = 1$ ，Engel's Aggregate condition 成立。

(3)在消費組合 $(x_1, x_2) = (1,6)$ 下，消費者效用水準 $u = \ln(1) + 6 = 6$ ，當價格為 $P = (2,10)$ 下，為了維持 6 單位效用水準，計算支出極小化的需求量：

$$\left.\begin{cases} Min\ \ E = 2x_1 + 10x_2 \\ s.t.\ \ \ u = \ln x_1 + x_2 = 6 \end{cases}\right\} x_1^* = 5\ ,\ x_2^* = 6 - \ln 5 = 4.39$$

此時需要的最小所得為 $I = 2(5) + 10(4.39) = 53.9$

58.假設有一個追求效用最大的消費者其效用函數為： $U(x_1, x_2, x_3, \cdots, x_n) = \ln(x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot x_3^{\alpha_3} \cdots\cdots x_n^{\alpha_n})$ 其中 $x_1, x_2, x_3, \cdots, x_n$ 為這個消費者所消費的各種財貨之數量， $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n$ 為固定的參數，財貨 $x_1, x_2, x_3, \cdots, x_n$ 的價格分別為 $P_1, P_2, P_3, \cdots, P_n$ ，並且這消費者的所得為 I 。

(1)試推導這個消費者對 x_1 財貨的需求函數(demand function)。 (10%)

(2)推導出財貨 x_1 與財貨 x_2 之間的交叉彈性(cross-price elasticity)。(5%)【98 北大財政所】

解： (1) $x_1 = \dfrac{\alpha_1 I}{(\alpha_1 + \alpha_2 + \cdots + \alpha_n)P_1} = \dfrac{\alpha_1}{\sum_{i=1}^{n}\alpha_i} \cdot \dfrac{I}{P_1}$

(2)交叉需求彈性 $\varepsilon_{12} = \dfrac{\partial x_1}{\partial P_2}\dfrac{P_2}{x_1} = 0$

59.Walter consumes two goods, X and Y. Walter's utility function can be represented by $U = 10X^2Y$. The price of good X is \$2, and the price of good Y is \$1. Walter has \$30 to spend on the purchases of goods X and Y. If Walter is maximizing his utility subject to his budget constraint, how many units of goods X and Y should he buy? (10%)【99 中山企研所丁組】

解： $\boxed{\begin{matrix} Max\,U = 10X^2Y \\ s.t.\ \ 2X + Y = 30 \end{matrix}} \Rightarrow X^* = 10\ ,\ Y^* = 10$

60.Charlie consumer two goods, professional baseball games (B) and mystery novels (N). The price of baseball games is P_B; the price of mystery novels is P_N . Charlie has an income of I to spend on these two goods. Charlie's utility function can be represented as $U = B^2N$, with $MU_B = 2BN$ and $MU_N = B^2$. What is the equation for the demand curve for mystery novels and baseball games? (10%)【99 中山企研所丁組】

解： $\boxed{\begin{matrix} Max\,U = B^2N \\ s.t.\ \ P_BB + P_NN = I \end{matrix}} \Rightarrow B^d = \dfrac{2I}{3P_B}\ ;\ N^* = \dfrac{I}{3P_N}$

61.某人的效用函數是 min{X, 5Y+4Z}。X 的價格是 1，Y 的價格是 10，Z 的價格是 3。此人的所得是 27，請問此人對 X 的需求量是多少單位？ 【中正國經所】

解：已知 $U = \min\{X, 5Y + 4Z\}$ ； $P_X = 1,\ \ P_Y = 10$ ， $P_Z = 3,\ \ M = 27$

$U=5Y+4Z$ 為直線型效用函數，且 $\frac{MU_Y}{P_Y}=\frac{5}{10}<\frac{MU_Z}{P_Z}=\frac{4}{3}$ ⇒ 全部買 Z 財貨，而不買 Y 財

⇒ 則 $U=\min(X,4Z)$　　均衡發生在 $X=4Z$，而 $Y^*=0$

∴ $X=4Z$ 代入限制式 $27=X+10Y+3Z$　　$27=X+3\left(\frac{X}{4}\right)$　　$27=\frac{7}{4}X$　　∴ $X^*=27\times\frac{4}{7}=\frac{108}{7}$

62.若兩種商品的價格分別為 $P_1=5$，$P_2=10$，所得 $m=120$，若消費者的效用函數如下：(10 分)【97 東吳企研所】

(1) $u(x_1,x_2)=\min\{3x_1,2x_2\}$　　　　(2) $u(x_1,x_2)=x_1+3x_2$

試求消費者對第一、二種商品的需求量 (x_1,x_2) 各應為多少？

看圖形

解：

(1)完全互補效用函數：$\begin{cases}3x_1=2x_2\\5x_1+10x_2=120\end{cases}$　$x_1^*=6$，$x_2^*=9$

(2)完全替代效用函數：$\frac{Mu_1}{P_1}=\frac{1}{5}<\frac{Mu_2}{P_2}=\frac{3}{10}\Rightarrow x_1^*=0$，$x_2^*=12$

63.某消費者消費 X_1 及 X_2 兩財，兩財價格分別為 P_1、P_2，所得為 I，其效用函數為 $U(X_1,X_2)=X_1^{\alpha}X_2^{1-\alpha}$，$0<\alpha<1$，則下列何者為真？ (A)效用極大消費組合為 $(X_1^*,X_2^*)=(\alpha I,(1-\alpha)I)$ (B)無異曲線凹向原點 (C)效用極大消費組合落於預算集合內側 (D)在效用極大消費組合 (X_1^*,X_2^*) 上，邊際替代率等於相對價格 (E)此消費者必不具理性偏好。(2 分)【97 交大科管所】

解：(D)；(A)效用極大化需求量 $X_1^*=\frac{\alpha I}{P_1}$；$X_2^*=\frac{(1-\alpha)I}{P_2}$

64.Assume there are two goods in the world: apples and oranges. Sam has a utility function of $u=7a+5o$, where a denotes the quantity of apples and o the quantity of oranges.

(1)Draw the indifference curves that are defined by the utility function?

(2)What is the marginal rate of substitution between apples and oranges when Sam consumes 100 apples and 50 oranges?

(3)What do the answers to question (B) imply about the type of apples and oranges are for Sam?

(4)If the unit price of the apples and oranges are \$5 and \$4, respectively, what bundle of apples and oranges would Sam buy with his income of \$200?【94 政大風管所】

解：(2) $MRS_{ao}=\frac{7}{5}$，與 Sam 的消費組合無關。

(3)Sam 的效用函數為完全替代。

(4)因為 $MRS_{ao}=\frac{7}{5}>\frac{P_a}{P_o}=\frac{5}{4}$；效用極大化需求量：$a^*=40$，$o^*=0$

$\frac{7}{5}>\frac{5}{4}$ ∴ all by apple　$\frac{200}{5}=40$個

65.【是非題】設某消費者的效用函數為 $U=X^{\alpha}Y^{\beta}$，$\alpha>0$，$\beta>0$，並設此消費者在預算限制下求效用極大，則我們可知道此消費者對 X 財貨需求的所得彈性值為 1。(2 分)【96 成大國

企所】

解：正確； Cobb-Douglas utility Function $U=X^{\alpha}Y^{\beta}$ ，X 的 Marshallian demand function 為 $X=\dfrac{\alpha}{(\alpha+\beta)}\dfrac{M}{P_X}$

X 財所得彈性： $\ln X=\ln\left(\dfrac{\alpha}{\alpha+\beta}\right)+\ln M-\ln P_X$ $\qquad \therefore \varepsilon_M^X=\dfrac{d\ln X}{d\ln M}=1$

66.【是非題】A 消費 X、Y 兩物，其邊際替代率$(=\Delta Y/\Delta X)$為 $MRS_A=Q_Y/Q_X$，Q_X、Q_Y 為 X、Y 的數量。如果 X、Y 的價格為 $P_X=20$ ，$P_Y=10$ ，A 用盡所有的所得，買 10 單位的 X 與 10 單位的 Y。這個 $Q_X=10$ ，$Q_Y=10$ 消費組合可以讓 A 的總效用最大。(8 分)【96 暨南國企所】

解：錯誤；利用消費者均衡條件 $MRS_{XY}=\dfrac{P_X}{P_Y}\Rightarrow\dfrac{Q_Y}{Q_X}=2$

預算限制條件 $P_XQ_X+P_YQ_Y=20(10)+10(10)=300$ ，將 $Q_Y=2Q_X$ 代入

$20Q_X+10Q_Y=300\Rightarrow Q_X^*=7.5$ ， $Q_Y^*=15$

67.Sammy has $60 in weekly income, the current price of clams is $5 per pound, and the current price of potatoes is $1 per pound. Both are normal goods for Sammy. For each of the following situations, construct a diagram shows the substitution effect alone and also shows the substitution and income effects together. Put the quantity of clams (in pounds) on the horizontal axis and the quantity of potatoes (in punds) on the vertical axis.

(1)The price of a pound of clams falls from $5 to $2.50 and the price of a pound of potatoes remains at $1. (10 分)

(2)The price of a pound of clams rises from $5 to $10 and the price of a pound of potatoes remains at $1. (10 分)【98 交大運管所】

解： (1)當蛤蜊價格由 5 元降到 2.5 元，圖示 SE 與 IE。

(2)當蛤蜊價格由 5 元上漲到 10 元，圖示 SE 與 IE。

logXY

68.June has two children named Mary and John and she loves her children equally. June has a total of $1,000 to give them.

(1)Suppose that June utility function is U(X, Y) = log X + log Y, where X is the amount of the money June gives to Mary and Y is the amount of the money June gives to John. How will June choose to divide the money?

(2)Suppose that June utility function is U(X, Y) = max {X, Y}. How will June choose to divide the money?

(3)Suppose that June utility function is $U(X,Y)=X^2+Y^2$. How will June choose to divide the money? 【97 高雄金融管理】

解：

(1) $\begin{cases} Max U=\log X+\log Y \\ s.t.\quad X+Y=1{,}000 \end{cases}$ $\quad |MRS_{XY}|=\dfrac{1/X}{1/Y}=\dfrac{1}{1}\Rightarrow X=Y$ 代入預算限制式可得：

$X^*=\dfrac{1}{2}\times1{,}000=500$ ， $Y^*=\dfrac{1}{2}\times1{,}000=500$

(2) $\begin{cases} MaxU = \max\{X,Y\} \\ s.t. \quad X+Y=1{,}000 \end{cases}$ 消費者為凹性偏好，無異曲線凹向原點，效用最大為角隅解

$\begin{cases} If \;\; Y^*=0,\; X^*=1{,}000 \Rightarrow U^*=1{,}000 \\ If \;\; Y^*=1{,}000,\; X^*=0 \Rightarrow U^*=1{,}000 \end{cases} \Rightarrow$ 全部所得買 X 或是全買 Y 財無差異。

(3) $\begin{cases} MaxU = X^2+Y^2 \\ s.t. \quad X+Y=1{,}000 \end{cases}$ $\quad |MRS| = \dfrac{2X}{2Y} \Rightarrow \dfrac{d|MRS|}{dX} = \dfrac{1}{Y^2}(Y - X\dfrac{dY}{dX}) = \dfrac{1}{Y^2}\left(Y+\dfrac{X^2}{Y}\right) > 0$，

MRS 遞增，無異曲線凹向原點，凹性偏好，效用最大為角隅解

$If \;\; Y^*=0,\; X^*=1{,}000 \Rightarrow U^*=1{,}000^2 \qquad If \;\; Y^*=1{,}000,\; X^*=0 \Rightarrow U^*=1{,}000^2$

→全部所得買 X 或是全買 Y 財無差異。

題型：普通需求函數計算

69.(1)【是非題】季芬財(Giffen goods)一定是劣等財(inferior goods)，劣等財也一定是季芬財。(5 分)【97 宜蘭應經所】

(2)【是非題】若恩格爾曲線(Engel curve)為垂直線，則普通需求線(ordinary demand)或(Marshallian demand)亦為垂直線。(5 分)【97 宜蘭應經】

解： (1)錯誤。

(2)錯誤；若恩格爾曲線為垂直線，表示財貨必為「中性財」，中性財必符合需求法則，普通需求曲線必為負斜率。

70.已知消費者的效用函數為 $U=(x+2)(y+1)$，x 與 y 為兩種財貨的消費量，而預算限制式為 $P_x x + P_y y = B$，此處，$P_x > 0$，$P_y > 0$，B>0分別表示 x、y 財的價格，與預算。

(1)寫出 Lagrangian 函數。 (2) 求出最適解 x*,y* 。

(3) 根據比較靜態的結果，回答 x 與 y 財是否為 Giffen goods？

(4) 延續(3)，兩財為正常財或劣等財？

(5) 延續(3)，兩財的關係為互補財或替代財？ 【97 高應大企研所】

解：(1) $Max\; U=(x+2)(y+1)=xy+x+2y+2$, st. $\;P_x x+P_y y=B$

$\Rightarrow Lagrange(\ell)=xy+x+2y+2+\lambda(B-P_x x-P_y y)$，其中λ為 Largrange 乘數

(2) $Lagrange(\ell)=xy+x+2y+2+\lambda(B-P_x x-P_y y)$

FOC $\quad \dfrac{\partial \ell}{\partial x}=0 \Rightarrow y+1-\lambda P_x=0 \quad \dfrac{\partial \ell}{\partial y}=0 \Rightarrow x+2-\lambda P_y=0 \quad \dfrac{\partial \ell}{\partial \lambda}=0 \Rightarrow P_x x+P_y y=B$

由 — 可得：$\dfrac{y+1}{x+2}=\dfrac{P_x}{P_y} \quad P_x x-P_y y=-2P_x+P_y$ 和預算限制式聯立求解：

$$\begin{bmatrix} P_x & P_y \\ P_x & -P_y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} B \\ -2P_x+P_y \end{bmatrix}$$

$$x^* = \frac{\begin{vmatrix} B & P_y \\ -2P_x + P_y & -P_y \end{vmatrix}}{\begin{vmatrix} P_x & P_y \\ P_x & -P_y \end{vmatrix}} = \frac{B + P_y - 2P_x}{2P_x} \qquad y^* = \frac{\begin{vmatrix} P_x & B \\ P_x & -2P_x + P_y \end{vmatrix}}{\begin{vmatrix} P_x & P_y \\ P_x & -P_y \end{vmatrix}} = \frac{B + 2P_x - P_y}{2P_y}$$

$\frac{d|MRS|}{dx} = \frac{\frac{dy}{dx}(x+2)-(y+1)}{(x+2)^2} = \frac{\left(-\frac{y+1}{x+2}\right)(x+2)-(y+1)}{(x+2)^2} = \frac{-2(y+1)}{(x+2)^2} < 0$（滿足效用極大化充分條件）

(3) $\frac{\partial x^*}{\partial P_x} = -\frac{B+P_y}{4P_x^2} < 0 \Rightarrow$ 符合需求法則，X 不為 Giffen goods

$\frac{\partial y^*}{\partial P_y} = -\frac{B+2P_x}{4P_y^2} < 0 \Rightarrow$ 符合需求法則，Y 不為 Giffen goods

(4) $\begin{cases} \frac{\partial x^*}{\partial B} = \frac{1}{2P_x} > 0 \\ \frac{\partial y^*}{\partial B} = \frac{1}{2P_y} > 0 \end{cases} \Rightarrow$ x, y 皆為正常財　　(5) $\begin{cases} \frac{\partial x^*}{\partial P_y} = \frac{1}{2P_x} > 0 \\ \frac{\partial y^*}{\partial P_x} = \frac{1}{P_y} > 0 \end{cases} \Rightarrow$ x, y 互為替代財

71.某甲將其所得全數消費在 X 與 Y 二種商品，已知 X、Y 二商品之所得彈性分別為 0.92 與 1.24，則甲之所得中有多少百分比花在 X 商品上？　【台大財金所】

解：利用恩格爾加總定理：$\sum \alpha_i E_{Mi} = 1 \Rightarrow 0.92\alpha_X + 1.24(1-\alpha_X) = 1 \Rightarrow \alpha_X = 0.75$

72.Emily has decided always to spend one-third of her income on clothing. (1)What is the price elasticity of clothing demand?
(2)What is her income elasticity of clothing demand? Suppose that Emily's preference changes and she decides to spend only one-fourth of her income on clothing now.
(3)What is her income elasticity now? Two drivers-Tom and Jerry-each drive up to a gas station. Before looking at the price, each places an order. Tom's says. "I'd like 10 gallons of gas." Jerry says, "I'd like $10 worth of gas.
(4)What is Tom's price elasticity of demand?
(5)What is Jerry's price elasticity of demand?【元智國企所】

解：(1)(2) $\frac{P_X X}{M} = \frac{1}{a} \Rightarrow X = \frac{M}{aP_X} \Rightarrow \ln X = \ln M - \ln a - \ln P_X \Rightarrow E_d = -\frac{\partial X}{\partial \ln P_X} = 1 \qquad E_M = \frac{\partial X}{\partial \ln M} = 1$

(3) $\because \frac{P_X X}{M} = \frac{1}{4} \Rightarrow X = \left(\frac{1}{4}\right)\left(\frac{M}{P_X}\right) \Rightarrow \varepsilon_M^X = 1$

(4)X 消費量固定不變，表示 X 財需求曲線為垂直線，需求彈性等於零。

(5)不管價格為何，X 財貨支出固定不變，X 財貨需求曲線為雙曲線，需求彈性等於一。

73.Suppose there just two goods, X and Y. If X accounts for 75% of the budget and has income elasticity $\eta_X = 0.8$ ，η_Y for Y?　【台科大企研所】

解：利用恩格爾加總定律，$\eta_Y = 1.6$。

74.設某消費者效用函數為 $U=XY$，且已知 $P_X=8$，$P_Y=2$，且 M 為所得

(1)X 和 Y 是否可以同時為劣等財？為什麼？

(2)導出 X 的 Engel 曲線。【淡江企研所】

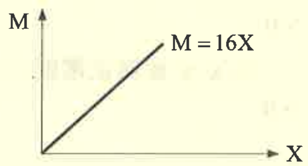
解：(1)社會上不可能二種財貨全為劣等財，利用 Engel 加總定律說明：$\varepsilon_M^x\alpha_X+\varepsilon_M^Y\alpha_Y=1$

若 X、Y 皆為劣等財,則 $\varepsilon_M^X<0$，$\varepsilon_M^Y<0$，而 α_X，α_Y 必大於零。加總之總和為負值，不可能等於 1⇒ X、Y 不可能同時為劣等財⇒ 若其中一個財貨為劣等財，則另一財貨必為奢侈品。

(2) $\boxed{\begin{array}{ll}\text{Max} & U=XY \\ \text{s.t} & M=8X+2Y\end{array}}$ ⇒利用消費者均衡條件 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}\;\dfrac{Y}{X}=\dfrac{8}{2}$ $\Rightarrow Y=4X$

代回所得限制式　$M=8X+8X$　$\therefore\ X=\dfrac{M}{16}$　X 財 Engel Curve 方程式為 $M=16X$，

Engel's Curve 為過原點直線⇒ $\varepsilon_M^X=1$



題型：Stone-Geary 效用函數

75.設效用函數為 Stone-Geary 的形式 $U=(X-\overline{X})^{\alpha}(Y-\overline{Y})^{1-\alpha}$，$1>\alpha>0$，$\overline{X}>0$，$\overline{Y}>0$

(1)求所得消費函數，並繪其圖形。

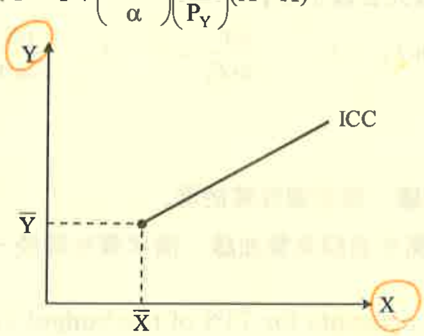
(2)設 $\alpha=\dfrac{1}{4}$，$P_X=1$，$P_Y=1$，$\overline{X}=2$，$\overline{Y}=2$，$I=10$，繪出 X 財貨所對應的 Engel 曲線。【淡江財金所】

解：(1)求 ICC 方程式，滿足 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}$ 軌跡線

$$\frac{\alpha(X-\overline{X})^{\alpha-1}(Y-\overline{Y})^{1-\alpha}}{(1-\alpha)(X-\overline{X})^{\alpha}(Y-\overline{Y})^{-\alpha}}=\frac{P_X}{P_Y}$$

$$\frac{\alpha(Y-\overline{Y})}{(1-\alpha)(X-\overline{X})}=\left(\frac{P_X}{P_Y}\right)\Rightarrow Y-\overline{Y}=\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{P_X}{P_Y}\right)(X-\overline{X})$$

$$\therefore Y=\overline{Y}+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{P_X}{P_Y}\right)(X-\overline{X})$$



$$\frac{MU_X}{MU_Y}=\frac{P_X}{P_Y}$$

$$\frac{\alpha(Y-\overline{Y})}{(1-\alpha)(X-\overline{X})}=\frac{P_X}{P_Y}$$

$$\therefore (Y-\overline{Y})=\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{P_X}{P_Y}\right)(X-\overline{X})$$

$$Y=\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{P_X}{P_Y}\right)(X-\overline{X})+\overline{Y}$$

$$\boxed{\begin{array}{ll}(2)\ Max & U=(X-\overline{X})^{\frac{1}{4}}(Y-\overline{Y})^{\frac{3}{4}} \\ s.t & X+Y=I\end{array}}$$

$\frac{MU_X}{MU_Y}=\frac{P_X}{P_Y}=\frac{1}{1}$

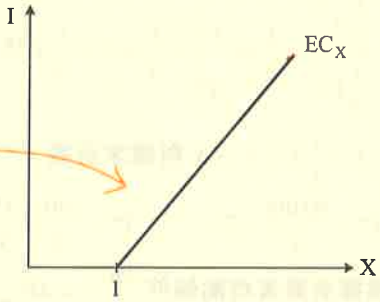
$$\frac{MU_X}{MU_Y}=\frac{P_X}{P_Y}\Rightarrow\frac{\frac{1}{4}(X-\overline{X})^{\frac{-3}{4}}(Y-\overline{Y})^{\frac{3}{4}}}{\frac{3}{4}(X-\overline{X})^{\frac{1}{4}}(Y-\overline{Y})^{\frac{-1}{4}}}=\frac{P_X}{P_Y}$$

$$\frac{1(Y-\overline{Y})}{3(X-\overline{X})}=\frac{P_X}{P_Y}\Rightarrow\frac{Y-2}{3(X-2)}=1$$

$\Rightarrow Y=2+3X-6$ 代回限制式

$X+(2+3X-6)=I\Rightarrow 4X=I+4$

$\Rightarrow X^*=\frac{I+4}{4}$ ……X 財恩格爾曲線



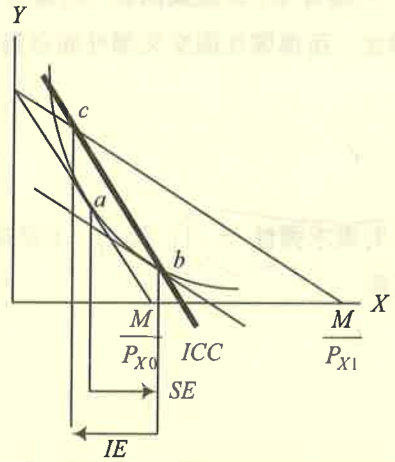
76.如果某消費者只消費 X、Y 兩種財貨。

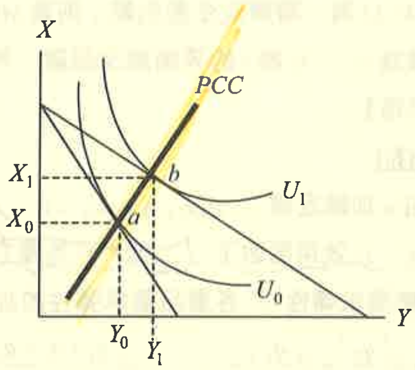
(1)請在 X 為季芬財之假設下，繪出該消費者之所得消費曲線。

(2)請在 X、Y 均為正常財，而且 X、Y 互為互補品之假設下，繪出 Y 價格變動時該消費者之價格消費曲線。　　　　【96 淡江管科所】

解：

X 財價格下跌時：　　　　(2) Y 財價格下跌





77.Kelly has the following utility function: $U(X,Y)=\sqrt{X}+\sqrt{Y}$, where X is her consumption of CDs, with price $P_X=\$1$, and Y is her consumption of DVDs, with $P_Y=\$3$.

(1) Derive Kelly's demand for DVDs and CDs.

(2) Assume that her income $I=\$100$. How many DVDs and CDs will Kelly consume?

(3) What is the marginal utility of income? 【96 海洋應經所】

解：(1)

$U(X,Y)=\sqrt{X}+\sqrt{Y}$

$MU_X=\frac{1}{2}X^{-\frac{1}{2}}$，$MU_Y=\frac{1}{2}Y^{-\frac{1}{2}}\Rightarrow\frac{MU_X}{MU_Y}=\frac{P_X}{P_Y}=\left(\frac{X}{Y}\right)^{-\frac{1}{2}}=\frac{1}{3}$

$X=9Y$

$X+3Y=I$，$9Y+3Y=I$，$12Y=I$，$Y=\frac{I}{12}$

$X+\frac{1}{3}X=I$，$X=\frac{3}{4}I$

(CD) $X=75$

(DVD) $Y=\frac{100}{12}=\frac{25}{3}$

$MU_m=\frac{MU_X}{P_X}=\frac{\frac{1}{2}X^{-\frac{1}{2}}}{1}$

$$\boxed{\begin{array}{l}\text{Max} \;\; U=\sqrt{X}+\sqrt{Y} \\ \text{s.t} \;\; X+3Y=I\end{array}}$$

利用消費者均衡條件 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}$ 求解

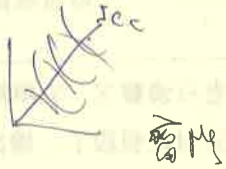
$$\frac{\frac{1}{2}X^{\frac{-1}{2}}}{\frac{1}{2}Y^{\frac{-1}{2}}}=\frac{1}{3}\Rightarrow\frac{Y^{\frac{1}{2}}}{X^{\frac{1}{2}}}=\frac{1}{3}\Rightarrow Y=\frac{1}{9}X \text{ 代回 } X+3Y=I$$

$X+\dfrac{1}{3}X=I\Rightarrow X$ 財需求函數 $X^M=\dfrac{3I}{4}$ ；$Y^M=\dfrac{I}{12}$

(2) $X^*=\dfrac{3(100)}{4}=75$ $\qquad Y^*=\dfrac{100}{12}=\dfrac{25}{3}$

(3) 根據消費者均衡條件 $\dfrac{MU_X}{P_X}=MU_m$ 求解

$$MU_m=\frac{MU_X}{P_X}=\frac{\frac{1}{2}X^{\frac{-1}{2}}}{P_X}=\frac{\frac{1}{2}(75)^{\frac{-1}{2}}}{1}=\frac{1}{2\sqrt{75}}=\frac{1}{10\sqrt{3}}$$



78.【複選題】下列敘述何者為錯誤？ (A)消費者所消費之各種財貨的所得彈性以各財貨支出佔所得比例為加權之平權和一定為 1 (B)以 Cobb-Douglas 效用函數所導出的 X 財貨的需求函數，其交叉彈性一定為零(C)若市場中每位消費者的需求彈性皆為 ε，則市場需求彈性亦為 ε (D)當 $F(K,L)$ 為一階齊次生產函數，則當 MP_L 為正時，則隱含 AP_K 為遞減函數 (E)當 X 財貨的需求函數為 P_X、P_Y 與 I 的零階齊次函數，則其價格彈性、所得彈性與交叉彈性和必為 1【95 北大經研所】

解：**(C)(E)**

(A)Engel's 加總定律： $\varepsilon_M^X\alpha_X+\varepsilon_M^Y\alpha_Y=1$

why? U=X·Y

(B) $C-D$ 效用函數下，X 與 Y 必為獨立品 $\Rightarrow$ X 財需求彈性 $=-1$，而 $\varepsilon_{XY}=0$ 必成立

(C)市場總需求彈性 = 各產品需求彈性的加權平均總和 ?

$$=\varepsilon\left(\frac{q_1}{Q}\right)+\varepsilon\left(\frac{q_2}{Q}\right)+\varepsilon\left(\frac{q_3}{Q}\right)+\cdots \qquad =\varepsilon\left[\frac{q_1+q_2+q_3+\cdots}{Q}\right]=\varepsilon$$

(D) $Q=f(L,K)$ 為 H. O. D. 1

$$\xrightarrow{\text{尤拉定理}} MPP_L L+MPP_K K=Q$$

$$MPP_L\frac{L}{K}+MPP_K=APP_K$$

$$MPP_L=\frac{K}{L}(APP_K-MPP_K)$$

當 $MPP_L>0\Rightarrow APP_K>MPP_K\Rightarrow$ 此時 APP_K 為遞減函數

(E)普通需求函數為零階齊次函數： $\varepsilon^d+\varepsilon_{XY}+\varepsilon_M^X=0$，因此價格彈性、所得彈性與交叉彈性和必為 0。

79.Ms. Caffeine enjoys coffee (C) and tea (T) according to the utility function $U(C,T)=3C+4T$

(1)What does her utility function say about her marginal rate of substitution (MRS) of coffee for tea?

(2)What do her indifference curve look like?

(3)If coffee and tea cost $3 each and Ms. Caffeine has $12 to spend on these products, how much coffee and tea should she buy to maximize her utility?

(4)How would consumption change if the price of coffee fell to $2? 【政大風險管理】

解：

$\frac{12}{2}=6$

(1) $U=3C+4T$， $MRS_{CT}=\frac{3}{4}$

(1) $\frac{MU_{X(C)}}{MU_{Y(T)}}=MRS_{CT}=\frac{3}{4}$

(2)無異曲線為直線，表示咖啡(C)與(T)為完全替代品

(2) $MRS_{CT}=\frac{3}{4}$ 完全替代(直線)

(3)

$$\boxed{\begin{array}{l}\text{Max} \quad U=3C+4T \\ \text{s.t} \quad 3C+3T=12\end{array}}$$

$C^*=0$， $T^*=4$

(3)(4) 皆為角隅解

(4)

$$\boxed{\begin{array}{l}\text{Max} \quad U=3C+4T \\ \text{s.t} \quad 2C+3T=12\end{array}}$$

$C^*=6$， $T^*=0$，當咖啡價格從 $P_T=3$ 下跌至 2 元，將使咖啡購買量增加 6 單位

80.【是非題】According to the substitution effect along an indifference curve, when the relative price of a good falls, the consumer always substitutes more of that good for the other good.【95 成大企研所】

解：不一定；若無異曲線凸向原點負斜率，滿足 MRS_{XY} 遞減條件下，則 SE 恆為負必定成立，即財貨相對價格下跌時，消費者必定多買便宜財貨來替代其它的財貨，然而無異曲線為直角形，消費者偏好為「完全互補」效用函數，則 $SE=0$，當財貨相對價格下跌時，需求量仍不變。

81.Allen consumes only two goods, X and Y . His utility function can be written as $u(x,y)=\min\{(x+2y)^2, (3x+y)^2\}$, where x and y represent the consumption amounts of X and Y, respectively. x and y can be any non-negative real numbers. Which of the following is (are) true? (A)X and Y are prefect complements to Allen. (B)Allen's preference over X and Y is strictly convex. (C)Every indifference curve of Allen is a straight line. (D)Allen's preference is monotonic. (E)None of the above.【96 台大經研所】

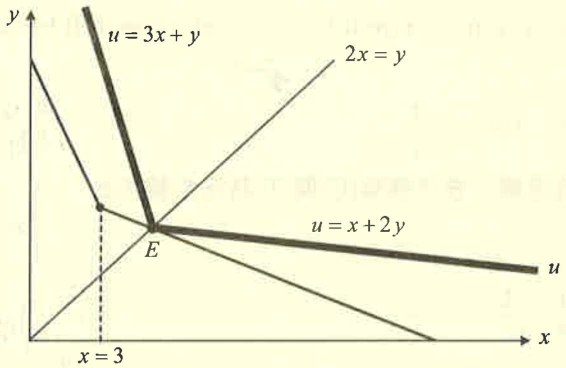
解：(D)；本題效用函數經由單調遞增轉換，等於效用函數 $u=\min(x+2y;3x+y)$ 為拗折型無異曲線，此種無異曲線的形狀不是嚴格凸性偏好(strictly convex)而是弱凸性偏好。

82.Continue from above. The unit price of X is $2 for the first three units and $1.5 for each additional unit of purchase. In other words, the payment of buying x units is

$$\begin{cases} 2x, & \text{if } x \le 3 \\ 6+1.5(x-3), & \text{if } x>3 \end{cases}$$

The unit price of Y is $1 regardless the units of purchase. Allen spends all of his income to buy these two goods. Which of the following is (are) true? (A)To maximize his utility, Allen will purchase 4 units of x if his income is \$15.5. (B)To maximize his utility, Allen will buy 5 units of Y if his income is \$10. (C)Allen will buy 3 units of X at several different income levels. (D)The slope of Allen's Engel curve of the consumption on X is discontinuous at the income level \$12. (E)None of the above. 【96 台大經研所】

解：(A)(B)；本題預算限制條件$\begin{cases} 若x \le 3\ ;\ 2x+y=m \\ 若x>3\ ;\ 1.5x+y=m-1.5 \end{cases}$



當$x=3$且由$2x+y=m$與$2x=y$聯立求解可得$m=12$，因此

❶ 當$m=12$時，效用最大均衡解恰好位於預算線拗折點上

❷ 當$m>12$時，效用最大均衡解恰好位於預算限制式$1.5x+y=m-1.5$與$2x=y$的聯立

❸ 當$m<12$時，效用最大均衡解恰好位於預算限制式$2x+y=m$與$2x=y$的聯立

(A)若$m=15.5 \Rightarrow 1.5x+y=14$與$2x=y$聯立求解得$x^*=4$。

(B)若$m=10 \Rightarrow 2x+y=10$與$2x=y$聯立求解可得$y^*=5$。

(C)當所得不同時，x 財最適消費量亦不同。

(D)當$m<12 \Rightarrow$x 財 Engel Curve 方程式$x=\dfrac{m}{4}$，$m=12$，$x=3$

當$m>12 \Rightarrow$x 財 Engel Curve 方程式$x=\dfrac{2m-3}{7}$，

由以上可知恩格爾曲線在$x=3$時呈現拗折狀，因此在$x=3$時，恩格爾曲線為連續不可微函數。

83.An inferior good is a good with negative income elasticity. A normal good is a good with positive income elasticity. A luxury is a good with income elasticity greater than one. Which of the following is (are) true? (A)It is possible that all of the goods you consume are inferior goods. (B)It is possible that all of the goods you consume are normal goods. (C)It is possible that all of the goods you consume are luxuries. (D)If the demand curve for a good is upward-sloping everywhere, the good must be a normal good. (E)None of the above. 【96 台大經研所】

解：(B)；(A)錯誤。根據恩格爾加總定理，不可能所有財貨皆為劣等財。 (B)對。有可能兩財皆為正常財。 (C)錯誤。根據恩格爾加總定理，不可能所有財貨皆為奢侈品。 (D)錯誤。若需求曲線正斜率，表示此財貨為 Giffen Goods，為 Inferior Goods 一種。

84.Suppose that ham and cheese are pure compliments—the will always be used in the ratio of one slice of ham on one slice of cheese to make a sandwich. Suppose also that

ham and cheese are the only goods that a consumer can buy and that bread is free. Assume that price of a slice of ham equals the price of a slice of cheese.
(1)Derive the demand functions of ham and cheese.
(2)Calculate the own-price elasticity of demand for ham and the cross-price elasticity of a change in the price of cheese on ham consumption. 【95 政大國貿所】

解：

(1)

$$\boxed{\begin{aligned}&\text{Max}\quad U=\min(X,Y)\\&\text{s.t}\quad M=P_XX+P_YY\end{aligned}}$$

均衡時 $\begin{cases}X=Y\\M=P_XX+P_YY\end{cases}\Rightarrow X^M=\dfrac{M}{P_X+P_Y}$ ； $Y^M=\dfrac{M}{P_X+P_Y}$

(2) $\varepsilon^d=\dfrac{-\partial X}{\partial P_X}\dfrac{P_X}{X}=\dfrac{M}{(P_X+P_Y)^2}\dfrac{P_X}{\dfrac{M}{P_X}+P_Y}=\dfrac{P_X}{(P_X+P_Y)}<1$

在 $P_X=P_Y$ 下，$\varepsilon^d=\dfrac{1}{2}$

$\varepsilon_{XY}=\dfrac{\partial X}{\partial P_Y}\dfrac{P_Y}{X}=\dfrac{-M}{(P_X+P_Y)}\cdot\dfrac{P_Y}{\dfrac{M}{P_X+P_Y}}=\dfrac{-P_Y}{(P_X+P_Y)}<0$

在 $P_X=P_Y$ 下，$\varepsilon_{XY}=\dfrac{-1}{2}<0\;\Rightarrow$ 表示 X 與 Y 為互補品

85.請回答下列問題：
(1)假若某甲只購買 X 與 Y 兩種財貨，已知其購買 X 財貨的支出佔所得的 40%，且 X 財貨的所得彈性為 0.25，試求 Y 財貨的所得彈性。 Y 60%
(2)承(1)，若 X 財貨的價格彈性為 0.8，試求 X 與 Y 財貨的交叉價格彈性。【95 中興應經所】

解：

(1)根據 Engel's Aggregate 定理： ← ICC

$\varepsilon_M^X\alpha_X+\varepsilon_M^Y\alpha_Y=1$

$0.4(0.25)+0.6(\varepsilon_M^Y)=1$

$\therefore\varepsilon_M^Y=1.5$

(2)根據 Coumot's Aggregate Condition：

$-\alpha_X=\varepsilon_{XX}\alpha_X+\varepsilon_{YX}\alpha_Y$

$\alpha_X=|\varepsilon_{XX}|\alpha_X-\varepsilon_{YX}\alpha_Y$

$\varepsilon_{YX}(0.6)=(0.8-1)(0.4)$

$\therefore\varepsilon_{YX}=-0.13$

0.4 0.2 0.6 $\alpha_X(1-|\varepsilon_X|)+\alpha_Y\varepsilon_{YX}$

86.若效用函數為 $U=XY$，預算線為 $100=10X+5Y$，則當 X 財貨價格下降為 5 時，請問下述何項答案為正確？ (A)以 Slutsky 定義，替代效果為 2.5。 (B)以 Slutsky 定義，所得效果為 2.5。(C)以 Hicks 定義，替代效果為 $\sqrt{50}$。 (D)以 Hicks 定義，所得效果為 $10-\sqrt{50}$。(E)價格效果為 5。(2 分)【96 台北經研所】

解：(A)(B)(D)(E)

普通需求函數：$X^M=\dfrac{M}{2P_X}$；$Y^M=\dfrac{M}{2P_Y}$

受補償需求函數：$X^H=\sqrt{\dfrac{P_Y}{P_X}U}$；$Y^H=\sqrt{\dfrac{P_X}{P_Y}U}$

原均衡：$X=5$，$Y=10$，$U_0=50$，新均衡：$X=10$，$Y=10$，$U_1=100$

Hicks 定義：$X^H=\sqrt{\dfrac{5}{5}\times 50}=5\sqrt{2}$，$SE_H=(5\sqrt{2}-5)$，$IE_H=(10-5\sqrt{2})$

Slutsky 定義：$CD=(10-5)5=25$，$X^S=\dfrac{75}{2(5)}=7.5$

$SE_S=7.5-5=2.5$，$IE_S=10-7.5=2.5$

$X^H=\sqrt{\frac{P_Y}{P_X}U}$ $\sqrt{\frac{4}{P_X}\times 200}$

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要多練習

57.若效用函數為$U=XY$，預算線為$160=8X+4Y$，請問下述何項答案為正確？ (A)X 財貨的 Marshall 需求函數為$X=\dfrac{80}{P_X}$。 (B)X 財貨的 Hick 需求函數為$X=\sqrt{\dfrac{800}{P_X}}$。 (C)X 財貨的 Slutsky 需求函數為$X=5+\dfrac{40}{P_X}$。 (D)X 財貨的 Marshall 需求函數之價格彈性為-1。 (E)X 財貨的 Hick 需求函數之價格彈性為-0.5。(2 分)。【96 北大經研所】

解：(A)(B)(C)(D)(E)

(A) $X^M=\dfrac{80}{P_X}$；$Y^M=\dfrac{80}{P_Y}$，$X_0=10$，$Y_0=20$，$U_0=200$

(B) $X^H=\sqrt{\dfrac{P_Y}{P_X}U_0}=\sqrt{\dfrac{800}{P_X}}\Rightarrow X^H=800^{\frac{1}{2}}\cdot P_X^{\frac{-1}{2}}$，受補償需求之價格彈性為$\dfrac{-1}{2}$

(C)Slutsky 需求函數：$\left\{\begin{matrix} Max\ U=XY \\ s.t\quad P_XX+4Y=10P_X+80\end{matrix}\right\}$ $X^S=5+\dfrac{40}{P_X}$

key 分配原則

88.當小丁消費 X, Y 及 Z 三種財貨分別為 x, y, 及 z 單位時，小丁所獲得的效用為 $U(x,y,z)=Ax^ay^bz^c$，令 p, q 及 r 分別代表 X, Y 及 Z 三種財貨的價格，而 m 為小丁的所得。假設小丁追求效用最大，且參數 A, a, b, c, p, q, r 及 m 均為正實數。

(1) 小丁對 X, Y 及 Z 三種財貨的需求函數依序為 ___(3)___ 。

(2) 在參數 a, b, c 滿足條件 ___(4)___ 時，第(1)小題的答案為最適解。

(3) 如第(1)小題。小丁的間接效用函數為 ___(5)___ 。【97 北大企研所】

解：(1) $x^M=\dfrac{am}{(a+b+c)p}$；$y^M=\dfrac{bm}{(a+b+c)q}$；$z^M=\dfrac{cm}{(a+b+c)r}$

(2) $a>0$；$b>0$；$c>0$

(3)間接效用函數$U=A\cdot\left(\dfrac{a}{a+b+c}\right)^a\left(\dfrac{b}{a+b+c}\right)^b\left(\dfrac{c}{a+b+c}\right)^c p^{-a}q^{-b}r^{-c}m^{a+b+c}$

89.A consumer has preferences over the single good x and all other goods m represented by the utility function, $u(x,m)=\ln(2x)+m$. Let the price of x of p, the price of m be unity, and let income be y. Fill in the answer to the following questions. (1)The Marshallian demand function for x is ______.(3 分) (2)The Marshallian demand function for m is ______ .(3 分) (3)The indirect utility function $v(p,y)$ is ______ .(2 分)【99.96 中正國經所】

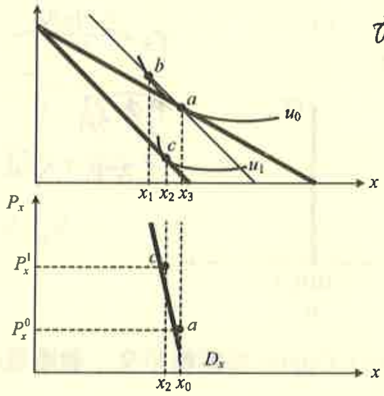
解：

(1)效用極大化模型 $\begin{cases} Max\ U = \ln 2x + m \\ s.t\ \ Px + m = y \end{cases}$ Marshallian 需求函數為 $x = \dfrac{1}{p}$

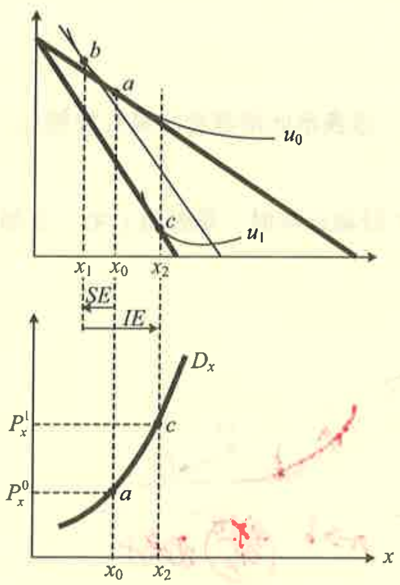
(2) $m = y - 1$　　　(3)間接效用函數 $u = \ln\left(\dfrac{2}{p}\right) + y - 1$

90.消費者消費兩種財貨，其中劣等財價格上升，產生替代效果與所得效果，試導出劣等財之需求曲線與價格變動之效果。【96 高雄第一科大行銷所】

Case1：$|SE| > |IE|$ 劣等財

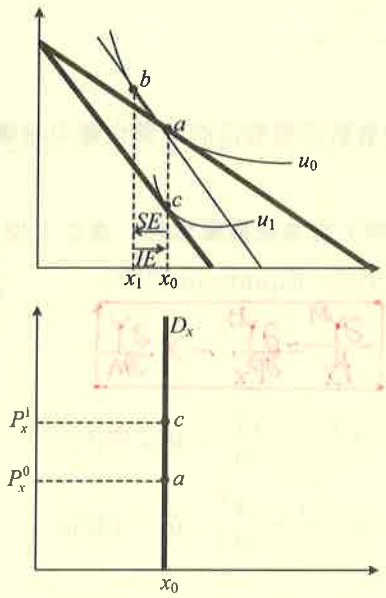


case 2：$|SE| < |IE|$ 劣等財



季芬財違反需求法則，D_X 為正斜率

case3：$|SE| = |IE|$ 劣等財



D_X 為垂直線

91.某人消費 X 和 Y，效用函數為 $U(X,Y)=\ln X+0.1Y$。若此人的所得為 200，商品 X 和 Y 的單價為 P_X 和 P_Y。

①【直接法】(1) 導出此此人對商品 X 的需求函數。

②預算式 (2) 畫出此人對 Y 的恩格爾曲線，並清楚標示截距與斜率的相關資訊。【96 淡江經研所】

③1⇒ $\boxed{\varepsilon_M^X d_X + d_Y \varepsilon_M^Y = 1}$

解：(1)利用消費者均衡條件 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}$ 求解

$$\frac{\frac{1}{X}}{0.1}=\frac{P_X}{P_Y}\Rightarrow X^M=\frac{10P_Y}{P_X}$$

(2)將 X 財需求量代回預算線 $M=P_XX+P_YY$

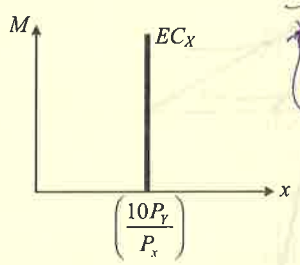
$$M=P_X\left(\frac{10P_Y}{P_X}\right)+P_YY$$

$\therefore Y$ 財需求函數：$Y^M=\dfrac{M-10P_Y}{P_Y}=\dfrac{M}{P_Y}-10$

(3) X 財為中性財 → 保證 Y 財必為奢侈品

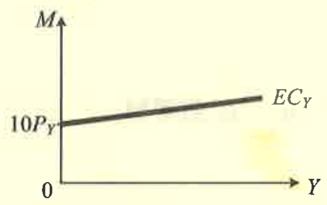
X 財 Engel's Curve 為垂直線

$\therefore \varepsilon_M^X=0$



直接預算式可得

$\boxed{M=P_YY+10P_Y}$



Y 財 Engel's Curve 和縱軸相交，截距項($10P_Y$)

Y 財為奢侈品，$\varepsilon_M^Y=>1$。

(1) $\begin{cases}\max U(x,y)=\ln x+0.1Y\\ 200=P_XX+P_YY\end{cases}$

$\dfrac{MU_X}{MU_Y}=\dfrac{\frac{1}{x}}{\frac{1}{10}}=\dfrac{P_X}{P_Y}\Rightarrow \boxed{X=\dfrac{10P_Y}{P_X}}$

$200=10P_Y+P_YY$

$\boxed{Y=\dfrac{200-10P_Y}{P_Y}}$

要記，X財貨支出的錢固定 $\boxed{P_XX}$

(2) $Y=\dfrac{I-10P_Y}{P_Y}=\dfrac{I}{P_Y}-10$

$\dfrac{\partial Y}{\partial M}>0$

又 $\alpha_X\varepsilon_M^X+\alpha_Y\varepsilon_M^Y=1$

$\therefore \varepsilon_M^Y>1$ (奢侈品)

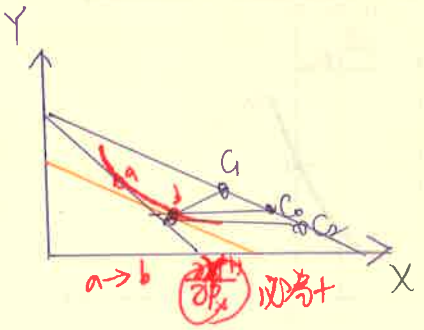
92.當所得增加時，消費者對兩種物品的消費也會同時增加，這表示此兩種物品是互換補品。【元智財金所】

解：當所得增加時，X 與 Y 財消費量會增加，表示 X 財與 Y 財為正常財，是否為（毛）互補品關係，利用交叉項 Slutsky Equatioin 判斷：

$$\underset{?}{\frac{\partial Y^M}{\partial P_X}}=\underset{(+)}{\frac{\partial Y^H}{\partial P_X}}-X\underset{(+)}{\frac{\partial Y}{\partial M}}$$

$\boxed{\dfrac{\partial Y^M}{\partial P_X}=\dfrac{\partial Y^H}{\partial P_X}-X\dfrac{\partial Y}{\partial M}}$

當 Y 為正常財時 $\begin{cases}\left|\dfrac{\partial Y^H}{\partial P_X}\right|>\left|X\dfrac{\partial Y}{\partial M}\right|\Rightarrow\dfrac{\partial Y^M}{\partial P_X}>0\Rightarrow 替代品\\ \left|\dfrac{\partial Y^H}{\partial P_X}\right|<\left|X\dfrac{\partial Y}{\partial M}\right|\Rightarrow\dfrac{\partial Y^M}{\partial P_X}<0\Rightarrow 互補品\end{cases}$



a→b $\dfrac{\partial Y^H}{\partial P_X}$ 必為+

93.【單選題】假設小王對財貨 X 與 Y 的效用函數為 $U(X,Y)=\ln(X)+3Y$，財貨 X 與 Y 的價格

分別為 6 元與 9 元，所得為 120 元，則下列何者為正確？ (A)小王對 X 財貨的均衡需求量為 2 (B)小王對 X 財貨的需求不具所得效果 (C)當財貨 X 的價格由 6 元下降為 3 元時，X 財貨的替代效果為 0.5，所得效果亦為 0.5 (D)當小王的所得上升為 240 元時，小王對 X 財貨的均衡需求量會上升為 4 (E)以上皆正確　　【95 北大經研所】

解：**(B)**

(A)利用 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}$ 均衡條件求解：$\dfrac{\frac{1}{X}}{3}=\dfrac{6}{9}\Rightarrow X^*=\dfrac{1}{2}$ 代回預算線：$120=6X+9Y$

$\therefore Y^*=\dfrac{120-3}{9}=\dfrac{117}{9}=13$

(B)準線性偏好，X 財為中性財，$IE=0$

(C) X 財需求函數 $X^M=\dfrac{P_Y}{3P_X}$

原均衡　　　　新均衡

$X_0=\dfrac{9}{3(6)}=0.5$　　$X_1=\dfrac{9}{3(3)}=1$

$\therefore IE=0,\ SE=PE=0.5$

(D) （$\because X$ 為中性財），表示所得增加，X 財需求量不受影響。

94.Assume that a utility function is given by $U=\ln X+Y$. What is the Slutsky equation that decomposes the change in the demand for X in response to a change in its price? What is the income effect? What is the substitution effect?【逢甲財金所】

解：$\begin{cases}\text{Max } U=\ln X+Y\\ \text{s.t}\quad P_X X+P_Y Y=M\end{cases}$

$\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}\Rightarrow\dfrac{1}{X}=\left(\dfrac{P_X}{P_Y}\right)\Rightarrow X^M=\left(\dfrac{P_Y}{P_X}\right),\quad Y^M=\dfrac{M-P_Y}{P_Y}$

(1)所得效果 $-X\dfrac{\partial X}{\partial M}=-\left(\dfrac{P_Y}{P_X}\right)(0)=0$（$\because X$ 為中性財）

(2)替代效果 $=\dfrac{\partial X^H}{\partial P_X}=\dfrac{\partial X^M}{\partial P_X}=\dfrac{-P_Y}{P_X^2}<0$

95.設間接效用函數為 $V(P_X,P_Y,I)=\dfrac{I}{P_Y}+\dfrac{P_Y}{4P_X}$ 利用 Roy 恆等式求普通需求函數 $X(P_X,P_Y,\text{I})=?$

【政大經研所】

解：$U=\dfrac{I}{P_Y}+\dfrac{P_Y}{4P_X}$

Roy's identity, $X^M=\dfrac{\frac{-\partial U}{\partial P_X}}{\frac{\partial U}{\partial I}}=\dfrac{-\left(\frac{P_Y}{4P_X^2}\right)}{\frac{1}{P_Y}}=\dfrac{P_Y^2}{4P_X^2}$

96.【是非題】The indirect utility function of a consumer for a two commodities, X and Y, would is given by $V = M(0.5/P_X)^{\frac{1}{2}}(0.5/P_Y)^{\frac{1}{2}}$, for V is the indirect utility, M is money income, P_X and P_Y are price of the commodities, X and Y. Then the Marshallian demand function for X is $Q_X^d = \dfrac{M}{P_X}$. 【中原國貿所】

解：錯誤。$U = \frac{1}{2}P_X^{\frac{-1}{2}}P_Y^{\frac{-1}{2}}M$

$$\frac{\partial U}{\partial P_X} = -P_X^{\frac{-3}{2}}P_Y^{\frac{-1}{2}}M \qquad \frac{\partial U}{\partial M} = \frac{1}{2}P_X^{\frac{-1}{2}}P_Y^{\frac{-1}{2}}$$

根據 Roy's identity 可知：$X^M = \dfrac{-\frac{\partial U}{\partial P_X}}{\frac{\partial U}{\partial M}}$ 　 $\therefore X^M = \dfrac{P_X^{\frac{-3}{2}}P_Y^{\frac{-1}{2}}M}{\frac{1}{2}P_X^{\frac{-1}{2}}P_Y^{\frac{-1}{2}}} = \dfrac{2M}{P_X}$

相差兩倍

97.Assume that the utility is given by $U(x, y) = x^a y^b$

(1)Find the uncompensated (Marshallian) demand for good x and y.

(2)Find the marginal utility of income. $MU_m = \frac{MUx}{x} = \frac{MUy}{y}$

(3)Find the compensated (Hicksian) demand for good x and y.【95 成大經研所】

解：(1)

$$\boxed{\begin{array}{l}\text{Max} \quad U = X^a Y^b \\ \text{s.t} \quad M = P_X X + P_Y Y\end{array}}$$

$$X^M = \frac{aM}{(a+b)P_X}\ ;\ Y^M = \frac{bM}{(a+b)P_Y}$$

(判斷為Cobb-Douglas)

(2)利用消費者均衡條件 $\dfrac{MU_X}{P_X} = MU_m$ 求解

$a \cdot a^{(a-1)} = a^a$ 同底相乘指數相加

$$MU_m = \frac{aX^{a-1}Y^b}{P_X} = \frac{a\left[\frac{aM}{(a+b)P_X}\right]^{a-1}\left[\frac{bM}{(a+b)P_Y}\right]^b}{P_X}$$

$$\therefore MU_m = a^{2a-1}b^b(a+b)^{-a-b+1}P_X^{-a}P_Y^{-b}M^{a+b-1}$$

?

$MU_m = \frac{MU_Y}{P_Y} = \frac{b\,x^a y^{b-1}}{P_Y}$

$\frac{b\left[\frac{aM}{(a+b)P_X}\right]^a\left[\frac{bM}{(a+b)P_Y}\right]^{b-1}}{P_Y}$

$\therefore MU_m = a^a b^b (a+b)^{-a-b+1} P_X^{-a} P_Y^{-b} M^{a+b-1}$

$MU_m = \left[\frac{a^a b^b}{(a+b)^{a+b-1}}\right] P_X^{-a} P_Y^{-b} M^{a+b-1}$

(3)

$$\boxed{\begin{array}{l}\text{Min} \quad E = P_X X + P_Y Y \\ \text{s.t} \quad U = X^a Y^b\end{array}}$$

利用消費者均衡條件 $\dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y}$ 求解

$$\frac{aY}{bX} = \frac{P_X}{P_Y} \Rightarrow Y = \left(\frac{bP_X}{aP_Y}\right)X \text{ 代回效用函數}$$

$\Rightarrow$ $P_X X : P_Y Y = a : b$ 直接拆且效用函數 $aP_Y Y = bP_X X$

$$U = X^a\left(\frac{bP_X}{aP_Y}\right)^b X^b \qquad U = X^{a+b}\left(\frac{bP_X}{aP_Y}\right)^b$$

$U = \left(\frac{bP_X}{aP_Y}\right)^b \cdot X^{a+b}$

$$X^H = \left(\frac{aP_Y}{bP_X}\right)^{\frac{b}{a+b}} U^{\frac{1}{a+b}} \qquad Y^H = \left(\frac{bP_X}{aP_Y}\right)^{\frac{a}{a+b}} U^{\frac{1}{a+b}}$$

$X^H = \sqrt[a+b]{\left(\frac{aP_Y}{bP_X}\right)^b \cdot U}$

98.設某乙消費 X 與 Y 兩種財貨的效用函數為：$U = AX^\alpha Y^\beta$，其中 A、α、β 為參數；又其預

$U=AX^{\alpha}Y^{\beta}$ （拉格朗日）

算限制式為：$P_XX+P_YY=M$，P_X、P_Y 分別表示 X 與 Y 兩種財貨的價格，M 表示所得。

(1)試求 X 與 Y 兩種財貨的需求函數。

(2)已知 $A=\alpha=\beta=1$、$M=100$、$P_Y=1$，若 X 財貨價格由 $P_X=2$ 降為 $P_X=1$，試依 Hicks 的實質所得概念，求 X 財貨價格變動的替代效果與所得效果。

(3)承(2)，試求其補償變量(compensating variation)。【95 中興應經所】

原均衡	$U=25\sqrt{2}\times25\sqrt{2}$	新
$X_0^M=25$	$X=\sqrt{\frac{P_Y}{P_X}U}=25\sqrt{2}$	$X_{new}=50$
$Y_0=50$	$Y=\sqrt{\frac{P_X}{P_Y}U}=25\sqrt{2}$	$Y_{new}=50$
$SE_X^H=25\sqrt{2}-25$, $SE_Y^H=25\sqrt{2}-50$	$IE_X^H=50-25\sqrt{2}$, $IE_Y^H=50-25\sqrt{2}$	

解：(1)　Marshallian demand function

key $X^M=\dfrac{\alpha M}{(\alpha+\beta)P_X}$；$Y^M=\dfrac{\beta M}{(\alpha+\beta)P_Y}$

(2)　Hicksian demand function

key

$$\boxed{\begin{aligned}&\text{Max}\ \ E=P_XX+P_YY\\&\text{s.t}\ \ U=XY\end{aligned}}$$

$$X^H=\sqrt{\frac{P_Y}{P_X}U}\text{；}Y^H=\sqrt{\frac{P_X}{P_Y}U}$$

支出 $25\times2+1\times50=100$　　支出 $50\sqrt{2}$　　支出 100 $50\times1+50\times1=100$

原均衡 a 點

$(P_X,P_Y,M)=(2,1,100)$

$X_0=\dfrac{100}{2(2)}=25$

$Y_0=\dfrac{100}{2(1)}=50$

$U_0=1250$

支出極小需求量 b 點

$(P_X',P_Y,U_0)=(1,1,1250)$

$X_1=\sqrt{\dfrac{1}{1}(1250)}=25\sqrt{2}$

$Y_1=\sqrt{\dfrac{1}{1}(1250)}=25\sqrt{2}$

支出 $=25\sqrt{2}+25\sqrt{2}=50\sqrt{2}$

$CV=50\sqrt{2}-100$

新均衡 c 點

$(P_X',P_Y,M)=(1,1,100)$

$X_2=\dfrac{100}{2(1)}=50$

$Y_2=\dfrac{100}{2(1)}=50$

支出 $25\sqrt{2}+25\sqrt{2}=50\sqrt{2}$

$SE_H(a\to b)=25\sqrt{2}-25$

$IE_H(b\to c)=50-25\sqrt{2}$

(3) $CV=\int_2^1\sqrt{\dfrac{P_Y}{P_X}U_0}\,dP_X=\int_2^1\sqrt{\dfrac{1}{P_X}(1250)}\,dP_X=25\sqrt{2}\int_2^1P_X^{-\frac{1}{2}}\,dP_X$

$=25\sqrt{2}(2P_X^{\frac{1}{2}}|_2^1)=50\sqrt{2}(1-\sqrt{2})=50\sqrt{2}-100$

題型：單一財貨替代效果、所得效果之計算

99.Suppose that the consumer has a demand function for milk of the form $x_1=10+\dfrac{m}{10p_1}$. Originally his income is \$120 per week and the price of milk is \$3 per quart. Now suppose that the price to milk falls to \$2 per quart. (1)Please find the substitution effect. (2)Please find the income effect. 【東華企研所】

解：$x=10+\dfrac{m}{10p}$

當 $p_0=3$，$m_0=120$，原均衡：$x_0=10+\dfrac{120}{10(3)}=14$

當 $p_1=2$，$m_0=120$，新均衡：$x_1=10+\dfrac{120}{10(2)}=16$　　$\therefore PE=x_1-x_0=2$

$CD=(3-2)(14)=14$　　$m_1=120-14=106$　　$x_2=10+\dfrac{106}{10(2)}=15.3$

$SE = x_2 - x_0 = 1.3 \quad IE = x_1 - x_2 = 0.7 \quad PE = x_1 - x_0 = 2$

100.設某消費者對豆漿的需求函數為 $X = 20 + \dfrac{Y}{20P}$；P 為豆漿價格，Y 為該消費者之貨幣所得。假設其每天原有貨幣所得 240 元，而豆漿原來價格為每單位 6 元。試求豆漿每單位價格下跌 2 元所造成該消費者之替代效果（substitution effect）與所得效果（income effect）。【中興企管所】

question 可否改變動百分比計算?

解：

原本

$X = 20 + \dfrac{Y}{20P} \qquad when\ Y = 240, P = 6, X_0 = 20 + \dfrac{240}{20\times 6} = 22 \qquad when\ Y = 240, P' = 4, X_2 = 20 + \dfrac{240}{20\times 4} = 23$

$CD = (6-4)\times 22 = 44 \qquad Y' = 240 - 44 = 196 \qquad when\ Y' = 196, P' = 4, X_1 = 20 + \dfrac{196}{20\times 4} = 22.45$

$SE = 22.45 - 22 = 0.45 \qquad IE = 23 - 22.45 = 0.55 \qquad PE = SE + IE = 1 \qquad or\ PE = 23 - 22 = 1$

101.黃君的所得(I)為 240 元，他對 X 商品的需求函數為 $X = 10 + (I/3P)$，其中 P 為 X 商品的價格。如果 P 從 10 元降為 8 元，請以 Sluskey Equation 的觀念計算替代效果、所得效果和價格效果。【淡江經研所】

(10-8)×18

解：原均衡點： $X_0 = 10 + \dfrac{240}{3(10)} = 18$ 　新均衡： $X_2 = 10 + \dfrac{240}{3(8)} = 20$

Cost Difference： $CD = (10-8)\times 18 = 36$ ； $I' = 240 - 36 = 204$

$X_1 = 10 + \dfrac{204}{3(8)} = 18.5$

SE($a \to b$) ： $\overline{X_0X_1} = 18.5 - 18 = 0.5$

IE($b \to c$) ： $\overline{X_1X_2} = 20 - 18.5 = 1.5$

PE($a \to c$) ： $\overline{X_0X_2} = 20 - 18 = 2$ 　　$CD = 36$

102.Ms. Lee consumes apple(A) and bananas(B), has the utility function: $U(A,B) = 10A^{\frac{1}{2}}B^{\frac{1}{2}}$, Originally the price of apples is \$1($P_A = 1$), the price of bananas is \$1($P_B = 1$), and Ms. Lee's income is \$40. However, the the price of bananas suddenly increases to \$2. What are the total, substitute and income effects of this price change for bananas?【交大經管所】

解：❶首先求出一般需求函數：
$$\begin{cases} \max U = 10A^{\frac{1}{2}}B^{\frac{1}{2}} \\ s.t.\quad P_AA + P_BB = M \end{cases} \Rightarrow \begin{cases} A^M = \dfrac{M}{2P_A} \\ B^M = \dfrac{M}{2P_B} \end{cases}$$

❷接著求出受補償需求函數：
$$\begin{cases} \min E = P_AA + P_BB \\ s.t.\quad 10A^{\frac{1}{2}}B^{\frac{1}{2}} = U \end{cases} \Rightarrow \begin{cases} A^h = \dfrac{1}{10}\left(\dfrac{P_A}{P_B}\right)^{-\frac{1}{2}} U \\ B^h = \dfrac{1}{10}\left(\dfrac{P_A}{P_B}\right)^{\frac{1}{2}} U \end{cases}$$

20 ⇒ 後來的效用　$A=20$　$B=10$

由 $U=10A^{\frac{1}{2}}B^{\frac{1}{2}}=10\sqrt{200}=100\sqrt{2}$

❸價格效果的分解

Hicks 定義	Slutsky 定義
(1)　原均衡： $(P_A,P_B M)=(1,1,40)$ $\Rightarrow A_0=20,B_0=20,U_0=200$ (2)　新均衡： $(P_A,P_B M)=(1,2,40)$ $\Rightarrow A_2=20,B_2=10,U_2=100\sqrt{2}$ (3)　替代效果的計算： $(P_A,P_B,U)=(1,2,200)\Rightarrow B_1=14.14$ $SE=14.14-20=-5.86$ $IE=10-14.14=-4.14$ $PE=10-20=-10$	(1)　原均衡： $A_0=20,B_0=20,U_0=200$ 新均衡：$A_2=20,B_2=10,U_2=100\sqrt{2}$ (2)　$CD=(2-1)\times 20=20$ $M'=40+20=60$ (3)　替代效果的計算： $(P_A,P_B M)=(1,2,60)$ $\Rightarrow B_1=15$ $SE=15-20=-5$ $IE=10-15=-5$ $PE=10-20=-10$

題型：準線性偏好效用函數價格效果拆解項

103.Susan is planning a trip to Paris and she has allocated \$80 of her vacation budget to spend on entrance fees to museums and monuments. Her preferences over bundles of museum visits (x) and monument visits (y) can be represented by the utility function $U(x,y)=4\ln x+0.5y$. The price of admission to a museum is P_x and the price of admission to a monument is P_y.

(1) Find Susan's demand functions for museum and monument visits.
(2) Suppose that the price of admission to a museum is \$10 and the price of admission to a monument is \$5. What are Susan's demands for museum and monument visits? What is the (own price) elasticity of Susan's demand for museum visits at these prices?
(3) Suppose that the price of admission to a museum falls to \$8. What are Susan's demands for museum and monument visits?
(4) Putting museum visits on the x-axis and monument visits on the y-axis, illustrate in an indifference curve diagram of Susan's optimal choice of museum and monument visits when the prices of each are \$ 10 and \$5 respectively. Illustrate in your diagram the effect of the decrease in the price of admission to museums (from \$10 to \$8) on her demands for museum and monument visits. Make sure to clearly indicate the income and substitution effects in your diagram for both activities. Which effect (the income or substitution) is larger on her demand for museum visits?【97 政大財管所】

解：(1)　Susan 的最適化決策如下：$\begin{cases} Max\,U=4\ln x+0.5y \\ s.t.\,P_x x+P_y y=M \end{cases}$

$L=4\ln x+0.5y+\lambda(M-P_x x-P_y y)$

f.o.c.　$\dfrac{\partial \ell}{\partial x}=0,\ \dfrac{4}{x}=\lambda P_x$　　$\dfrac{\partial \ell}{\partial y}=0,\ 0.5=\lambda P_y$　　$\dfrac{\partial \ell}{\partial \lambda}=0,\ M-P_x x-P_y y=0$

$$\frac{4/x}{1/2}=\frac{\lambda P_x}{\lambda P_y}\Rightarrow \frac{8}{x}=\frac{P_x}{P_y}\Rightarrow x^*=\frac{8P_y}{P_x}\qquad y^*=\frac{M-P_x x}{P_y}=\frac{1}{P_y}\left(M-P_x\cdot\frac{8P_y}{P_x}\right)=\frac{1}{P_y}(M-8P_y)=\frac{M}{P_y}-8$$

因此可知 Susan 對於博物館與紀念館參觀的需求函數分別為 $\dfrac{8P_y}{P_x}$ 與 $\dfrac{M}{P_y}-8$。

(2)將 $P_x=10,\ P_y=5,\ M=80$ 代回 x^*, y^*；$x^*=\frac{40}{10}=4, y^*=\frac{80}{5}-8=8$；因此可知當博物館與紀念館入場卷的價格分別為\$10 與\$5 時，Susan 對於博物館與紀念館參觀的需求數量分別為 4 與 8。$\varepsilon_{XX}=-\frac{\partial x}{\partial P_x}\frac{P_x}{x}=\frac{8P_y}{P_x^2}\frac{10}{4}=1$

(3)當 P_x 下跌為\$8 時，$x'=\frac{40}{8}=5 \quad y'=\frac{80}{5}-8=8$

(4)因為 x 為中性財，$\therefore P_x$ 變動時，所得效果 = 0，價格效果 = 替代效果。$SE=PE=5-4=1$

題型：Cobb-Douglas 間接效用與支出函數對偶理論

104.(1)The Cobb-Douglas utility function is given by: $u(x_1, x_2)=x_1^a x_2^{1-a}$. Can we also write $u(x_1, x_2)=a\ln x_1+(1-a)\ln x_2$? Please explain why?

(2)Please derive the Marshallian demand functions and the indirect utility function (which is a utility function at given prices (p_1 and p_2) and income(m)) by solving the following problem subject to the budget constraint: Max $a\ln x_1+(1-a)\ln x_2$ such that $p_1x_1+p_2x_2=m$

(3)If the indirect utility function is given by: $v(p_1,p_2,m)=(p_1^r+p_2^r)^{-1/r}m$, please find the demand function of good 1: $x_1(p_1,p_2,m)$. 【96 北大合經所】

解：(1) Cobb-Douglas $\Rightarrow u(x_1,x_2)=x_1^a x_2^{1-a}$、$\ln u(x_1,x_2)=a\ln x_1+(1-a)\ln x_2$

$\therefore\ a\ln x_1+(1-a)\ln x_2$ 為 $x_1^a x_1^{1-a}$ 取對數後之結果，並不影響其性質，x_1、x_2 之邊際替代率相同於 $x_1^a x_2^{1-a}$，因此可做此一轉換。

(2) Max $u(x_1x_2)=a\ln x_1+(1-a)\ln x_2$

s.t $\quad p_1x_1+p_2x_2=m$

$\Rightarrow L=a\ln x_1+(1-a)\ln x_2+\lambda[m-p_1x_1-p_2x_2]$

$$\text{F.O.C}\begin{cases}\dfrac{\partial L}{\partial x_1}=0\,,\ a\dfrac{1}{x}-\lambda p_1=0\\[2mm] \dfrac{\partial L}{\partial x_2}=0\,,\ (1-a)\dfrac{1}{x_2}-\lambda p_2=0\\[2mm] \dfrac{\partial L}{\partial x_3}=0\,,\ p_xx_1+p_2x_2=m\end{cases}$$

$\Rightarrow\left(\frac{a}{1-a}\right)\left(\frac{x_2}{x_1}\right)=\frac{p_1}{p_2}$，$x_2=\left(\frac{1-a}{a}\right)\left(\frac{p_1}{p_2}\right)x_1$ 代入

$p_1x_1+p_2\left(\frac{1-a}{a}\right)\left(\frac{p_1}{p_2}\right)x_1=m$，$\left(1+\frac{1-a}{a}\right)p_1x_1=m$，$\left(\frac{1}{a}\right)p_1x_1=m$

$x_1^M=a\left(\frac{m}{p_1}\right)$，$x_2^M=\left(\frac{1-a}{a}\right)\left(\frac{p_1}{p_2}\right)a\left(\frac{m}{p_1}\right)=(1-a)\left(\frac{m}{p_2}\right)$

$$\therefore \text{Marshallian demand function}\Rightarrow\begin{cases}x_1^M=a\left(\dfrac{m}{p_1}\right)\\[2mm] x_2^M=(1-a)\left(\dfrac{m}{p_2}\right)\end{cases}$$

$X_1^M\ \frac{am}{P_1}$

$X_2^M=\frac{(1-a)m}{P_2}$

間接效用函數： $\Rightarrow v(p_1, p_2, m) = a\ln\left(a\left(\dfrac{m}{p_1}\right)\right) + (1-a)\ln\left((1-a)\left(\dfrac{m}{p_2}\right)\right)$

(3) $v(p_1, p_2, m) = (p_1^r + p_2^r)^{-\frac{1}{r}} m$ 利用 Roy's identity 求 $x_1^M \Rightarrow x_1^M = -\dfrac{\partial v(\cdot)/\partial p_1}{\partial v(\cdot)/\partial m}$

$$= \frac{\left(-\frac{1}{r}\right)(p_1^r + p_2^r)^{-\frac{1}{r}-1} \cdot r p_1^{r-1} m}{(p_1^r + p_2^r)^{-\frac{1}{r}}} = (p_1^r + p_2^r)^{-1} p_1^{r-1} m \quad = \frac{p_1^{r-1}}{(p_1^r + p_2^r)} m$$

105.A consumer's preferences over two goods X and Y are described by the following utility function, $U(X,Y) = X^2Y^5$. The prices for goods X and Y are P_x and P_Y. This consumer has M dollars income.

(1)Find the Marshallian demand functions for goods X and Y.

(2)Find the indirect utility function.

(3)Show that the indirect utility function is homogeneous of degree 0 in (P_x, P_Y, M).

(4)Find the expenditure function. (5)Find the Hicksian demand functions for goods X and Y. 【96 暨南財金所】

解：(1)依題意，設立消費者最適化問題如下：

$$\begin{cases} Max\, U = X^2Y^5 \\ s.t.\ \ P_X X + P_Y Y = M \end{cases}$$

由 Largrange Method 可得：

$L = X^2Y^5 + \lambda(M - P_X X - P_Y Y)$

F.O.C $\dfrac{\partial L}{\partial X} = 0$ ， $2XY^5 - \lambda P_X = 0$ $\quad \dfrac{\partial L}{\partial Y} = 0$ ， $5X^2Y^4 - \lambda P_Y = 0$ $\quad \dfrac{\partial L}{\partial \lambda} = 0$ ，

$M - P_X X - P_Y Y = 0$

$\dfrac{2XY^5}{5X^2Y^4} = \dfrac{\lambda P_X}{\lambda P_Y} \Rightarrow \dfrac{2Y}{5X} = \dfrac{P_X}{P_Y} \Rightarrow P_Y Y = \dfrac{5}{2} P_X X$ 帶回預算限制式

$M = P_X X + \dfrac{5}{2} P_X X = \dfrac{7}{2} P_X X \Rightarrow \boxed{X^* = \dfrac{2}{7}\dfrac{M}{P_X}\ ,\ Y^* = \dfrac{5}{7}\dfrac{M}{P_Y}}$ Marshallian D function

(2) $v = u(X^* \cdot Y^*) = \left(\dfrac{2}{7}\dfrac{M}{P_X}\right)^2 \left(\dfrac{5}{7}\dfrac{M}{P_Y}\right)^5 = \left(\dfrac{2}{7}\right)^2 \left(\dfrac{5}{7}\right)^5 P_X{}^{-2} P_Y{}^{-5} M^7$

(3) $v(\lambda P_X, \lambda P_Y, \lambda M) = \left(\dfrac{2}{7}\right)^2 \left(\dfrac{5}{7}\right)^5 (\lambda P_x)^{-2} (\lambda P_y)^{-5} (\lambda M)^7 = \left(\dfrac{2}{7}\right)^2 \left(\dfrac{5}{7}\right)^5 (P_x{}^{-2})(P_y{}^{-5}) M^7 (\lambda^{-2+(-5)+7})$

$= v \cdot \lambda^0 \Rightarrow v(P_X, P_Y, M)$ 為(P_x ， P_Y ，M)的零次齊次函數

(4) 根據對偶理論，間接效用函數的反函數(inverse)即為支出函數

$U = \left(\dfrac{2}{7}\right)^2 \left(\dfrac{5}{7}\right)^5 P_x^{-2} P_y^{-5} M^7 \qquad M^7 = U \cdot \left(\dfrac{2}{7}\right)^{-2} \left(\dfrac{5}{7}\right)^{-5} P_x^2 P_y^5 \qquad$ let M = E ，

$E = U^{\frac{1}{7}} \left(\dfrac{2}{7}\right)^{\frac{-2}{7}} \left(\dfrac{5}{7}\right)^{\frac{-5}{7}} (P_x^{\frac{2}{7}})(P_y^{\frac{5}{7}})$

(5) 根據 Shephard's lemma：

$U = X^a Y^b$

(a) $\boxed{X^M = \dfrac{a}{(a+b)}\dfrac{M}{P_x} \qquad Y^M = \dfrac{b}{(a+b)}\dfrac{M}{P_y}}$

$X^h = \sqrt[a+b]{\left(\dfrac{aP_y}{bP_x}\right)^b U}$

$Y^h = \sqrt[a+b]{\left(\dfrac{bP_x}{aP_y}\right)^a U}$

$$X^H = \frac{\partial E}{\partial P_x} = \left(\frac{2}{7}\right)^{\frac{5}{7}}\left(\frac{5}{7}\right)^{-\frac{5}{7}} u^{\frac{1}{7}} (P_x^{-\frac{5}{7}})(P_y^{\frac{5}{7}})$$

$$X^H = \sqrt[7]{\left(\frac{2P_y}{5P_x}\right)^5 U}$$

$$Y^H = \frac{\partial E}{\partial P_y} = \left(\frac{5}{7}\right)^{\frac{2}{7}}\left(\frac{2}{7}\right)^{-\frac{2}{7}} u^{\frac{1}{7}} (P_x^{\frac{2}{7}})(P_y^{-\frac{2}{7}})$$

$$Y^H = \sqrt{\left(\frac{5P_x}{2P_y}\right)^2 U}$$

106.若小明的支出函數為 $E(P_1,P_2,U) = UP_1^{\frac{1}{4}}P_2^{\frac{3}{4}}\left(3^{-\frac{3}{4}}+3^{\frac{1}{4}}\right)$，其中 U 為效用水準，而 P_1、P_2 分別為財貨 X_1、X_2 的價格，則小明的間接效用函數為__(一)__，對 X_1 財貨之 Hicksian 需求函數__(二)__，對 X_1 財貨之需求價格需求彈性為__(三)__。　【中原企管所】

解：

(1)間接效用函數為支出函數的反函數(Inverse)，令 E = M：

間接效用函數：$V = M\left(3^{-\frac{3}{4}}+3^{\frac{1}{4}}\right)^{-1} P_1^{-\frac{1}{4}}P_2^{-\frac{3}{4}}$

(2)利用 Shephard's Lemma，可求解受補償需求函數：$X_1^h = \frac{\partial E}{\partial P_1} = \frac{1}{4}UP_1^{-\frac{3}{4}}P_2^{\frac{3}{4}}\left(3^{-\frac{3}{4}}+3^{\frac{1}{4}}\right)$

(3)利用 Roy's identity 可求解一般需求函數：$X_1^M = -\frac{\partial V/\partial P_1}{\partial V/\partial M} = \frac{M}{4P_1} \Rightarrow \varepsilon_{XX} = \frac{d\ln X_1}{d\ln P_1} = -1$

107.Consider the indirect utility function given by v(p_1, p_2, I) = $\frac{I^2}{p_1 p_2}$

(1)What are the demand functions $x_1(p_1, p_2, I)$ = ________, $x_2(p_1, p_2, I)$ = ______?

(2)Find the expenditure function $E(p_1, p_2, U_0)$ = ________.

(3)Derive the direct utility function $U(x_1, x_2)$ = ________.　【95 成大交管所】

解：(1) 利用 Roy's identity：

$$X_1^M = -\frac{V_{P_1}}{V_I} = -\frac{-\frac{I^2}{p_1^2 p_2}}{\frac{2I}{p_1 p_2}} = \frac{I}{2p_1} \qquad X_2^M = -\frac{V_{P_2}}{V_I} = -\frac{-\frac{I^2}{p_2^2 p_1}}{\frac{2I}{p_1 p_2}} = \frac{I}{2p_2}$$

(2)間接效用函數與支出函數互為反函數，令 I = E、V = U：$V = \frac{I^2}{p_1 p_2} \Rightarrow E^2 = Up_1p_2 \Rightarrow E = \sqrt{Up_1p_2}$

(3) 利用 Shephard's lemma：

$$X_1^h = E_{P_1} = \frac{1}{2}\sqrt{U\frac{p_2}{p_1}} \qquad X_2^h = E_{P_2} = \frac{1}{2}\sqrt{U\frac{p_1}{p_2}}$$

$$X_1^h \times X_2^h = \frac{1}{2}\sqrt{U\frac{p_2}{p_1}} \times \frac{1}{2}\sqrt{U\frac{p_1}{p_2}} = \frac{1}{4}U \Rightarrow U = 4X_1X_2$$

108.假定效用函數為 $U = X^{0.5}Y^{0.5}$，所得為 I。 (1)求出 X 和 Y 的 Marshall 需求函數 (2)間接效用函數 (3)支出函數 (4)利用支出函數求出 X 商品受補償需求函數 【96 銘傳國企所】

解：

(1) $X^M = \dfrac{I}{2P_X} \qquad Y^M = \dfrac{I}{2P_Y}$

(2) $V = U\left(X^M, Y^M\right) = \left(\dfrac{I}{2P_X}\right)^{\frac{1}{2}} \times \left(\dfrac{I}{2P_Y}\right)^{\frac{1}{2}} = I\left(\dfrac{1}{2P_X}\right)^{\frac{1}{2}} \times \left(\dfrac{1}{2P_Y}\right)^{\frac{1}{2}}$

(3) $E = U\left(\dfrac{1}{2P_X}\right)^{-\frac{1}{2}} \times \left(\dfrac{1}{2P_Y}\right)^{-\frac{1}{2}} = 2UP_X^{0.5}P_Y^{0.5}$

(4)利用 Shephard's lemma： $X^h = E_{P_X} = UP_X^{-0.5}P_Y^{0.5}$ $\qquad X^h = \sqrt{\dfrac{P_Y}{P_X}} \cdot U$

109.假設消費者效用函數為 $U(X) = [\alpha_1 X_1^\rho + \alpha_2 X_2^\rho]^{\frac{1}{\rho}}$：

(1)請證明 $\rho = 1$，則無異曲線為直線。

(2)若 $\alpha_1 = \alpha_2 = 1$，請寫出馬歇爾需求函數(Marshallian Demand function)及間接效用函數(Indirect utility function)。 【銘傳金融所、管科】

$V\ (\alpha_1 X_1 + \alpha_2 X_2)$

解：(1) $MRS = \dfrac{MU_{X_1}}{MU_{X_2}} = \dfrac{\frac{1}{\rho}\left[\alpha_1 X_1^\rho + \alpha_2 X_2^\rho\right]^{\frac{1}{\rho}-1}(\alpha_1 \rho X_1^{\rho-1})}{\frac{1}{\rho}\left[\alpha_1 X_1^\rho + \alpha_2 X_2^\rho\right]^{\frac{1}{\rho}-1}(\alpha_2 \rho X_2^{\rho-1})}$

$MU_X = \alpha$

$MU_Y = \alpha_2$

$\therefore MRS = \dfrac{\alpha_1}{\alpha_2}\left(\dfrac{X_1}{X_2}\right)^{\rho-1}$ 若 $\rho = 1$ 代入，則 $MRS = \dfrac{\alpha_1}{\alpha_2}\left(\dfrac{X_1}{X_2}\right)^0 = \dfrac{\alpha_1}{\alpha_2}$

$\therefore MRS = \dfrac{\alpha_1}{\alpha_2}$, Constant

$\because \alpha_1$，α_2 為常數 $\quad \therefore$ MRS 為一固定數，則無異曲線為直線

(2) $\boxed{\begin{array}{ll} Max & U = (X_1^\rho + X_2^\rho)^{\frac{1}{\rho}} \\ s.t & M = P_1 X_1 + P_2 X_2 \end{array}}$ $\qquad \because MRS = \dfrac{\alpha_1}{\alpha_2}\left(\dfrac{X_1}{X_2}\right)^{\rho-1} = \left(\dfrac{X_1}{X_2}\right)^{\rho-1}$

利用消費者均衡條件 $\dfrac{MU_{X_1}}{MU_{X_2}} = MRS = \dfrac{P_1}{P_2}$ 求解

$\dfrac{MU_x}{MU_Y} = \dfrac{\frac{1}{\rho}(\ )^{\frac{1}{\rho}-1}\rho\left(X_1^{\rho-1}\right)}{\frac{1}{\rho}(\ )^{\frac{1}{\rho}-1}\rho\left(X_2^{\rho-1}\right)} \qquad \left(\dfrac{X_1}{X_2}\right)^{\rho-1}$

$\left(\dfrac{X_1}{X_2}\right)^{\rho-1} = \left(\dfrac{P_1}{P_2}\right) \Rightarrow X_1 = \left(\dfrac{P_1}{P_2}\right)^{\frac{1}{\rho-1}} X_2$ 代回預算式 $M = P_1\left(\dfrac{P_1}{P_2}\right)^{\frac{1}{\rho-1}} X_2 + P_2 X_2$

$\Rightarrow X_2^M = \dfrac{M}{P_1\left(\dfrac{P_1}{P_2}\right)^{\frac{1}{\rho-1}} + P_2}$ ， $X_1^M = \dfrac{M}{P_1 + P_2\left(\dfrac{P_1}{P_2}\right)^{\rho-1}}$

Indirect utility function： $V(P_1, P_2, M) = \left[\left(\dfrac{M}{P_1 + P_2\left(\dfrac{P_1}{P_2}\right)^{\rho-1}}\right)^\rho + \left(\dfrac{M}{P_1\left(\dfrac{P_1}{P_2}\right)^{\frac{1}{\rho-1}} + P_2}\right)^\rho\right]^{\frac{1}{\rho}}$

直 無法直接看出 直 只能推導

110.A consumer has a utility function of the form $U(X_1, Y_2) = -X_1^{-1} - X_2^{-1}$, and a budget line $P_1X_1 + P_2X_2 = 100$. Show the indirect utility function.【朝陽企研、財金、保研所】

解.
$$\boxed{\begin{array}{ll} \underset{X_1, X_2}{Max} & U = -\dfrac{1}{X_1} - \dfrac{1}{X_2} \\ s.t & P_1X_1 + P_2X_2 = 100 \end{array}}$$

$$X^M = \frac{100}{P_{X_1} + P_{X_1}^{\frac{1}{2}} P_{X_2}^{\frac{1}{2}}}$$

$$X^M = \frac{100}{P_{X_1} + P_{X_1}^{\frac{1}{2}} P_{X_2}^{\frac{1}{2}}}$$

good

$$\Rightarrow \text{消費者均衡條件} \frac{MU_{X_1}}{MU_{X_2}} = \frac{P_1}{P_2} \Rightarrow \frac{P_1}{P_2} \Rightarrow \frac{\dfrac{1}{X_1^2}}{\dfrac{1}{X_2^2}} = \frac{P_1}{P_2}$$

$$\left(\frac{X_2}{X_1}\right)^2 = \frac{P_1}{P_2} \Rightarrow X_2 = \sqrt{\frac{P_1}{P_2}} X_1 \text{ 代入 } P_1X_1 + P_2X_2 = 100$$

$$P_1X_1 + P_2\sqrt{\frac{P_1}{P_2}}X_1 = 100 \qquad P_1X_1 + (P_1P_2)^{0.5}X_1 = 100 \Rightarrow X_1^* = \frac{100}{P_1 + P_1^{0.5}P_2^{0.5}}$$

$$X_2^* = \frac{100}{P_1 + P_1^{0.5}P_2^{0.5}} \frac{P_1^{0.5}}{P_2^{0.5}} = \frac{100P_1^{0.5}}{P_1P_2^{0.5} + P_1^{0.5}P_2} = \frac{100}{P_1^{0.5}P_2^{0.5} + P_2}$$

$P_X\uparrow$

111.Suppose that X and Y are the only two goods consumed, with P_X increased, I and P_Y unchanged. With convex indifference curves, use the Slutsky equation to evaluate the inferiority or normality of X and Y for each of the following cases:①X increases, Y decreases; ②X and Y both decreases; ③X decrease, Y increase. 【中央財金所】

解：$\dfrac{\partial X^M}{\partial P_X} = \dfrac{\partial X^H}{\partial P_X} - X\dfrac{\partial X}{\partial I} \Rightarrow$ 替代效果恆為負，$\dfrac{\partial X^H}{\partial P_X} < 0$

$\dfrac{\partial Y^M}{\partial P_X} = \dfrac{\partial Y^H}{\partial P_X} - X\dfrac{\partial Y}{\partial I} \Rightarrow$ 社會上只有兩個財貨，X、Y 必為淨替代，$\dfrac{\partial Y^H}{\partial P_X} > 0$

(1) $\dfrac{\partial X}{\partial I} = \dfrac{1}{X}\left[\underset{(-)}{\dfrac{\partial X^H}{\partial P_X}} - \underset{(+)}{\dfrac{\partial X^M}{\partial P_X}}\right] < 0 \because P_X \uparrow \rightarrow X \uparrow \Rightarrow \dfrac{\partial X^M}{\partial P_X} > 0$， 此時 $\dfrac{\partial X}{\partial I} < 0$，X 為劣等財。

$\dfrac{\partial Y}{\partial I} = \dfrac{1}{X}\left[\underset{(+)}{\dfrac{\partial Y^H}{\partial P_X}} - \underset{(-)}{\dfrac{\partial Y^M}{\partial P_X}}\right] > 0 \because P_X \uparrow \rightarrow Y \downarrow \Rightarrow \dfrac{\partial Y^M}{\partial P_X} < 0$， 此時 $\dfrac{\partial Y}{\partial I} > 0$，Y 為正常財（奢侈品）

(2)

$\dfrac{\partial X}{\partial I} = \dfrac{1}{X}\left[\dfrac{\partial X^H}{\partial P_X} - \dfrac{\partial X^M}{\partial P_X}\right] \because P_X \uparrow \rightarrow X \downarrow \Rightarrow \dfrac{\partial X^M}{\partial P_X} < 0$， 此時 $\dfrac{\partial X}{\partial I} \gtrless 0$

$\Rightarrow \text{if } \left|\underset{(-)}{\dfrac{\partial X^H}{\partial P_X}}\right| \lesseqgtr \left|\underset{(-)}{\dfrac{\partial X^M}{\partial P_X}}\right|$ $\quad\therefore$ X 財性質未定

$$\frac{\partial Y}{\partial I}=\frac{1}{X}\left[\frac{\partial Y^{H}}{\partial P_X}-\frac{\partial Y^{M}}{\partial P_X}\right]>0 \quad \because\ P_X\uparrow\rightarrow Y\downarrow\Rightarrow\frac{\partial Y^{M}}{\partial P_X}<0 \quad \text{此時}\ \frac{\partial Y}{\partial I}>0$$

，<u>Y 為正常財</u>

(+)　(−)

(3) $$\frac{\partial X}{\partial I}=\frac{1}{X}\left[\frac{\partial X^{H}}{\partial P_X}-\frac{\partial X^{M}}{\partial P_X}\right]\gtrless 0 \quad \because\ P_X\uparrow\rightarrow X\downarrow\Rightarrow\frac{\partial X^{M}}{\partial P_X}<0$$

此時 $\frac{\partial X}{\partial I}\gtrless 0 \ \Rightarrow \text{if}\ \left|\frac{\partial X^{H}}{\partial P_X}\right|\lesseqgtr\left|\frac{\partial X^{M}}{\partial P_X}\right|$　　$\therefore$ <u>X 財貨性質未定</u>

$$\frac{\partial Y}{\partial I}=\frac{1}{X}\left[\frac{\partial Y^{H}}{\partial P_X}-\frac{\partial Y^{M}}{\partial P_X}\right]\gtrless 0$$

(+)　(+)

$\because\ P_X\uparrow\rightarrow Y\uparrow\Rightarrow\frac{\partial Y^{M}}{\partial P_X}>0$ 此時 $\frac{\partial Y}{\partial I}\gtrless 0 \ \Rightarrow \text{if}\ \left|\frac{\partial Y^{H}}{\partial P_X}\right|\gtreqless\left|\frac{\partial Y^{M}}{\partial P_X}\right|$

<u>$\therefore$ Y 財貨性質未定，但 X、Y 財不可能同時為劣等財。</u>

第四章　無異曲線分析法之應用

主題一：福利變動之測度

1.假設效用函數為 U = XY 時，所得 M = 100，兩種財貨的價格分別為 $P_X^0=1, P_Y^0=1$，請問：

(1) X 財貨的普通需求函數為何? Hicks 受補償需求函數為何?

(2) 若 P_X 由 1 元下降至 0.25 元，為了維持實質所得不變，按 Hicks 與 Slutsky 定義法，他需要的貨幣所得為何? 替代效果與所得效果分別為何？

(3) 若 P_X 由 1 元下降至 0.25 元，其補償變量(compensating variation)為何?均等變量(equivalent variation)為何? 消費者剩餘的變動為何?　【97 文化經研所】

解：(1) $\boxed{\begin{array}{ll} \text{Max} & U = XY \\ \text{s.t} & M = P_X X + P_Y Y \end{array}}$　　$L = XY + \lambda[M - P_X X - P_Y Y]$

F.O.C　$\dfrac{\partial L}{\partial X} = 0 \Rightarrow Y - \lambda P_X = 0$　$\dfrac{\partial L}{\partial Y} = 0 \Rightarrow X - \lambda P_Y = 0$

$\dfrac{\partial L}{\partial \lambda} = 0 \Rightarrow M - P_X X - P_Y Y = 0$

$\Rightarrow \dfrac{Y}{X} = \dfrac{P_X}{P_Y}$，得 $X^M = \dfrac{M}{2P_X}$，$Y^M = \dfrac{M}{2P_Y}$

$\boxed{\begin{array}{ll} \text{Min} & E = P_X X + P_Y Y \\ \text{s.t} & U = XY \end{array}}$　　$L = P_X X + P_Y Y + \lambda[U - XY]$

F.O.C　$\dfrac{\partial L}{\partial X} = 0 \Rightarrow P_X - \lambda Y = 0$　$\dfrac{\partial L}{\partial Y} = 0 \Rightarrow P_Y - \lambda X = 0$　$\dfrac{\partial L}{\partial \lambda} = 0 \Rightarrow U - XY = 0$

$\Rightarrow \dfrac{Y}{X} = \dfrac{P_X}{P_Y} \Rightarrow U = \dfrac{P_X}{P_Y} X^2 \Rightarrow X^H = \sqrt{\dfrac{P_Y}{P_X} U}$，$Y^H = \sqrt{\dfrac{P_X}{P_Y} U}$

(2)Step 1：先計算原均衡 a 點(50, 50)　$X_0 = \dfrac{100}{2(1)} = 50$　$Y_0 = \dfrac{100}{2(1)} = 50$　$U_0 = 2500$

Step 2：再計算新均衡 d 點(10 , 10)　$X_1 = \dfrac{100}{2(0.25)} = 200$　$Y_1 = \dfrac{100}{2(1)} = 50$　$U_2 = 20000$

Step 3：再計算 b 點：$X_2 = \sqrt{\dfrac{1}{0.25}(2500)} = 100$　$Y_2 = \sqrt{\dfrac{0.25}{1}(2500)}$

E = 0.25(100)+1(25)=50；CV=50-100=-50

Step 4：Slutsky 分析法先找出 c 點均衡點，新預算線通過原先購買組合 $(X_0, Y_0) = (50, 50)$

求出 $m' = 0.25(50) + 1(50) = 62.5$　　CD = 100 - 62.5 = 37.5

$\therefore X_3 = \dfrac{62.5}{2(0.25)} = 125$，$Y_3 = \dfrac{62.5}{2(1)} = 31.25$

Hicks 分析法：　SE (a → b) $\begin{cases} X = 100 - 50 = 50 \\ Y = 25 - 50 = -25 \end{cases}$　IE(b → d) $\begin{cases} X = 200 - 100 = 1 \\ Y = 50 - 25 = 25 \end{cases}$

PE(a → d) $\begin{cases} X = 20 - 50 = 1 \\ Y = 50 - 50 = 0 \end{cases}$

Slutsky 分析法：　SE(a → c) $\begin{cases} X = 125 - 50 = 75 \\ Y = 31.25 - 50 = -18.75 \end{cases}$

IE(c → d) $\begin{cases} X = 200 - 125 = 75 \\ Y = 50 - 31.25 = 18.75 \end{cases}$　　PE(a → d) $\begin{cases} X = 20 - 50 = 150 \\ Y = 50 - 50 = 0 \end{cases}$

(3)

$$\Delta CS = \int_{0.25}^{1} \frac{M}{2P_X} dP_X = 50\int_{0.25}^{1} \frac{1}{P_X} dP_X = 50 \ln P_X \Big|_{0.25}^{1} = -50 \ln 0.25$$

$$CV = \int_{1}^{0.25} \sqrt{\frac{P_Y}{P_X} U_0} dP_X = 50\int_{1}^{0.25} P_X^{\frac{-1}{2}} dP_X = 100 P_X^{\frac{1}{2}} \Bigg|_{1}^{0.25} = 100(\sqrt{0.25} - 1) = -50$$

$$EV = \int_{0.25}^{1} \sqrt{\frac{P_Y}{P_X} U_1} dP_X = 100\int_{0.25}^{1} P_X^{\frac{-1}{2}} dP_X = 200 P_X^{\frac{1}{2}} \Bigg|_{0.25}^{1} = 200(1 - \sqrt{0.25}) = 100$$

2.Suppose the utility function U(x, y) = XY.　Let Px = 1.
(1)Compare the slope of compensated demand curve for x with that of ordinary demand curve.　(2)Suppose income is $100 and the Px falls from $1 to $0.25 due to a success of a good project. Determine the consumers' gain in terms of Compensating variation (CV) and Equivalent variation (EV).
(3)Set the pre-project price index at unity, what is the price index for U = U⁰ after the project?　(4)Suppose the price of y had risen to $2.25 at the same time as consumers' gain (in terms of CV) occurs. Is this sufficient to cancel out the advantages of the fall in the price of X?　【96 交大財金所】

解：(1)普通需求函數：　$P_x x + P_x x = M \Rightarrow x = \dfrac{M}{2p_x}$　　$y = \dfrac{M}{2P_y} \Rightarrow \dfrac{\partial x}{\partial P_x} = \dfrac{-M}{2P_x^{\,2}} < 0$

受補償需求函數：$X = \sqrt{\dfrac{P_y}{p_x} u}$　　$y = \sqrt{\dfrac{P_x}{P_y} u} \Rightarrow \dfrac{\partial x^H}{\partial P_x} = \dfrac{-u^{\frac{1}{2}}}{2P_x^{\frac{3}{2}}} < 0$

(2)　CV 表價格改變後，維持原效用，所需補償或扣除數額。

原均衡：$X = \dfrac{100}{2 \times 1} = 50$，　$Y = \dfrac{100}{2 \times 1} = 50$　　$u_0 = 50 \times 50 = 2500$

新均衡：$X = \dfrac{100}{2 \times 0.25} = 200$，　$Y = \dfrac{100}{2 \times 1} = 50$　　$u_1 = 200 \times 50 = 10000$

$$CV = \int_{1}^{0.25} \sqrt{\frac{P_Y}{P_X} U_0} dP_X = 50\int_{1}^{0.25} P_X^{\frac{-1}{2}} dP_X = 100 P_X^{\frac{1}{2}} \Bigg|_{1}^{0.25} = 100(\sqrt{0.25} - 1) = -50$$

$$EV = \int_{0.25}^{1} \sqrt{\frac{P_Y}{P_X} U_1} dP_X = 100\int_{0.25}^{1} P_X^{\frac{-1}{2}} dP_X = 200 P_X^{\frac{1}{2}} \Bigg|_{0.25}^{1} = 200(1 - \sqrt{0.25}) = 100$$

(3)在 $u_0 = 50$ 下：$(P_x^0, P_y^0) = (1,1)$　　$(x_0, y_0) = (50,50)$　　$(P_x^1, P_y^1) = (0.25,1)$　　$(x_1, y_1) = (100,25)$

$$\text{真實物價指數} = \frac{0.25 \times 10 + 1 \times 25}{1 \times 50 + 1 \times 50} \times 100 = 50$$

$$(4)\ CV = \int_{2.25}^{1} \sqrt{\frac{P_x}{P_y} u_0} dP_y = 50\int_{2.25}^{1} P_y^{\frac{-1}{2}} dP_y = 100 P_y^{\frac{1}{2}} \Bigg|_{2.25}^{1} = 100(1 - \sqrt{2.25}) = -50$$

因此當 $P_y=1$ 上漲到 $P_y=2.25$ 時，偏好將 X 財貨價格由 $P_x=1$ 下跌到 $P_x=0.25$ 所產生的消費者福利增加量完全抵銷掉。

3.A consumer purchases two goods with a utility function $U(x_1, x_2) = X_1^{0.5}X_2^{0.5}$. The prices and income are $p_1 = 1$, $p_2 = 1$ and $I = 10$, respectively. Suppose that the price of good 1 increases to $p_1' = 2$. Calculate the following three impacts on consumer's welfare due to this price change. (1)What is the change in Consumer Surplus $\Delta CS =$ ______? (2)Compensated Variation is defined by"the quantity of income needed to be changed under the new prices, in order to maintain the original utility level before the prices change". What is CV = ______? (3)Equivalence Variation is defined by"the quantity of income needed to be changed before the prices change, in order to have the utility level under the new prices". What is EV = ______? 【96 成大交管所】

解：(1)一般需求函數：$X_1^M = \dfrac{I}{2P_1}, X_2^M = \dfrac{I}{2P_2}$ $\quad CS_1 = \int_1^2 \dfrac{10}{2P_1} dP_1 = 5\ln P\Big|_1^2 = 5(\ln 2 - \ln 1) = 5\ln 2$

(2)受補償需求函數：$X_1^h = \sqrt{\dfrac{P_2}{P_1}} \times U, X_2^h = \sqrt{\dfrac{P_1}{P_2}} \times U,$

原均衡：$(P_1 = 1, P_2 = 1, I = 10) \Rightarrow X_1^h = X_2^h = 5, U_0 = 5$

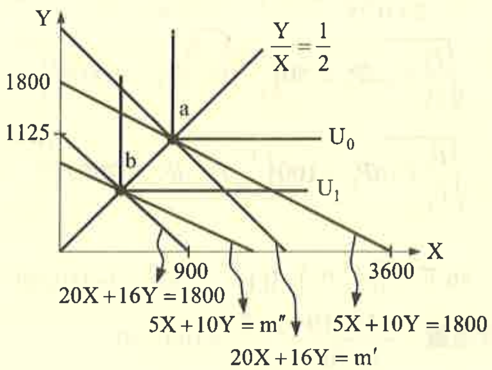
新均衡：$(P_1 = 2, P_2 = 1, I = 10) \Rightarrow X_1^h = 2.5, X_2^h = 5, U_1 = \sqrt{12.5}$

$CV = \int_1^2 5\sqrt{\dfrac{1}{P_1}} dP_1 = 5 \times \left(2P_1^{\frac{1}{2}}\right)\Bigg|_1^2 = 10\left(\sqrt{2} - 1\right)$ $\quad$ (3) $EV = \int_1^2 \sqrt{12.5}\sqrt{\dfrac{1}{P_1}} dP_1 = \sqrt{12.5} \times \left(2P_1^{\frac{1}{2}}\right)\Bigg|_1^2 = 10 - 5\sqrt{2}$

4.老王的效用函數為 $U(X,Y) = \min(X, 2Y)$，在雲林縣工作的他月薪為 18,000 元，所面對的物價為 $P_X = 5$，$P_Y = 10$。老王的老闆想將他送到台南分公司上班，薪水照舊，但台南的物價較貴，$P_X = 20$，$P_Y = 16$。(1)老王說只要適當的調薪，他不介意到台南分公司上班。請問：要維持他在雲林縣的效用水準，老闆應該給他多少薪水？
(2)老王說他願意接受減薪，以求留在雲林工作。請問：他最多願意減多少薪水？

解：雲林：$5X + 10Y = 18000$

台南：$20X + 16Y = 18000$



(1)Step 1：先計算 a 點均衡點

$$\boxed{\begin{array}{ll} \text{Max} & U = \min(X, 2Y) \\ \text{s.t} & 50X + 10Y = 18000 \end{array}} \text{ 均衡時} \begin{cases} X = 2Y \\ 5X + 10Y = 18000 \end{cases} \Rightarrow \begin{cases} X = 1800 \\ Y = 900 \end{cases}$$

Step 2：價格上漲後，維持原效用，所必需補償之金額，即求補償變量(CV)

$$\boxed{\begin{array}{ll} \text{Min} & m' = 20X + 16Y \\ \text{s.t} & U_0 = \min(X, 2Y) = 1800 \end{array}}$$

將$(X = 1800\ ,\ Y = 900)$代入上式 $m' = 36000 + 14400 = 50400$

$CV = 50400 - 18000 = 32400$

P	U	X	Y	M
b	1800	1800	900	18000
b	1800	1800	900	
a	642	642	321	18000
a	642	642	321	

(2)Step 1：先計算 b 點均衡點 $\boxed{\begin{array}{ll} \text{Max} & U = \min(X, 2Y) \\ \text{s.t} & 20X + 16Y = 18000 \end{array}}$

20X+8X

$$均衡時 \begin{cases} X = 2Y \\ 20X + 16Y = 18000 \end{cases} \Rightarrow \begin{cases} X = 642 \\ Y = 321 \end{cases}$$

Step 2：價格上漲後，維持新效用水準，所必需取走金額，即求均等變量(EV)

$$\boxed{\begin{array}{ll} \text{Min} & m'' = 5X + 10Y \\ \text{s.t} & U_1 = \min(642, 642) = 642 \end{array}}$$

~~642×15+6×321~~

~~9630+1950~~

將 $X = 642,\ \ Y = 321$ 代入 $m'' = 5X + 10Y$

$m'' = 5(642) + 10(321) = 6420 \qquad \therefore$ 取走(減薪) $18000 - 6420 = 11580$

5.若某消費者的效用函數為 $U = (X_1X_2)^{\frac{1}{2}}$，他原來所面對的 X_1 與 X_2 價格分別為 $P_1 = 1$，$P_2 = 1$，而所得為 100。若現在所得不變，但 $P_1 = 2$，$P_2 = 1$，請求算其 CV(compensating variation)與 EV(equivalent variation)。 **【中正國經所】**

解：Step 1：計算 Hicksian demand function $\boxed{\begin{array}{ll} \text{Min} & E = P_1X_1 + P_2X_2 \\ \text{s.t} & U = (X_1X_2)^{\frac{1}{2}} \end{array}}$

$$L = P_1X_1 - P_2X_2 + \lambda[U - X_1^{\frac{1}{2}}X_2^{\frac{1}{2}}]$$

$$\frac{\partial L}{\partial X_1} = 0 \ \Rightarrow\ P_1 - \frac{\lambda}{2}X_1^{-\frac{1}{2}}X_2^{\frac{1}{2}} = 0 \qquad \frac{\partial L}{\partial X_2} = 0 \ \Rightarrow\ P_2 - \frac{\lambda}{2}X_1^{\frac{1}{2}}X_2^{-\frac{1}{2}} = 0$$

$$\Rightarrow \frac{X_2}{X_1} = \frac{P_1}{P_2} \quad \Rightarrow X_2 = \frac{P_1}{P_2}X_1 \Rightarrow U = \sqrt{\frac{P_1}{P_2}X_1^2} \ \Rightarrow\ X_1^H = \sqrt{\frac{P_2}{P_1}}U\ ,\ X_2^H = \sqrt{\frac{P_1}{P_2}}U$$

Step 2：再求出 Marshallian demand function $\boxed{\begin{array}{ll} \text{Max} & U = (X_1X_2)^{\frac{1}{2}} \\ \text{s.t} & M = P_1X_1 + P_2X_2 \end{array}}$

$$L = X_1^{\frac{1}{2}}X_2^{\frac{1}{2}} + \lambda[M - P_1X_1 - P_2X_2]$$

$$\frac{\partial L}{\partial X_1} = 0 \ \Rightarrow\ \frac{1}{2}X_1^{-\frac{1}{2}}X_2^{\frac{1}{2}} - \lambda P_1 = 0 \qquad \frac{\partial L}{\partial X_2} = 0 \ \Rightarrow\ \frac{1}{2}X_1^{\frac{1}{2}}X_2^{-\frac{1}{2}} - \lambda P_2 = 0$$

$\Rightarrow \dfrac{X_2}{X_1} = \dfrac{P_1}{P_2}$ 代回預算限制式 $\qquad \therefore\ X_1^M = \dfrac{M}{2P_1}\ ,\ X_2^M = \dfrac{M}{2P_2}$

Step 3：計算 CV，EV

(1)原均衡點 $(P_1, P_2, M) = (1, 1, 100) \Rightarrow (X_1, X_2) = (50, 50) \qquad U_0 = 50$

(2)新均衡點 $(P_1', P_2, M) = (2, 1, 100) \Rightarrow (X_1, X_2) = (25, 50) \qquad U_1 = 25\sqrt{2}$

(3) $CV=\int_1^2\left(\frac{P_2}{P_1}\right)^{\frac{1}{2}}U_0dP_1=\int_1^2(P_1)^{-\frac{1}{2}}\ 50dP_1=50\int_1^2 P_1^{-\frac{1}{2}}dP_1=50\left[2P_1^{\frac{1}{2}}\Big|_1^2\right]=100(\sqrt{2}-1)$

$EV=\int_2^1\left(\frac{P_2}{P_1}\right)^{\frac{1}{2}}U_1dP_1=\int_2^1(P_1)^{-\frac{1}{2}}U_1dP_1$

$=25\sqrt{2}\int_2^1 P_1^{-\frac{1}{2}}dP_1=25\sqrt{2}\left[2P_1^{\frac{1}{2}}\Big|_2^1\right]=50\sqrt{2}(1-\sqrt{2})=50\sqrt{2}-100$

$\boxed{\text{CV}}$ ⇒ P 上漲後，維持原效用，必需補償$100(\sqrt{2}-1)$金額

$\boxed{\text{EV}}$ ⇒ P 上漲後，維持新效用，必需取走$(50\sqrt{2}-100)$金額

6.設$U(X,Y)=X^{\frac{1}{2}}+Y$ (1)求 X 財貨的普通需求函數 (2)求 X 財貨的補償需求函數 (3)設$P_Y^0=1$，求P_X由 1 元增至 2 元的補償變量。　　　　【淡江財金所】

解：(1) $\boxed{\begin{array}{ll}\text{Max} & U=X^{\frac{1}{2}}+Y\\ \text{s.t} & M=P_XX+P_YY\end{array}}$

$L=X^{\frac{1}{2}}+Y+\lambda[M-P_XX-P_YY]$

F.O.C $\frac{\partial L}{\partial X}=0 \Rightarrow \frac{1}{2}X^{-\frac{1}{2}}-\lambda P_X=0$ ……❶

$\frac{\partial L}{\partial Y}=0 \Rightarrow 1-\lambda P_Y=0$ ……………❷

$\frac{\partial L}{\partial \lambda}=0 \Rightarrow M-P_XX-P_YY=0$ …❸

— $\Rightarrow X=\frac{1}{4}\left(\frac{P_Y}{P_X}\right)^2$，即為普通需求函數

(2)根據 Slutsky equation 可知，由於 X 為中性財，故所得效果為零

則受補償需求函數會與普通需求函數相同；則$X^H=\frac{1}{4}\left(\frac{P_Y}{P_X}\right)^2$

(3) $CV=\int_2^1\frac{1}{4}P_X^{-2}dP_X=-\frac{1}{4}P_X^{-1}\Big|_2^1=-\frac{1}{8}$

6.一消費者消費兩種商品，商品X與商品Y。

效用函數為$U(X,Y)=\frac{1}{2}X^2+Y$。其每週總消費預算為\$128，商品X原價格為\$4，商品Y價格為\$8。若商品X價格下降為\$2，求補償變量(compensating variation)與對等變量(equivalent variation)。必須明示計算過程及相關效用函數值、消費預算值。答案先行書寫（補償變量及相關效用函數值、消費預算值；對等變量及相關效用函數值、消費預算值）。之後詳列計算細節。【中山財管所】

解：$u=\frac{1}{2}X^2+Y$

$$MRS_{XY}=\frac{dY}{dX}=\frac{-MU_X}{MU_Y}=-X<0$$

$\frac{d|MRS_{XY}|}{dX}=1>0$, MRS_{XY} 遞增 $\rightarrow$ 凹性偏好，無異曲線凹向原點

(1)原均衡：	(2)新均衡：
$Max\,U=\frac{1}{2}X^2+Y$ s.t $4X+8Y=128$ $X_0=32,\ Y_0=0,\ U_0=512$	$\mathrm{Max}\,U=\frac{1}{2}X^2+Y$ s.t $2X+8Y=128$ $X_1=64,\ Y_1=0,\ U_1=2048$
(3) CV（原效用，新價格）	(4) EV（新效用，原價格）
Min $E=2X+8Y$ s.t $U_0=\frac{1}{2}X^2+Y=512$ Corner Solution: $X^H=32,\ Y^H=0$ 支出 $=2(32)=64$ $CV=128-64=64$	Min $E=4X+8Y$ s.t $U_1=\frac{1}{2}X^2+Y=2048$ Corner Solution: $X^H=64,\ Y^H=0$ 支出 $=4(64)=256$ $EV=256-128=128$

7.設某市場的供給函數為 $Q^S=-9+\frac{P}{2}$，需求函數為 $Q^d=36-\frac{P}{3}$，課征 5 元從量稅後消費者剩餘等於_____元？ 【中原企研所】

解：

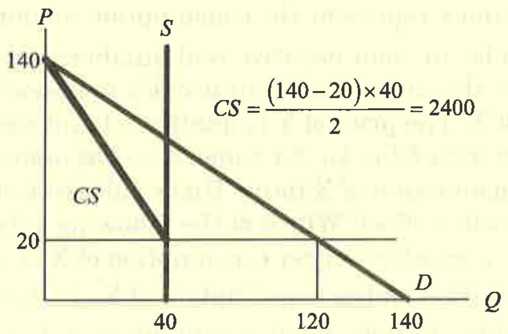
稅前均衡：$Q^S=Q^d\Rightarrow -9+\frac{P}{2}=36-\frac{P}{3}\Rightarrow P^*=54,Q^*=18$

稅後均衡：$P^d-P^s=t\Rightarrow 108-3Q-18-2Q=5\Rightarrow Q^{**}=17,P^{**}=57$

稅後消費者剩餘：$CS=\frac{(108-57)\times 17}{2}=433.5$

8.假設台北市民對路邊停車位的需求為 $Q^d=140-P$，而台北市政府對路邊停車位的定價為 $P_0=20$，供給量為 $Q^S=40$，求在所有潛在需求者皆有相同機率獲得停車位之假定下，車位可提供之消費者剩餘是多少呢？ 【政大科管所】

解：



9.一位喜歡收集模型的上班族，其主要的收集模型以迪士尼系列的卡通人物(X)和合金機器人(Y)為主，而他每個月會用 6,000 元來收集模型，假設其收集模型的消費決策為：

Max $\quad U=f(X,Y)=X^{\frac{1}{3}}Y^{\frac{2}{3}}$

subject to $\quad 6000=500X+2000Y$

(1)當迪士尼系列的卡通人物模型價格上升為 1,000 元時，該上班族需增加多少收集模型的預算才能購買原有的消費組合？ (10%)

(2)承第(1)題，該上班族需增加多少收集模型的預算才能夠維持原來的效用？【98 中興應經所】 (15%)

解：(1)價格上漲前：

$$\begin{cases}\max\ U=X^{\frac{1}{3}}Y^{\frac{2}{3}}\\ \text{s.t.}\quad 500X+2000Y=6000\end{cases}\Rightarrow\begin{cases}X^*=\dfrac{1}{3}\times\dfrac{6000}{500}=4\\ Y^*=\dfrac{2}{3}\times\dfrac{6000}{2000}=2\end{cases}$$

價格上漲後，支出差額：

$\Delta M=(P_{X_1}-P_{X_0})X_0=500\times4=2000$

$M_1=M_0+\Delta M=6000+2000=8000$

價格上漲後，上班族的所得需要增加 2,000(新所得$M_1=8000$)才買得起原組合($X^*=4$，$Y^*=2$)

(2)承(1)原均衡之效用水準為$U_0=4^{\frac{1}{3}}\times2^{\frac{2}{3}}=4^{\frac{2}{3}}$

在維持原效用水準不變之下，消費者追求支出極小化

$$\begin{cases}\min\ E=1000X+2000Y\\ \text{s.t.}\quad X^{\frac{1}{3}}Y^{\frac{2}{3}}=4^{\frac{2}{3}}\end{cases}$$

$$\text{f.o.c.}\quad\begin{cases}MRS=\dfrac{P_X}{P_Y}\Rightarrow\dfrac{Y}{2X}=\dfrac{1000}{2000}\Rightarrow X=Y\text{ ....(a)}\\ X^{\frac{1}{3}}Y^{\frac{2}{3}}=4^{\frac{2}{3}}\qquad\qquad\text{..........(b)}\end{cases}$$

將(a)代入(b) $X^H=4^{\frac{2}{3}}=Y^H$

$E^*=1000X^H+2000Y^H=3000\times4^{\frac{2}{3}}$

$CV=E^*-M=(3000\times4^{\frac{2}{3}})-6000$

10.Grace consumes only two goods, X and Y. Her utility function can be written as $u(x,y)=\sqrt{x}+\sqrt{y}$, where x and y represent the consumption amounts of X and Y, respectively. x and y can be any non-negative real numbers. Grace has $24 to spend on these two goods. Suppose the city government decides to impose a new regulation on the production process of X. The price of Y is unaffected and remains at $6, but the price of X would increase from $2 to $a, for some $a>2$.Decompose the total effect of the regulation on Grace's consumption of X into a Hicks substitution effect (holding the utility level fixed) and income effect. Which of the following is (are) true? (A)The substitution effect of this regulation on her consumption of X is $-16/9$ if $a=3$. (B)The income effect of this regulation on her consumption of X is $-16/9$ if $a=3$ (C)The substitution effect of this regulation on her consumption of X must be negative for any $a>2$. (D)The income effect of this regulation on her consumption of X must be negative for any $a>2$. (E)None of the above.【96 台大經研所】

解：**(B)(C)(D)**

◆效用極大化條件：

$$\text{Max } u(x,y)=\sqrt{x}+\sqrt{y} \quad x\geq 0,\ y\geq 0$$

$$\text{s.t. } P_x x+P_y y=24$$

$$\therefore MRS_{XY}=\frac{MU_x}{MU_y}=\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}=\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}=\frac{P_x^0}{P_y^0}=\frac{2}{6}=\frac{1}{3}\Rightarrow x=9y \text{ 代入預算限制式} \Rightarrow 18y+6y=24\Rightarrow y^*=1,\ x^*=9$$

$$\therefore x^*=9,\ y^*=1$$

◆由支出極小化條件：

$$\text{Min } E=P_x x+P_y y$$

$$\text{s.t } u=\sqrt{x}+\sqrt{y}$$

$$\therefore MRS_{XY}=\frac{MU_x}{MU_y}=\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}=\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}=\frac{P_x}{P_y}\Rightarrow y^{\frac{1}{2}}=\frac{P_x x^{\frac{1}{2}}}{P_y} \text{ 代入效用函數：} \quad u=x^{\frac{1}{2}}+(\frac{P_x}{P_y})x^{\frac{1}{2}}=(1+\frac{P_x}{P_y})x^{\frac{1}{2}}$$

$$x^H=[\frac{u}{1+(\frac{P_x^1}{P_y})}]^2=[\frac{\sqrt{9}+\sqrt{1}}{1+(\frac{3}{6})}]^2=\frac{64}{9}$$

If a 3，$MRS_{xy}=\frac{P_x^1}{P_y}=\frac{3}{6}=\frac{1}{2}$ $\therefore x'=\frac{16}{3},\ y'=\frac{4}{3}$

$$SE=\frac{64}{9}-9=-\frac{17}{9} \qquad IE=\frac{16}{3}-\frac{64}{9}=-\frac{16}{9}$$

(A)錯。$SE=-\frac{17}{9}$。(B)對。$IE=-\frac{16}{9}$。(C)對。對於任何 a>2 的情況消費 x 財的替代效果一定為負 (D)對。對於任何 a>2 的情況消費 x 財的所得效果一定為負。

11.Continue from above. Suppose the price of X rises to \$3 after the new regulation being imposed (i.e. a = 3). Which of the following is (are) true? (A)Grace's consumer surplus declines \$24 ln (3/2) because of the regulation. (B)If Grace can bribe the mayor to prevent the regulation being imposed, the maximal amount Grace is willing to bribe is \$6. (C)If Grace can bribe the mayor to prevent the regulation being imposed, the maximal amount Grace is willing to bribe is \$8. (D)None of the above.【96 台大經研所】

解：**(D);**由上題可知 $MRS_{XY}=\frac{MU_x}{MU_y}=\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}=\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}=\frac{P_x}{P_y}\Rightarrow y=(\frac{P_x}{P_y})^2x$ 代入預算限制式

$$P_x x+P_y(\frac{P_x}{P_y})^2x=M\Rightarrow x^*=\frac{M}{P_x+\frac{P_x^2}{P_y}}\Rightarrow x^*=\frac{24}{P_x+\frac{P_x^2}{6}}$$

$$\Delta CS=\int_2^3\frac{24}{P_x+\frac{P_x^2}{6}}=24\int_2^3\frac{1}{1+\frac{P_x}{3}}\ln(P_x+\frac{P_x^2}{6})=3.912$$ ，故可知 Grace 願意出 3.912 去賄賂官員。

12.若效用函數為 $U(X,Y)$，預算線為 $I=P_X X+P_Y Y$，請根據下列敘述之相關資料，判斷何項為正確？(A) 若 X 財貨為季芬財，P_X 下跌，則 $EV>\Delta CS>CV$。(B) 若 X 財貨為劣等財，P_X 下跌，則 $EV>\Delta CS>CV$。(C)若 X 財貨為正常財，P_X 下跌，則 $EV<\Delta CS<CV$。(D) 若 X 財貨為劣等財，P_X 上漲，則 $EV<\Delta CS<CV$。(E) 以上皆錯誤。【96 北大經研所】

解：(E)

財貨性質	$P_X \downarrow$	$P_X \uparrow$
正常財	$EV>\Delta CS>CV$	$CV>\Delta CS>EV$
劣等財	$CV>\Delta CS>EV$	$EV>\Delta CS>CV$
中性財	$EV=\Delta CS=CV$	$EV=\Delta CS=CV$

13.Questions 1-3. A person consumes two goods: goods 1 and 2. The amounts of goods consumed are x_1 and x_2. Suppose that the person's utility function is $u(x_1,x_2)=x_1+\sqrt{x_2}$. The prices of the two goods are p_1 and p_2, and the person's income is w. The person always maximizes his utility.

1.When $x_1=x_2=4$, the person is willing to get 1 more unit of good 1 by giving up at most y units of good 2. When $x_1=8$ and $x_2=4$, the person is willing to get 1 more unit of good 1 by giving up at most z units of good 2. How much is $y-z$?

2.Suppose that when $p_1=2$, $p_2=1$, and $w=20$, the person gets utility U_1. When $p_1=p_2=2$, if he wants to have utility U_1, what is the minimal amount of income he needs to have?

3.Suppose that $p_1=2$, $p_2=1$, and the person cannot consume more than 5 units of good 1 $(x_1\le 5)$. If he wants to have utility U_1, what is the minimal amount of income he needs to have?【98 中興財金所】

解：

(1) $u=x_1+\sqrt{x_2}$ ， $MRS_{12}=\dfrac{Mu_1}{Mu_2}=\dfrac{1}{\frac{1}{2}x_2^{\frac{-1}{2}}}=2\sqrt{x_2}$

當 $(x_1,x_2)=(4,4)$ ， $MRS_{12}=y=4$

當 $(x_1,x_2)=(8,4)$ ， $MRS_{12}=z=4$

$\therefore y-z=4-4=0$

(2) 消費者最適化模型 $\begin{cases} Max\ u=x_1+\sqrt{x_2} \\ s.t.\ \ p_1x_1+p_2x_2=w \end{cases}$

$L=x_1+\sqrt{x_2}+\lambda(w-p_1x_1-p_2x_2)$

F.O.C $\dfrac{\partial L}{\partial x_1}=0\Rightarrow 1+\lambda(-p_1)=0 \ldots❶$

$\dfrac{\partial L}{\partial x_2}=0\Rightarrow \dfrac{1}{2}x_2^{\frac{-1}{2}}+\lambda(-p_2)=0 \ldots❷$

$\dfrac{\partial L}{\partial \lambda}=0\Rightarrow w-p_1x_1-p_2x_2=0 \ldots❸$

可得 $2\sqrt{x_2}=\dfrac{p_1}{p_2}\Rightarrow x_2^*=\dfrac{p_1^2}{4p_2^2}$

$w=p_1x_1+p_2\left(\dfrac{p_1^2}{4p_2^2}\right)$

$\therefore x_1^*=\dfrac{w}{p_1}-\dfrac{p_1}{4p_2}$

若 $p_1=2$ ， $p_2=1$ ， $w=20 \Rightarrow x_1^*=9.5$ ， $x_2^*=1$ ， $u^*=9.5+\sqrt{1}=10.5$

支出極小化模型 $\begin{cases} Min\ \ E=2x_1+2x_2 \\ s.t.\ \ \ u=x_1+\sqrt{x_2}=10.5 \end{cases}$

$x_2^H=\dfrac{p_1^2}{4p_2^2}=\dfrac{4}{16}=0.25$ 代入效用限制式可得 $x_1^H=10$，維持效用水準 10.5 不變下，最少所得為 $2(10)+2(0.25)=20.5$

(3) 為了維持效用水準 10.5 且 $x_1^*=5$ 條件下，利用效用函數 $10.5=5+\sqrt{x_2}$

$\therefore x_2^*=30.25$，此時最少所得為 $2(5)+1(30.25)=40.25$

14.假設消費者的偏好滿足完整性(completion)、遞移性(transitivity)、反身性(reflexivity)與單調性(monotonicity)，消費者同時購買 X 財貨與 Y 財貨，消費者的預算限制式 $P_xX+P_yY=m$ ， P_x、 P_y 與 m 分別為 X 財貨的價格、Y 財貨的價格與消費者的所得。請回答下列問題：

(1)當兩財貨均為正常財時，試由無異曲線與預算限制式求得財貨 X 的 Marshall 未補償需求線與 X 財貨的 Hicks 受補償需求函數？ (10%)

(2)由於景氣不好，政府擬對 X 財貨課稅，政府目前考慮兩種課稅方式：(1)政府對每單位 X 財貨課徵 t 元的從量稅；(2)政府擬對消費者課徵與從量稅收入 (tX) 等量的所得稅 I ，即 $I=tX$。試以無異曲線說明，在政府稅收不變下，政府採用何者課稅方式下的消費者效用較高？

(10%)【98 淡江財金、保險、國貿】

解：(1)：正常財：若財貨價格下降，未受補償需求曲線比受補償需求曲線平坦

利用 Slutsky Equation 判斷需求曲線之形狀

❶替代效果必為負數。

❷正常財(隨著實質所得增加，消費量隨之增加，$\dfrac{\partial X}{\partial M}>0$)，因此，所得效果 $(-X\dfrac{\partial X}{\partial M})$ 必為負；而替代效果必為負數，所以 SE 與 IE 呈現同方向變動，價格效果必為負數，因此正常財之普通需求曲線必為負斜率，必定符合需求法則。

(2)前提：維持二種稅制稅收相同情況下，站在民眾角度而言，課所得稅稅後效用高於貨物稅。

15.Consider two commodities and answer the following questions by drawing clear pictures.

(1)How does the budget constraint of a consumer change if the government imposes a quantity tax on one of the commodities? (3 分)

(2)How does the budget constraint change if the government imposes an income tax that raises the same amount of revenue as the quantity tax does? (7 分)

(3)Which taxing scheme is better? (10 分)【98 清華經研所】

解：參閱講義。

16.Consider a consumer who purchases two goods: gasoline (X) and other goods (Y).

Her preferences can be described by the utility function: $U(X_1, X_2) = X^{\frac{1}{3}}Y^{\frac{2}{3}}$

Her weekly income is $48. The price per gallon of gasoline is $2, and the price per unit of other goods is $1.

(1) Drive her Marshallian (ordinary) demand curves for gasoline and other goods. What are the amounts of gasoline and other goods this consumer will purchase?

(2)Now suppose that due to environmental concerns the government considers a rationing scheme for gasoline, which would impose a consumption limit on this consumer of no more that 5 gallons per week. To ensure that the consumer becomes no worse off as a result of the rationing, by how much will the government have to increase her income? 政府補

【95 政大國貿所】

purchase 購買

amount 數量

concern 關心

worse 更差的

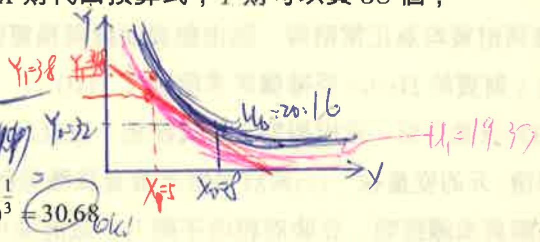
解：(1) Marshallian demand function： $X^M = \dfrac{M}{3P_X}$；$Y^M = \dfrac{2M}{3P_Y}$

若 $P_X = 2$，$P_Y = 1$，$M = 48$，則 $X_0 = 8$，$Y_0 = 32$，$U_0 = 8^{\frac{1}{3}} \times 32^{\frac{2}{3}} = 20.16$

(2)政府限量政策下，消費者只能購買 5 個 X 財代回預算式，Y 財可以買 38 個，

$U_1 = 5^{\frac{1}{3}} \times 38^{\frac{2}{3}} = 19.33$

$$\boxed{\begin{array}{ll}\text{Min} & E = 2X + Y \\ \text{s.t} & U_0 = X^{\frac{1}{3}}Y^{\frac{2}{3}} = 20.16\end{array}}$$



$\frac{Y}{X} = 4$　$Y = 4X$

$X^H = (451.25)^{\frac{1}{3}} = 7.67$ ok!　　$Y^H = 4(451.25)^{\frac{1}{3}} = 30.68$ ok!

支出 $= 2(7.67) + (30.68) = 46.02$　　補貼金額 $= 48 - 46.02 = 1.98$

表示政府為了使消費者在限量之後仍維持 $U_0 = 20.16$

原來效用水準，則政府需要補貼 1.98 所得

17.T 國的國民消費 X、Y 兩種財貨，每位國民的效用函數為 $U(X,Y) = XY$，其中 X 與 Y 為該國民財貨 X 與 Y 的消費量。財貨 X 的單位價格為 9 元，Y 的單位價格為 1 元，每位國民的所得為 120 元。T 國政府想用以下兩種方法之一來提高國民福祉。方法一：國民每買一單位財貨 X 可由政府獲得 5 元的補貼（即對 T 國國民來說，財貨 X 的單位價格由 9 元降至 4 元）。方法二：給每位國民 R 元的現金。假設無論政府採取方法一或方法二，均不會對民眾徵稅。

(1)若政府決定採取方法一，則政府花費在補貼每位國民上的金額為：__(1)__元。

(2)若政府決定採取方法二，且希望採取兩種方法時國民的效用水準相同，則 $R =$ __(2)__。

【98 台大財金所】

解：

(1)價格補貼 $\begin{cases} Max & U = XY \\ s.t & 4X + Y = 120 \end{cases}$

$X^* = 15$，$Y^* = 60$，$U_1 = 900$

補貼金額 $= 5 \times 15 = 75$

$X(P_{X0} - P_{X1})$ = 補
$= 15 \times (9-4) = 15 \times 5 = 75$

(2) $\begin{array}{ll} Min & E = 9X + Y \\ s.t & U_1 = XY = 900 \end{array}$

$X_1^H = \sqrt{\frac{1}{9}(900)} = 10$，$Y_1^H = \sqrt{\frac{9}{1}(900)} = 90$

$E = 9(10) + 90 = 180$，表示所得補貼必須給予消費者 60 元，才能維持與價格補貼有相同的效用水準。

$Y^H = \sqrt{\left(\frac{P_{X0}}{P_Y}\right) U}$　$\sqrt{9 \times V}$

18.今年(2009)年初台灣絕大部分的人都領到 3600 元的消費券，極少數人則可能因為條件不符合而沒有領到。如果您已經領到 3600 元的消費券，請利用經濟理論分析您使用消費券的行為。如果您無法領取消費券，請利用經濟理論分析您的同學或朋友使用消費券的行為。(10 分)【98 中山亞太】

解：參閱講義。

19.Consider an employee who does not receive employer-based health insurance and must divide his $7000 per week in after-tax income between wealth insurance and "other goods"

(1)Draw this worker's opportunity set if the price of health insurance is $1000 per week and the price of"other goods"is $1000 per week.

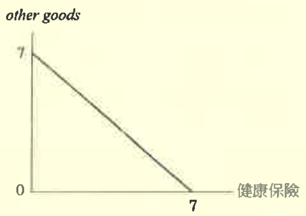
(2)On the same graph, illustrate how the opportunity set would change if the employer agreed to give this employee $1000 worth of health insurance per week (under current tax laws, this form of compensation is nontaxable).

(3)Would this employee be better or worse off if, instead of the health insurance, the employer gave him a $1000 per week raise that was taxable at a rate of 25 percent?

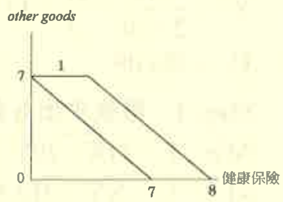
【96 中興企研所】

解：

(1)

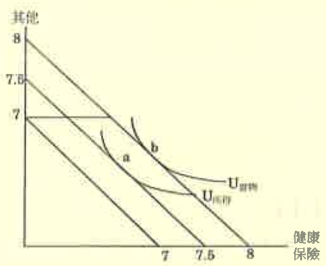


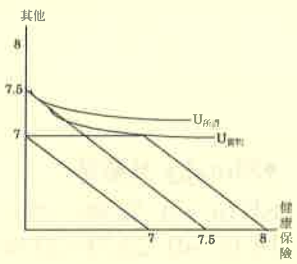
(2) 實物補貼



(3)若雇主直接支付 1000 薪水，但課徵 25%稅率，因此稅後所得只有 750 元，所得補貼 750 元，至於效用高低的討論，無法確定，分述如下：

情況 1：員工偏好健康保險：實物補貼較佳　情況 2：並不特別偏好員工保險，所得補貼較佳

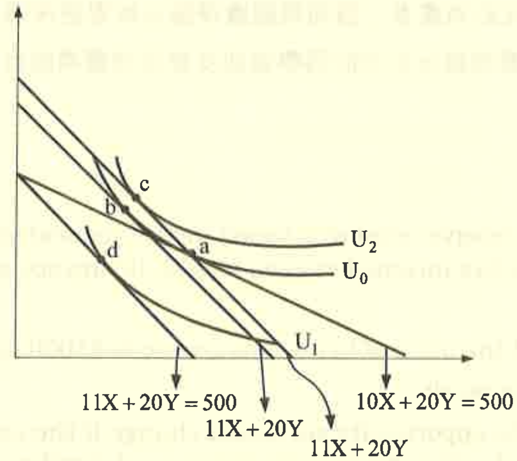




20.假設某國僅有一位追求效用極大的消費者，他的效用函數為 $U=X\cdot Y$，其中 X 財貨價格為 10，Y 財貨價格為 20，他的所得為 500。假設現在政府對 X 財按價格課徵百分之十的貨物稅，請計算此一貨物稅所造成的替代效果與所得效果（請以 X 的數量變化表示之）。請問政府可以課到多少稅收？如果政府取消貨物稅，但改課以相同稅收的定額稅，請問該消費者

的效用水準比起課貨物稅時的效用水準增加（或減少）多少？【北大經研所】

解：



(1)先計算課徵貨物稅造成 SE、IE

Step 1：先求 a 點(25, 12.5)

$$\boxed{\begin{array}{ll} \text{Max} & U=XY \\ \text{s.t} & 10X+20Y=500 \end{array}}$$

$$X^*=\frac{500}{2\times10}=25$$

$$Y^*=\frac{500}{2\times20}=12.5$$

$$U_0=312.5$$

Step 2：再求出 d 點(22.73, 12.5)

$$\boxed{\begin{array}{ll} \text{Max} & U=XY \\ \text{s.t} & 11X+20Y=500 \end{array}}$$

$$X^*=\frac{500}{2\times11}=\frac{250}{11}=22.73$$

$$Y^*=\frac{500}{2\times20}=\frac{25}{2}=12.5$$

$$U_1=284.09$$

Step 3：再求 c 點

$$\boxed{\begin{array}{ll} \text{Max} & U=XY \\ \text{s.t} & 11X+20Y=m' \end{array}}$$

將$(X_0,Y_0)=(25,12.5)$代入

$$275+250=m'=525$$

$$\therefore X^*=\frac{525}{2\times11}=23.86$$

$$Y^*=\frac{525}{2\times20}=13.125$$

Step 4：最後求出 b 點

$$\boxed{\begin{array}{ll} \text{Min} & E=11X+20Y \\ \text{s.t} & U=XY=312.5 \end{array}}$$

$$X^*=23.84$$

$$Y^*=13.11$$

$$E=524.44$$

Step 5：結論

◆<u>Hicksian 分析法</u>

SE $(a\to b)$ $23.84-25=-1.16$

IE $(b\to d)$ $22.73-23.84=-1.11$

PE $(a\to d)$ $22.73-25=-2.27$

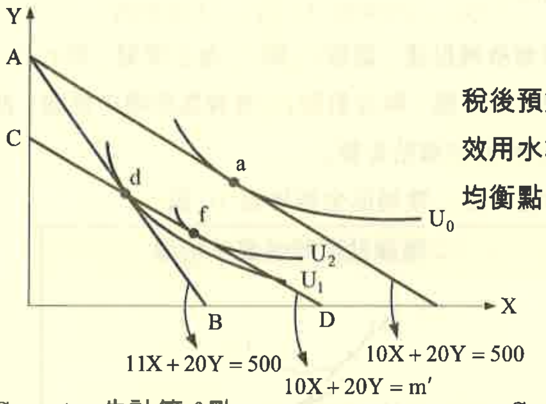
◆<u>Slutsky 分析法</u>

SE $(a\to c)$ $23.86-25=-1.14$

IE $(c\to d)$ $22.73-23.86=-1.13$

PE $(a\to d)$ $22.73-25=-2.27$

(2)取消貨物稅，改課定額稅



	貨物稅	定額稅
稅後預算線：	$\overline{AB}$	$\overline{CD}$
效用水準：	U_1	U_2
均衡點：	d	f

Step 1：先計算 f 點

$$\boxed{\begin{array}{ll} \text{Max} & U = XY \\ \text{s.t} & 10X + 20Y = m' \end{array}}$$

將 d 點(22.73，12.5)代入，求出 $m' = 477.3$

$$\therefore\ X^* = \frac{477.3}{2\times10} = 23.865\ Y^* = \frac{477.3}{2\times20} = 11.9325$$

$U_2 = 284.769$

Step 2：課定額稅

$U_2 = 284.769$

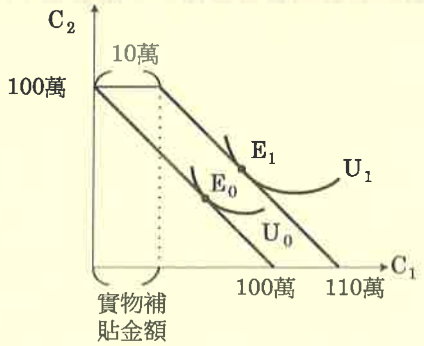
課貨稅，$U_1 = 284.09$

效用提昇 0.679

21.In order to spur consumer spending in 1998, the Japanese government considered an \$85 billion check system whereby every Japanese consumer would receive a shopping check that could be used to purchase Japanese products. For simplicity, assume the following: each consumer has wealth of 1 million yen, consumers must allocate this wealth consumption now (C_1) and consumption later (C_2), the interest rate is zero, the check is worth 10000 yen, and it can be spent only in the current period. If it is not spent, it is lost.
(1)Plot a budget line for a representative consumer both before and after the check program.
(2)Do you expect that current consumption of a typical consumer will inerease by the full 100000 yen of the check? Explain. (3)How does the impact of this 100000-yen check differ from simply giving the individual 100000yen?

解：1998 年因為日本物價上漲非常嚴重，因此日本政府發行購物券(Shopping check)，價值 10 萬日圓，為實物補貼一種，假設代表性消費者擁有 100 萬日圓所得，利率為零。

(1)

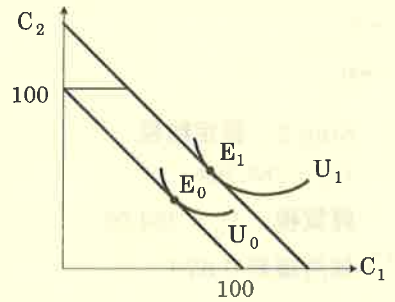
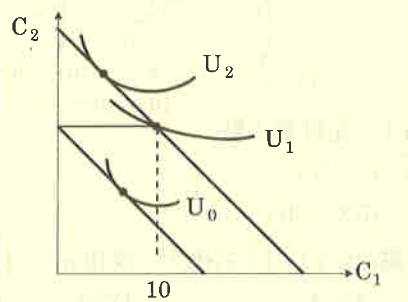


補貼前，消費者預算限制式為 $C_1 + C_2 = 1000000$

實施購物券補貼後，預算線呈拗折狀 $\begin{cases} 若C_1 \leq 100000, & C_2 = 1000000 \\ 若C_1 > 100000, & C_1 + C_2 = 1100000 \end{cases}$

(2)補貼前，最適消費均衡點 E_0，實施購物券補貼後，假設 C_1 與 C_2 為正常財，則 $C_1\uparrow$，$C_2\uparrow$，因此 C_1 增加金額必小於 100000 的實物補貼金額；除非假設 C_2 消費為所得中性物，此時才有可能 C_1 增加的消費金額恰好等於 10000 的實物補貼金額。

(3)實物補貼與所得補貼比較：……...前提：設二種補貼金額皆為 10 萬

二種補貼經濟效果相同	二種補貼經濟效果不相同
	
$\begin{cases} 消費金額（C_1,C_2）相同 \\ 補貼後效用相同 \end{cases}$	現金補貼效用高於實物補貼

相同稅收

22.【是非題】Other things being equal, the same amount of tax in the form of "income tax" will bring higher utility to tax payer, compared with "ad valorem tax." (2.5 分)【97 台大 B】

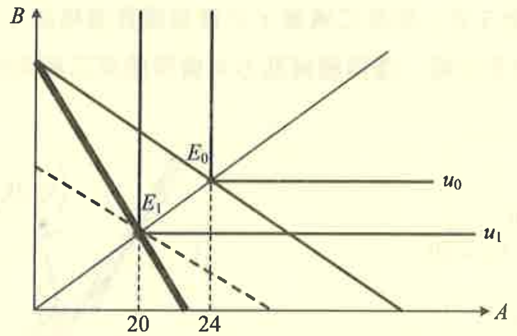
解：正確。維持兩種稅收相同情況下，課徵所得稅稅後效用仍高於貨物稅稅後效用。

23.當一消費者以全部的所得去消費 X、Y 兩種商品，請以無異曲線分析法畫圖比較並說明對 X 課貨物稅及相同課稅金額之定額稅對此人效用的影響。(25 分)【97 中央經研所】

解：參閱講義。

24.當 A、B 兩商品為完全互補情況下，若 A 商品價格(P_A)上升 40 元後，使得 A 商品由原來消費 24 單位降為 20 單位，請問價格上升效果相當於對該消費者課徵多少定額稅？(15 分)【97 嘉義應經所】

解：

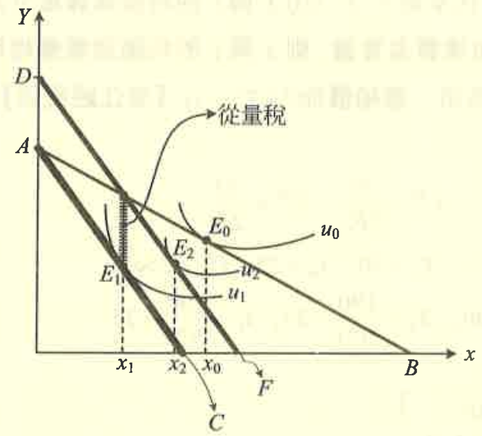


$$\begin{array}{l} (P_A^0+40)20+P_B^0B_1=M_0 \\ P_A^0\cdot 20+P_B^0B_1=M_1 \\ \hline 800=(M_0-M_1) \end{array}$$

表示價格上漲效果相當於對消費者課 800 元定額稅。

25.為因應京都議定書協定及國際油價高漲，若環保署擬提出政策建議政府對使用汽油課徵能源稅，惟為不影響民眾所得和消費，建議政府將能源稅稅收以退稅方式還給民眾，試以消費者理論分析，請問環保署所擬這項政策建議，若實際推行有何影響？(20 分)【97 嘉義應經所】

解：



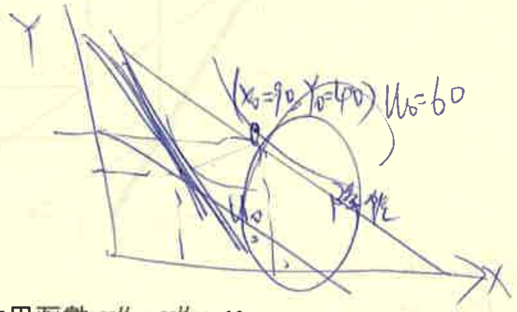
政府將民眾所繳納從量稅稅收退稅給民眾，使得預算線從 $\overline{AC}$ 平行右移到 $\overline{DF}$，退稅後消費量可能增加或減少，但效用水準必下降。

		預算線	效用水準	消費量
原均衡	E_0	$\overline{AB}$	u_0	x_0
課稅後	E_1	$\overline{AC}$	u_1	x_1
退稅後	E_2	$\overline{DF}$	u_2	x_2

26.某乙效用函數為 $U(X,Y)=X^{0.5}Y^{0.5}$，X 之單位價格為 9 元，Y 之單位價格為 9 元，某已知所得為 720 元，今政府為改善某乙之生活，考慮以下列兩種補助方式。方式一：規定某乙每

買一單位 X ，政府便補助 5 元（即某乙購買 X 之實質購買價格為 4 元），方式二：則是直接給某乙 T 元。請問 T 等於多少時，這兩種補貼方式會帶給某乙相同的效用？

解：(1)



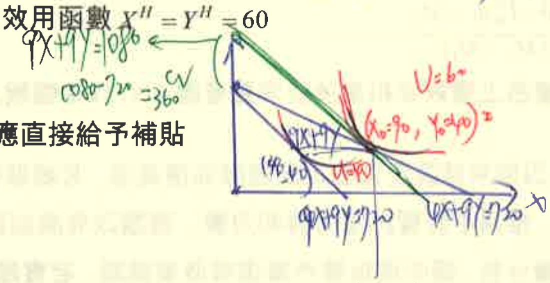
$$\boxed{\begin{array}{l}\text{Max } U=X^{\frac{1}{2}}Y^{\frac{1}{2}}\\ \text{s.t } 4X+9Y=720\end{array}}$$

$X_0=90$，$Y_0=40$，$U_0=60$

(2)

$$\boxed{\begin{array}{l}\text{Min } E=9X+9Y\\ \text{s.t } U_0=\sqrt{XY}=60\end{array}}$$

根據 $\dfrac{MU_X}{MU_Y}=\dfrac{P_X}{P_Y}\Rightarrow\dfrac{Y}{X}=1\Rightarrow Y=X$　　代回效用函數 $X^H=Y^H=60$



支出 $=9(60)+9(60)=1080$

(3)為了維持相同效用水準 $U_0=60$，政府應直接給予補貼

補貼金額 $(CV)=1080-720=360$

27.【複選題】假設某人有所得 100 元，其效用函數為 $U=X^{0.5}Y^{0.5}$，X 與 Y 的價格分別為 1 元與 2 元(A) X 與 Y 的邊際替代率是 Y/X (B) X 與 Y 的均衡消費量分別是 25 與 50 (C)如果政府單位 X 課徵 1 元的稅，且由消費者負擔，則 X 與 Y 的均衡消費量相等 (D)在課徵稅的情況下，如果要使某人維持稅前的效用，應補償他 $25(2^{0.5}-1)$【淡江經研所】

解：<u>(A)(C)</u>

(A) $MRS=\dfrac{-dY}{dX}=\dfrac{MU_X}{MU_Y}=\dfrac{Y}{X}$，$X^M=\dfrac{M}{2P_X}$，$Y^M=\dfrac{M}{2P_Y}$

(B) $P_X^0=1$，$P_Y^0=2$，$M_0=100$，$X_0=50$，$Y_0=25$，$U_0=25\sqrt{2}$

(C) $P_X'=2$，$P_Y^0=2$，$M_0=100$，$X_1=\dfrac{100}{2(2)}=25$，$Y_1=\dfrac{100}{2(2)}=25$

(D)

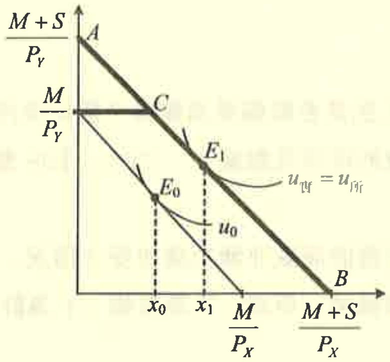
$$\boxed{\begin{array}{l}\text{Min } E=2X+2Y\\ \text{s.t } U_0=\sqrt{XY}=25\sqrt{2}\end{array}}$$

$X^H=Y^H=25\sqrt{2}$　　支出 $=2(25\sqrt{2})+2(25\sqrt{2})=100\sqrt{2}$　　$CV=100\sqrt{2}-100$

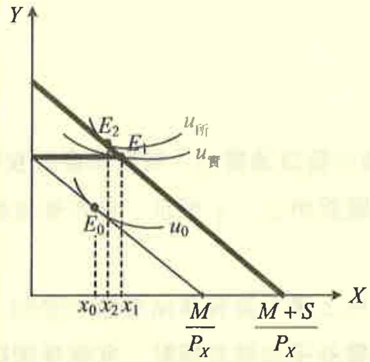
28.Assuming that there is public interest in insuring that all persons receive a minimum amount of education. Given that many indigent families would spend little or no resources on their children's education if they are to finance the cost of education themselves. There are two ways proposed for the government to help those families. ① Providing lump sum money payments to families in amounts sufficient to purchase the minimal requirements per child. ② Providing a non-transferable voucher to each child in a family entitling him/ her to receive the minimum educational benefits free of charge. (1)Which of the two methods of public support of education would be more effective in insuring the minimum requirement of education?
(2)Which of the two methods would generate a larger increase in the family's welfare?
【95 成大國企所】

解：$\begin{cases}\text{所得補貼→提供家庭一筆固定金額學費補貼} \\ \text{實物補貼→提供教育券，不得移轉}\end{cases}$

case 1：在相同補貼金額下，二者補貼有相同經濟效果，即有相同效用水準與財貨消費量



case 2：在相同補貼金額下，二者補貼經濟效果不同



實物補貼均衡點 E_1

所得補貼均衡點 E_2

站在政府立場，實物補貼優於所得補貼，因實物補貼較能鼓勵家庭接受教育；然而對家庭而言，所得補貼優於實物補貼，可以得到較高效用水準。

29.假設某甲有 10 萬元現金，將於今、明兩年用完，若政府於今年初發放 3,600 元消費券，並限定於今年年底前使用，過期作廢，當利率為零時，

(1)請畫出某甲在政府未發放消費券時，今、明兩年的預算線。

(2)請畫出某甲在政府發放消費券時，今、明兩年的預算線。【98 嘉大管研所】

解：

30.面對全球金融風暴，政府採取了發放消費券的政策，請分別從個體經濟分析（實物 VS.現金），畫圖並詳細論述此一消費券之政策效果。【98 成大企研所】 (25%)

解：

31.2008 年底，由於全球金融風暴所致，台灣的景氣亦受到極大的影響，為使國內的經濟活動愈趨活絡，政府於 2009 年 1 月 18 日發放「振興經濟消費券」，希望藉由人民的消費增加來促進經濟成長。然而，在政府設計消費券的當時，曾經討論過「直接發予現金」而不發消費券的方式，請回答以下問題：

(1)若是政策目標係要「促成國內消費金額增加」的前提下，直接發現金與發消費券兩種方式，

何者較能達成效果？(15 分)

(2)若是政策目標係要「使民眾的福利達到極大」的前提下，直接發現金與發消費券兩種方式，何者較能達成效果？(15 分)

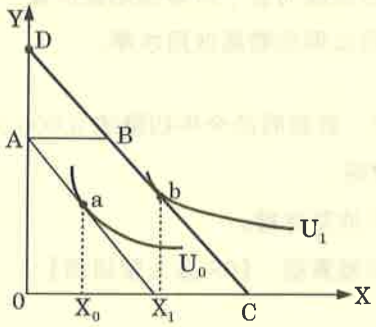
（需以圖形和文字說明始予計分。提示：圖形橫軸為消費財，縱軸為消費財以外事物）【98 銘傳管研所】

解：

32.政府為了刺激經濟而發放消費券，相較於發放食物券，人民是否較偏愛消費券？請以效用函數及預算限制式繪圖說明之。（提示：食物券有限定消費的種類及數量） (20%)【98 暨南國企所】

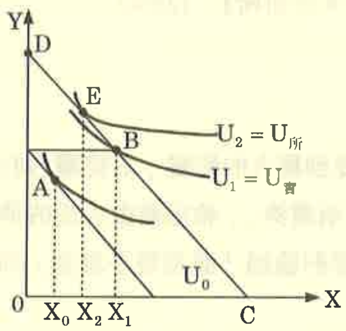
解：由於消費券在使用上幾乎沒有商品種類的限制，決大多數的商家亦無拒絕收受之情況，因此本題中我們將消費券視為現金補貼，食物券則視為實物補貼。假設：X 為食物，Y 為計價財

case 1：如果消費者比較偏好現在消費，則無異曲線較陡，因此所得補貼與食物補貼有相同經濟效果；$U_{所}=U_{實}$；$X_{所}=X_{實}$



實物補貼		所得補貼
拗折線 ABC		DC
均衡點 b		均衡點 b
補貼後效用 U_1	=	補貼後效用 U_1
均衡消費量 X_1	=	均衡消費量 X_1

case 2：如果消費者比較不偏好現在消費，則無異曲線較平坦：



實物補貼		所得補貼
拗折線 ABC		DC
均衡點 B 點		均衡點 E 點
補貼後效用 U_1	<	補貼後效用 U_2
均衡消費量 X_1	>	均衡消費量 X_2

33.In order to spur consumer spending, the Taiwanese government implemented a voucher system whereby every Taiwanese consumer would receive a shopping voucher that could be used to purchase products or services. For simplicity, assume the following: each consumer has wealth of NT$1,000,000, consumers must allocate this wealth between consumption now (c_1), and consumption later (c_2). the interest rate is zero, the voucher is worth NT$3,600, and it can be spent only in the current period. If it is not spent, it is lost. (15%)

(1) Plot budget lines for a representative consumer both before and after the vocher program (c_1 and c_2 are on the axes).

(2) Do you expect that current consumption of a typical consumer will increase by the full NT$3,600 of the voucher? Explain your answer by indifference curve and budget line diagram.

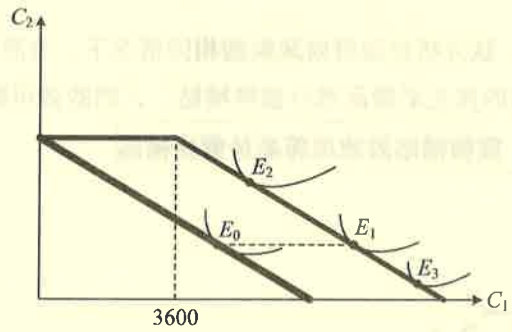
(3) Can this NT$3,600 voucher induce more consumption now (c_1) than simply giving the individual NT$3,600 cash? Explain your answer by indifference curve and budget line diagram. 【98 台科大企研所】

解：

(1)政府發放 3600 元消費券，為「實物補貼」，補貼後預算線：

$C_1 + C_2 = 1{,}003{,}600$ ， $C_1 \ge 3600$

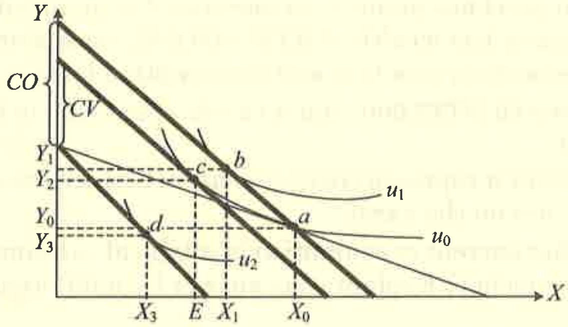
(2)



發放消費券之後，本期消費量增加幅度 $\begin{cases} E_0 \to E_1 : \Delta C = 3600 \\ E_0 \to E_2 : \Delta C < 3600 \\ E_0 \to E_3 : \Delta C > 3600 \end{cases}$

34.The old people who depend on government security payment as the major income source are usually hurt by high rate of inflation. In order to compensate for their loss, two proposals are raised. Proposal I argues that the old people should be compensated to the level that they are able to maintain the original consumption level, while proposal 2 suggests that the old should be compensated to the level that they are able to maintain the original utility level. Which proposal benefits more to the old? Use utility graph to back up your answer. 【成大企管、國企】

解：



(1) 原均衡 a 點，效用極大化需求量$(X_0,\ Y_0)$

(2) 新均衡 d 點，$(X_3,\ Y_3)$

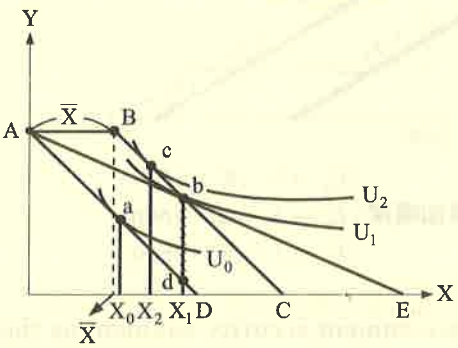
(3) 為了使老人可以維持原購買組合$(X_0,\ Y_0)$不變，均衡點 b，對應$(X_1,\ Y_1)$，須補貼消費者 CD「成本差額」

(4) 為了維持原來滿足水準不變，均衡點 c，對應$(X_2,\ Y_2)$，須補貼消費者 CV「補償變量」

(5) 站在老人立場，方案一可以帶來更高效用水準$(U_1 > U_0)$，不過政府補貼支出卻較高。

35.政府為照顧低收入者，試分析在政府財政負擔相同情況下，政府採直接發放民生必需品，抑或採對低收入戶所購買的民生必需品進行價格補貼，人們的效用較高？【政大金融所】

解：在相同補貼金額下，實物補貼的效用將高於價格補貼



比較項目	價格補貼		實物補貼
模型建立	Max $U = U(X, Y)$ s.t $(P_X - S)X + P_Y Y = M$		Max $U = U(X, Y)$ s.t $P_X(X - \overline{X}) + P_Y Y = M$ 其中 $S = P_X \overline{X}$
補貼後預算線	$AD \to AE$		$AD \to ABC$
補貼後均衡點	$a \to b$		$a \to c$
補貼後均衡消費量	X_1	>	X_2
補貼金額	$S = s \cdot X_1 = \overline{bd}$	=	$S = P_X \cdot \overline{X} = \overline{bd}$
補貼後效用	U_1	<	U_2

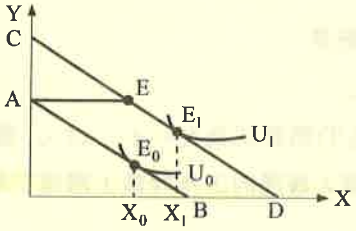
36.假說政府給予某甲二種援助的選擇：一種是實物補貼，提供 100 單位食物(相當於$1,000)的補助；另一種是現金補貼，提供$1,000 現金的補助，試以橫座標為食物，縱座標為其他物品，利用無異曲線分析，繪圖說明：

(1)若甲相對較偏好食物，則何種補貼對甲而言較佳？

(2)若甲相對較偏好其他物品，則何種補貼對甲而言較佳？

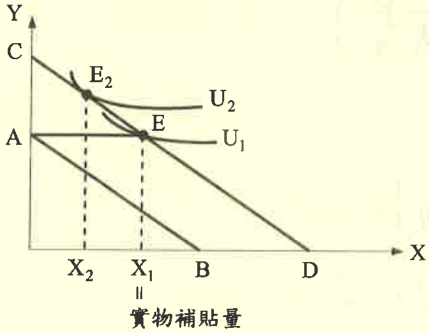
解：令 X 為食物，Y 為其它物品

(1)若甲相對較偏好食物，則對甲而言，二種補貼具有相同效用水準



現金補貼	實物補貼
CD	AED
$E_0 \rightarrow E_1$	$E_0 \rightarrow E_1$
X_1	X_1
$U_0 \rightarrow U_1$	$U_0 \rightarrow U_1$

(2)若甲較偏好其它物品，則對甲而言，現金補貼優於實物補貼



現金補貼		實物補貼
CD		AED
E_2		E
X_2		X_1=實物補貼量
$U_0 \rightarrow U_2$	>	$U_0 \rightarrow U_1$

主題三：休閒與工作選擇

1.假設個人消費兩種財貨：休閒(L)和商品(X)，效用函數是 $U(X,L)=X^{0.5}L^{0.3}$，休閒(L)和商品(X)的價格分別為 w 和 1。假設個人所得完全來自工作，個人可工作時間最多為 T(預算式為 $X+wL=wT$)。個人選擇最適的工作時數來極大化效用。請問工資(w)上漲時個人最適的工作時數上漲或下跌？【台大財金所、台大農經所】

解：

$$\boxed{\begin{array}{ll} \text{Max} & U=X^{0.5}L^{0.3} \\ \text{s.t} & X+wL=wT \end{array}}$$

$$L=X^{0.5}L^{0.3}+\lambda[wT-X-wL]$$

$C=W(T-L)+\pi$

F.O.C $\dfrac{\partial L}{\partial L}=0 \Rightarrow 0.3X^{0.5}L^{-0.7}-\lambda w=0$

$\dfrac{\partial L}{\partial X}=0 \Rightarrow 0.5X^{-0.5}L^{0.3}-\lambda=0$

$\dfrac{\partial L}{\partial \lambda}=0 \Rightarrow wT-X-wL=0$

$C=wT-wL$

$wL+C=wT$

$$\boxed{\begin{array}{l} X=\dfrac{5}{5+3}\dfrac{WT}{1}=\dfrac{5}{8}WT \\ L=\dfrac{3}{5+3}\cdot\dfrac{WT}{W}=\dfrac{3}{8}T \end{array}}$$

$\dfrac{\partial W}{\partial L}=0$

$$—\Rightarrow \frac{3X}{5L}=w \Rightarrow X=\frac{5}{3}Lw \text{ 代入}$$

$$\frac{5}{3}Lw+wL=wT \Rightarrow \frac{5}{3}L+L=T \Rightarrow L^{*}=\frac{3}{8}T$$

$$N^{S}=\frac{5}{8}T\ (\text{工作時數})\text{；}\ X^{*}=\frac{5}{3}\left(\frac{3}{8}T\right)w=\frac{5}{8}Tw$$

而 $\frac{dN^{S}}{dw}=0 \Rightarrow$ 工資率上漲時，個人最適勞動供給仍為 $\frac{5}{8}T$

不受工資率上漲之影響。

2.假設個人消費兩種財貨：休閒(L)和商品(X)，效用函數是

$U(X,L)=X^{a}L^{b}$，休閒(L)和商品(X)的價格分別為 w 和 1。

假設個人所得完全來自工作，個人可工作的時間最多為 T(預算式為 $X+wL=wT$)。個人選擇最適的工作時數來自極大化效用。請問工資(w)上漲時個人最適的工作時數上漲或下跌？【台大財金所】

解：Max $u=X^{a}L^{b}$

s.t $X+wL=wT$

$$\boxed{X=\frac{a}{a+b}\cdot\frac{wT}{P_x}=\frac{awT}{a+b} \quad Y=\frac{b}{a+b}\cdot\frac{wT}{P_y}=\frac{bT}{a+b}}$$

$£=X^{a}L^{b}=\lambda(X+wL-wT)$

F.O.C $\frac{\partial l}{\partial L}=0 \Rightarrow bX^{a}L^{b-1}+w\lambda=0$

$\frac{\partial l}{\partial X}=0 \Rightarrow aX^{a-1}L^{b}+w\lambda=0$

$\frac{\partial l}{\partial L}=0 \Rightarrow X+wL=0$

$\frac{b}{a}wL+wL=wT$

$\frac{(1)}{(2)} \Rightarrow \frac{bX}{aL}=W$ $X=\frac{a}{b}wL$

$L=\frac{b}{a+b}T$

$L^{*}=\frac{bT}{a+b}$ $N^{*}=T-L=\frac{aT}{a+b}$，$X^{*}=\frac{awT}{a+b}$

$\frac{\partial N^{s}}{\partial w}=0 \Rightarrow$ 當工資率上漲時，個人最適工作時數不受影響。

$400+(T-40)\times$

3.李小姐其效用函數(Utility function)如下：$U=I^{0.25}L^{0.75}$，其中 I 為李小姐每週的消費總金額，L 為其每週的休閒小時數，李小姐可以自由決定其上班時數，而其鐘點費為每十元。(1)請問李小姐每週上班時數。(2)若其上司決定將她高於 40 小時的工作時數部分，鐘點費調高為一倍半，請問新政策下，李小姐每週上班時數為何？【淡江國貿所】

$U=IL^{3}$ $I+10L=10T$

解：(1) $\boxed{\begin{array}{ll} Max & U=I^{0.25}L^{0.75} \\ s.t & I=10(T-L) \end{array}}$

$$\boxed{I=\frac{1\times 10T}{4\times 1}=\frac{5}{2}T \quad L=\frac{3\times 10T}{4\times 10}=\frac{3}{4}T}$$

$L=I^{0.25}L^{0.75}+\lambda[I-10T+10L]$

休閒

F.O.C $\frac{\partial L}{\partial L}=0 \Rightarrow 0.75I^{0.25}L^{-0.25}+10\lambda=0$

$\frac{\partial L}{\partial I}=0 \Rightarrow 0.25I^{-0.75}L^{0.75}+\lambda=0$

$N=T-L=T-\frac{3}{4}T=\frac{1}{4}T$

$I+15L=400+(T-40)\times 15$ if $L\le T-40$

$$\frac{\partial L}{\partial \lambda}=0 \Rightarrow I-10T+10L=0$$

$$\Rightarrow 3\times\frac{I}{L}=10 \text{ 代入} \qquad \therefore L^*=\frac{3}{4}T \qquad \text{工作時間為 } T-L^*=\frac{T}{4}$$

(2) $\boxed{\begin{array}{l} Max \quad U=I^{0.25}L^{0.75} \\ s.t \quad I=400+15(T-40-L),\ if\ L\le T-40 \end{array}}$

$$\Rightarrow \quad Max\ U=I^{\frac{1}{4}}L^{\frac{3}{4}}$$

$$s.t \quad I+15L=400+15(T-40)$$

$$L=I^{0.25}L^{0.75}+\lambda[15T-200-15L-I]$$

F.O.C $\frac{\partial L}{\partial L}=0 \Rightarrow 0.75I^{0.25}L^{-0.25}-15\lambda=0$

$$\frac{\partial L}{\partial I}=0 \Rightarrow 0.25I^{-0.75}L^{0.75}-\lambda=0$$

$$\frac{\partial L}{\partial \lambda}=0 \Rightarrow 15T-200-15L-I=0$$

$$L=\frac{3}{4}\left[\frac{400+15(T-40)}{15}\right]$$

$$\cancel{L=20+\frac{3}{4}T} \qquad L=\frac{3}{4}\left[\frac{400+15(T-40)}{15}\right]$$

$$\Rightarrow 3\times\frac{I}{L}=15 \text{ 代入} \qquad \therefore L^*=\frac{3}{4}T-10 \qquad \text{工作時間為 } T-L^*=\frac{T}{4}+10$$

$$L=20+\frac{3}{4}(T-40)$$

$$\boxed{L=20+\frac{3}{4}T-30=\frac{3}{4}T-10}$$

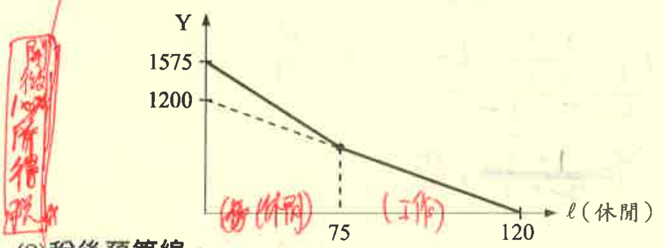
4.假設每週你有 120 小時可以花於工作或休閒，每小時工資為 10 元，若每週的工作時數超過 45 小時，則超過 45 小時的工作時數視為加班，每小時可以領較高的加班工資 15 元。

(1)請畫出預算線(橫軸為休閒時數、縱軸為工作所得)，並寫出預算限制式。

(2)若政府開徵 10%的所得稅率，則對預算線會造成何種影響？對你的工作意願又會造成何種影響？請說明你的理由。 【元智企研所】

解：(1)預算限制式為：

$$\begin{cases} Y=10(120-\ell),\ 75\le\ell\le120 \\ Y=450+15(75-\ell),\ 0\le\ell<75 \end{cases} \quad \text{Y 為所得，}\ell\text{ 為休閒} \quad \begin{cases} M=1200-10\ell,\ 75\le\ell\le120 \\ M=1575-15\ell,\ 0\le\ell<75 \end{cases}$$



(2)稅後預算線：

$$\begin{cases} Y=9(120-\ell),\ 75\le\ell\le120 \\ Y=405+13.5(75-\ell),\ 0\le\ell<75 \end{cases} \quad \begin{cases} Y=1080-9\ell,\ 75\le\ell\le120 \\ Y=1417.5-13.5\ell,\ 0\le\ell<75 \end{cases}$$

當政府課徵比例工資所得稅時，對勞動供給影響，需視休閒為正常財或劣等財而定。

稅後工資下降
- SE：休閒機會成本減少，消費者選擇多休閒，少工作
- IE：稅後勞動所得減少
  - 休閒為正常財 ⇒ 少休閒，多工作
  - 休閒為劣等財 ⇒ 多休閒，少工作
  - 休閒為中性財 ⇒ 休閒，工作

總效果之影響未定

5.Mr. B's utility function is $U(C,R)=CR^2$, where C is the amount of money he spends on consumption each day (the price of C is \$1), and R is the number of hours a day he spends not working (his leisure hours). He has no earning other than his labor income. If his wage rate increase from \$5 to \$10 an hour, how many more hours does he choose work per day?【中興財金所】

$U=CR^2$

解： Max $U(C,R)=CR^2$

s.t $C=W(24-R)$

$C+WR=24W$

$L=CR^2+\lambda(C+WR-24W)$

$C=\frac{1}{3}\times\frac{24W}{1}=8W \Rightarrow R=\frac{2\times 24W}{3\times W}=16$

F.O.C $\frac{\partial L}{\partial C}=0\Rightarrow R^2+\lambda(1)=0$ $\qquad \frac{\partial L}{\partial R}=0\Rightarrow 2CR+\lambda(W)=0$

$N=24-16=8$

$\therefore R^*=16$， $C^*=8W$ $\qquad L^s=24-R^*=8$

由於最適勞動供給函數不受工資高低之影響，因此當工資上漲時，消費者最適工作時數不變。

6.Take the case of an individual who drives utility from income (Y) and leisure (L). Denote hours worked in a day by W: the consumer has only T total hours in a day to divide between work and leisure. Income is derived from working, at the wage rate of r per hour. Assume the individual's utility function $U(L,Y)=LY$.

(1)Write down the budget constraint for this consumer. 【成大工管所】

(2)Find income and leisure at the optimum.

(3)Suppose the wage rate r goes up, how does this affect the amount of leisure?

解：(1)預算線：$W+L=T$， $Y=rW$

$\therefore Y=r(T-L)$

work

(2)

Max: $U=LY$

$Y+L\cdot r=T\cdot r$

$$\boxed{\begin{array}{l}\text{Max } U=LY\\ \text{s.t } Y+rL=rT\end{array}}$$

$L=LY+\lambda(r+rL-rT)$

$\Rightarrow L=\frac{1\cdot Tr}{2\cdot r}=\frac{1}{2}T \rightarrow \frac{\partial L}{\partial r}=0$

F.O.C $\frac{\partial L}{\partial L}=0\Rightarrow Y+\lambda(r)=0$

$W=T-\frac{1}{2}T=\frac{1}{2}T$

$\frac{\partial L}{\partial Y}=0\Rightarrow L+\lambda(1)=0$

$Y=\frac{1\cdot Tr}{2\cdot 1}=\frac{rT}{2}$

聯立求解 $L^*=\frac{T}{2}$，$Y^*=\frac{rT}{2}$

(3) $L^*=\frac{T}{2}$； $\frac{\partial L^*}{\partial r}=0$

$C+\ldots=(T-l\ldots)\times 150+4000$

$\therefore$工資提高不影響消費的最適休閒時間。

$C+150l=150T+$

加班費的考慮

7.若甲每週共有 120 小時可分配於工作(N)或休閒(ℓ)工作所得則可用於消費(c)，每週工作時數若低於 40 小時，則每小時工資為 100 元，若超時工作(工作時數超過 40 小時的部分)則每小時的加班工資為 150 元，若甲的效用函數為$U(\ell,C)=\ell^3C^3$，則甲會選擇每週工作______小時。

4000

【中央人管所】

解：Max $U=\ell^3C^3$

$$\begin{cases} C=100(120-\ell), & 80\le \ell\le 120\\ C=4000+150(80-\ell), & 80\ge \ell\end{cases}$$

$$\text{s.t}\quad \begin{cases} C=100(120-\ell),\ 80\le \ell \le 120 \\ C=4000+150(80-\ell),\ \ell<80 \end{cases} \Rightarrow \begin{cases} C=12000-100\ell,\ 80\le \ell \le 120 \\ C=16000-150\ell,\ \ell<80 \end{cases}$$

(1)若 $80\le \ell \le 120$ 時 $\boxed{\begin{array}{ll} \text{Max} & U=\ell^3C^3 \\ \text{s.t} & C=12000-100\ell \end{array}}$ $\therefore\ \ell^*=60$（不合）

(2)若 $\ell<80$ 時 $\boxed{\begin{array}{ll} \text{Max} & U=\ell^3C^3 \\ \text{s.t} & C=16000-150\ell \end{array}}$ $\therefore\ \ell^*=\dfrac{160}{3}=53.33$

$C^*=8000$ $\qquad n^*=120-53.33=66.67$

8.凱毓每週可支配使用的時間秉賦(endowment)有 T 小時,而她目前工作的報酬為每小時 W 元，又凱毓的效用函數為：$U=72n+nc-n^2$，其中 c 為消費金額，n 為休閒時間，則：在受預算限制而追求效用極大的目標下，請導出凱毓的勞動供給函數。

解：$\boxed{\begin{array}{ll} \text{Max} & U=72n+nc-n^2 \\ \text{s.t} & c=W(T-n) \end{array}}$ →非 CD function

$\Rightarrow \text{Max}\quad U=72n+WnT-Wn^2-n^2$

$$\text{F.O.C}\quad \frac{\partial U}{\partial n}=0 \quad \Rightarrow \quad 72+WT-2Wn-2n=0 \qquad \therefore\ n^*=\frac{72+WT}{2(W+1)}$$

$$L^S=T-n^*=\frac{2T-72}{2(W+1)}+\frac{WT}{2(W+1)}$$

9.最近台灣股市大漲,假設台灣民眾平均投機利得皆增加,請問你會增加工作還是減少工作？會增加儲蓄還是減少儲蓄？假設休閒為正常品，請利用無異曲線分析法，討論這兩情況下的替代效果(substitution effect)與所得效果(income effect).。

解：(1)此時，最適化模型為

$\boxed{\begin{array}{ll} \text{Max} & U=U(C,R) \\ \text{s.t} & PC=W(T-R)+N+100 \end{array}}$ $\qquad PC+WR=(WT+N+100)$

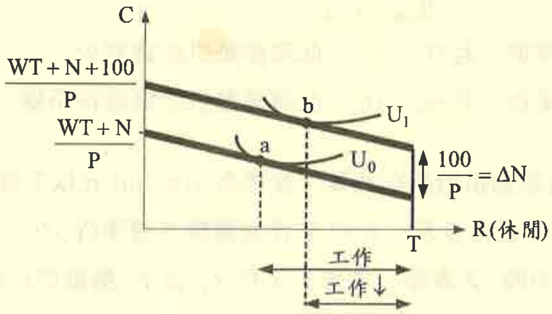
突然機會中獎 100 萬，非勞動所得增加 100 萬，但消費者之實質工資並未改變，因此，預算線平行右移，只會產生所得效果，**不會造成替代效果。**（因休閒之相對價格不受影響）

(2) N↑⇒消費者實質所得增加，在休閒為正常財情況下，休閒數量↑,工作↓，消費↑，儲蓄減少。

(3)

IE(a → b)

使工作↓，C↑，S↓



微分推導 $\frac{\partial U/\partial L}{\partial U/\partial C} = \frac{1}{2\times 5}L^{-\frac{1}{2}} = \frac{P}{W}$ $P=W\sqrt{L}$ $L=\left(\frac{P}{W}\right)^2$

10.Consider the following utility function of a consumer : $U = C + 2L^{\frac{1}{2}}$

Where C is goods and L is leisure. Derive his labor supply function 【政大國貿所】

解 $\boxed{\begin{array}{ll} Max & U = C + 2L^{\frac{1}{2}} \\ s.t & PC = W(24-L) \end{array}}$ $L = C + 2L^{\frac{1}{2}} + \lambda[PC - 24W + WL]$

$PC+WL=24W$ $\lambda = \frac{-1}{P}$ $\lambda = \frac{-L^{-\frac{1}{2}}}{W}$

F.O.C $\frac{\partial L}{\partial C} = 0 \Rightarrow 1 + \lambda P = 0$ $\frac{\partial L}{\partial L} = 0 \Rightarrow L^{-\frac{1}{2}} + \lambda W = 0$ $\frac{\partial L}{\partial \lambda} = 0 \Rightarrow PC - 24W + WL = 0$

$\Rightarrow L^{\frac{1}{2}} = \frac{P}{W} \Rightarrow L^* = \left(\frac{P}{W}\right)^2$ ………休閒需求函數 代回 $N^S = 24 - L^*$，

即可求出勞動供給函數 $N^S = 24 - \left(\frac{P}{W}\right)^2$ $N^S = 24 - L = 24 - \left(\frac{P}{W}\right)^2$ 勞動

11.某甲忽獲得 500 萬元遺產贈與，並同時得到 20%的加薪，假設某甲工作時數可彈性調整，分析上述事件將如何影響某甲之工作時數。 【台大財金所】

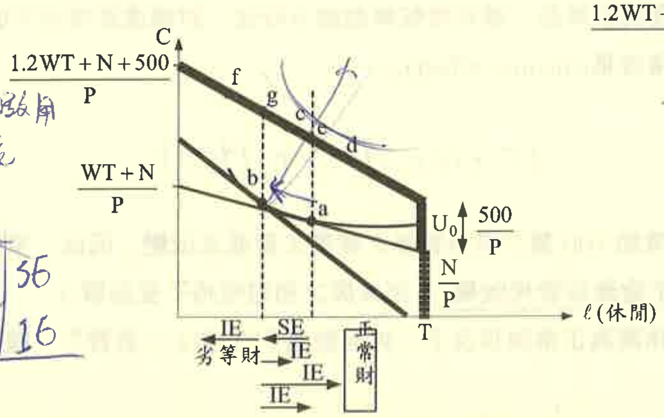
解：(1)模型建立

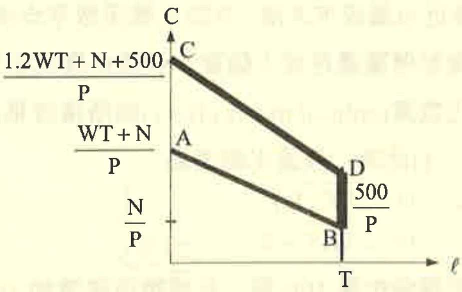
$\boxed{\begin{array}{ll} Max & U = U(C, \ell) \\ s.t & PC = W(T-\ell) + N \end{array}} \Rightarrow \boxed{\begin{array}{ll} Max & U = U(C, \ell) \\ s.t & PC = 1.2W(T-\ell) + N + 500 \end{array}}$

$PC + 1.2W\ell = 1.2WT + N + 500$

(2)加薪 20%⇒W 提高

500 萬遺產⇒非勞動所得增加，使預算線從 ABT→CDT，產生 SE 與 IE，對勞動供給意願未定，預算線斜率變陡，且往右移





休閒 加薪變陡

C↑ ℓ↓ 36

C↑ ℓ↑ 16

若 IE=SE

C ℓ↑

❶若休閒為正常財，且 IE > SE ，此時勞動供給會減少

❷若休閒為正常財，且 SE = IE ，此時勞動供給會維持不變

12.假設某國的勞動供給線在實質工資率為 100000 元以下時可以用 $(Y-70000)^2 + (110X)^2 = 40000^2$ 來近似表示，其中 Y 代表實質工資率(Y > 0)，X 代表工作天(X > 0)。問當實質工資率低於多少時，大家都不會想去工作？(提示：勞動供給函數為圓形的一部份。)(A)40000 元(B)70000 元(C)20000 元(D)30000 元【98 交大經管所】

解：(B)；本題勞動供給曲線為圓形，將 $X=0$ 代入勞動供給函數可得 $(Y-70{,}000)^2=40{,}000^2 \Rightarrow Y=30{,}000$ 或是 $Y=110{,}000$（不合）表示 $Y=30{,}000$ 以下的勞動供給量 $=0$。

13.假設王老五的勞動供給線在實質工資率為 100000 元以下時可以用 $(Y-70000)^2+(150X)^2=40000^2$ 來近似表示，其中 Y 代表實質工資率 $(Y>0)$，X 代表年工作天數 $(X>0)$。問王老五一年最少會休假幾天？（一年 365 天）（提示：勞動供給函數為圓形的一部份。） (A)105 天。 (B)81 天。 (C)120 天。 (D)98 天。(2 分)【96 交大經管所】

解：(D)，將勞動供給函數全微分可得 $2(Y-70000)dY+45000\times dX=0$

$$\therefore \frac{dX}{dY}=\frac{2(70000-Y)}{45000X}=0 \Rightarrow Y=70000$$ 代入勞動供給函數可得 $X=266.7$，故至少會休息 $365-266.7=98.3$ 天

14.假設老李正在研究「睡覺經濟學」，他和你我一樣每天都有 24 小時，其中 X 小時用於消費、S 小時用於睡覺，如果老李追求效用 $U=X^2S$ 最大，其中 $X=W(24-S)$，式中 W 為工資率，請問老李最佳之睡覺與消費時間分別為多少。　【台大農經所】

解：
$$\boxed{\begin{array}{ll}\text{Max} & U=X^2S \\ \text{s.t} & X=W(24-S)\end{array}} \qquad L=X^2S+\lambda[24W-WS-X]$$

$$\frac{\partial L}{\partial X}=0 \Rightarrow 2XS-\lambda=0 \qquad \frac{\partial L}{\partial S}=0 \Rightarrow X^2-\lambda W=0 \qquad \frac{\partial L}{\partial \lambda}=0 \Rightarrow 24W-WS-X=0$$

$$\Rightarrow \frac{2S}{X}=\frac{1}{W} \Rightarrow X=2SW \quad \text{代入 } 24W=WS+2SW$$

$$\therefore \begin{cases} S^*=8 \\ X^*=16W \end{cases} \Rightarrow \begin{cases} \text{睡覺8小時，工作16小時} \\ \text{消費16W} \end{cases}$$

24小時

$X+2SWS=24W$

$S=\frac{1}{3}\frac{24W}{W}=8$ (小時)

工作 16小時

$X=W\times(24-8)$

$X=16W$

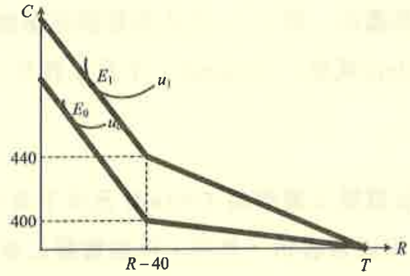
15. If an individual's labor supply curve slopes forward at low wages and bends backward at high wages, is leisure a Giffen good? If so, at high or low wage rates? And why? (10 分)【98 中山經研所】

解：錯誤，通常在低工資階段，勞動供給曲線呈現前彎形狀，而在高工資階段，勞動供給曲線呈現後彎形狀，表示休閒為正常財並且所得效果大於替代效果所致，並非休閒為季芬財。

IE > SE

16.Henry is paid \$100 and hour for the first 40 hours per week that he works. If he works more than 40 hours in a week, the extra hours are counted as overtime and are paid \$150 per hour. For example, if he works 50 hours, then his salary is $40\times\$100+10\times\$50=\$5{,}500$. Leisure is a normal good for Henry, and he is working more than 40 hours now. If his hourly wage for the first 40 hours per week rises to \$110, and the wage for overtime hours remain at \$150, then how will his working hours change?【96 中興財金所】

解：



前 40 小時工資率提高，對一個最適工作時間超過 40 小時的消費者而言，預算線平行右移，只有所得效果，沒有替代效果，假設休閒為正常財情況下，休閒量增加，工作量會減少。

主題四：跨期消費理論

1.Robinson Cursoe has the utility function $U(C_1, C_2) = C_1^{0.7}C_2^{0.3}$. C_1 and C_2 are the consumptions at day 1 and day 2, respectively. He owns endowments of 10 and 10 units at day 1 and 2, respectively. If the endowment is durable but does not appreciate, and if he is the only person who lives on the island, please find the optimal consumption C_1 day 1. ______【99 元智財金所】

解：

$C_1^* = 10$，因為稟賦量可以貯藏，但是沒有利息（令 $r = 0$），表示不存在借貸市場，因此效用極大化決策 $\begin{cases} Max\, U = C_1^{0.7}C_2^{0.3} \\ s.t\ \ C_1 + C_2 = 20 \end{cases}$ $C_1 \le 10$，表示此人只能儲蓄，不能借錢

$C_1^* = \dfrac{0.7\,(20)}{(0.7+0.3)\,1} = 14$，$C_2^* = \dfrac{0.3\,(20)}{(0.7+0.3)\,1} = 6$，但是題目限制條件為 $C_1 \le 10$，因此此人效用極大化消費量為 $C_1^* = 10$，$C_2^* = 10$，為「借貸平衡者」。

2.考慮某人的兩期消費模型。令兩期商品價格均為 1（即 $P_1 = P_2 = 1$），所得原賦為 $(Y_1, Y_2) = (1000, 540)$，名目利率為 8%。假設兩期消費的邊際效用分別為 $MU_1 = 1/C_1$ 與 $MU_2 = 0.7/C_2$。請問：

(1)該消費者的終身預算式為何？並畫出對應的預算線。

(2)此人在第一期與第二期最多可以消費多少？

(3)此人在第一期與第二期的最適消費各為多少？(15 分)【98 輔大管研所】

解：

(1) $C_1 + \dfrac{C_2}{1+8\%} = 1000 + \dfrac{540}{1+8\%} \Rightarrow 1.08C_1 + C_2 = 1620$

(2) 第一期最多可消費量 $C_1 = \dfrac{1620}{1.08} = 1500$

第二期最多可消費量 $C_2 = 1620$

(3) 消費者均衡條件：$\dfrac{MU_1}{MU_2} = (1+r) \Rightarrow \dfrac{1/C_1}{0.7/C_2} = 1+8\% \Rightarrow C_2 = 0.756C_1$

$\therefore 1.08C_1 + 0.756C_1 = 1620 \Rightarrow C_1^* = 882.3529$, $C_2^* = 667.0588$

3.Consider a model of only two periods: the current period and the future period. Let y and y^f denote the current real income and the future real income, respectively. Also, Let c, c^f and r denote the current real consumption. The future real consumption and the real interest rate, respectively.

(1) Draw a diagram (including the budget line and indifferent curves) to indicate the optimal consumption point for a saver and for a borrower, respectively. (6%)

(2) Draw a diagram to illustrate how the current real consumption of a saver would change if the real interest rate raise. (6%)

(3) Suppose that the government distributes consumption vouchers which are only effective for the current period to every resident. Draw a diagram to illustrate how the budget line of a borrower would change. Is it possible that the current real consumption of a borrower does not change at all? Explain. 【98 中央經研所】

解：

(1)模型之建立：
$$\boxed{\begin{array}{ll} \text{Max} & U = U(C_1, C_2) \\ \text{s.t} & C_1 + \dfrac{C_2}{1+r} \le Y_1 + \dfrac{Y_2}{1+r} \end{array}}$$

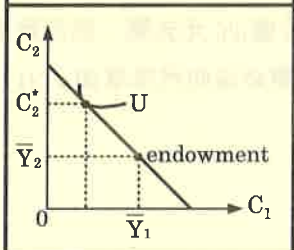
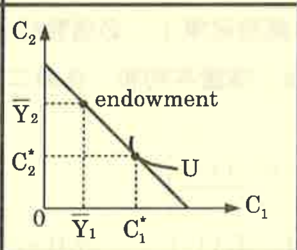
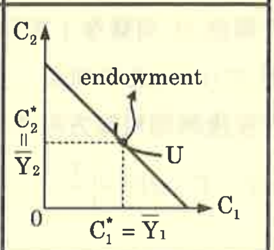
消費者面對完全借貸市場，且無任何借貸限制，在市場利率 r 固定情況下，透過跨期消費決策，追求一生效用最大。

最適化模型為 $L = U(C_1, C_2) + \lambda\left[Y_1 + \dfrac{Y_2}{1+r} - C_1 - \dfrac{C_2}{1+r}\right]$

F.O.C
$$\frac{\partial L}{\partial C_1} = 0 \quad \Rightarrow U_{C_1} - \lambda = 0 \cdots\cdots\cdots 1$$
$$\frac{\partial L}{\partial C_2} = 0 \quad \Rightarrow U_{C_2} - \frac{\lambda}{1+r} = 0 \cdots\cdots\cdots 2$$
$$\frac{\partial L}{\partial \lambda} = 0 \quad \Rightarrow Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} = 0 \cdots\cdots\cdots 3$$

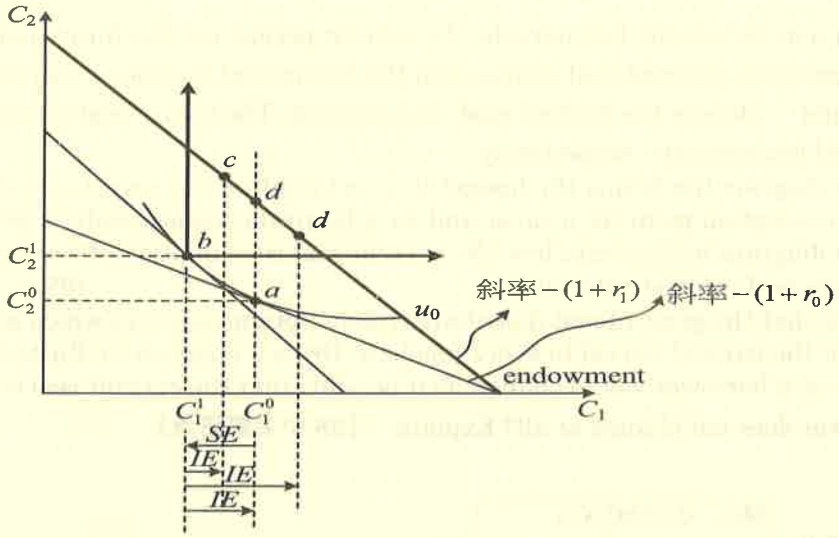
$\longrightarrow \Rightarrow \dfrac{MU_{C_1}}{MU_{C_2}} = (1+r)$ 代入

求出最適消費決策為 $C_1^* = f(r, x)$　$C_2^* = f(r, x)$ ；其中 $x = Y_1 + \dfrac{Y_2}{1+r}$

IF $C_1^* < \overline{Y}_1 \Rightarrow$ Saver	IF $C_1^* > \overline{Y}_1 \Rightarrow$ 債務人	IF $C_1^* = \overline{Y}_1$
Lender（貸出者）	Borrower（借取者）	借貸平衡者
		
$S_1 = (\overline{Y}_1 - C_1^*)$ $C_2^* = (Y_1 - C_1^*)(1+r) + \overline{Y}_2$	$B_1 = (C_1^* - \overline{Y}_1)$ $C_2^* = \overline{Y}_2 - (C_1^* - \overline{Y}_1)(1+r)$	$C_1^* = \overline{Y}_1$ $C_2^* = \overline{Y}_2$

(2)**實質利率上漲之比較靜態分析**

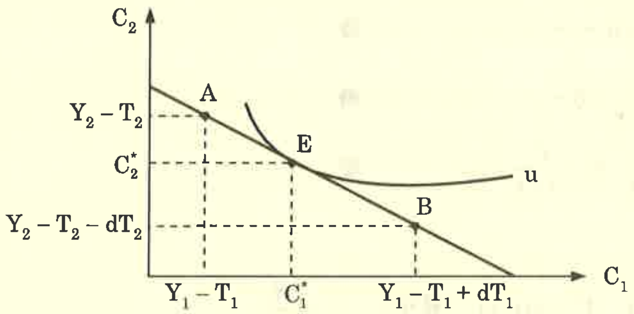
◆**假設此人為 Lender**



(3)李嘉圖 - 貝洛等值定理(Ricardian-Barro Equivalence Theorem)

利用跨期消費決策說明消費券政策不影響消費者跨期消費決策：

若政府在第一期、第二期課徵T_1, T_2定額稅，所得稟賦$(\overline{Y}_1, \overline{Y}_2)$財富現值為 W，在跨期預算限制下，選擇$C_1^*, C_2^*$消費量，E 為最適消費組合，A 點為稅後稟賦點，個人最適化模型如下：



$$\text{Max } u = u(C_1) + \frac{1}{1+P}u(C_2)$$

$$\text{s.t } C_1 + \frac{C_2}{1+r} = (Y_1 - T_1) + \frac{Y_2 - T_2}{1+r} = W$$

若現在政府發放dT_1消費券（如同減稅政策），必須發行公債dT_1元支應，然而到了第二期政府必須償還$dT_1(1+r)$之本利和，為了償還本利和，在第二期必須向民眾增加$dT_1(1+r)$之定額稅，消費者稅後跨期預算方程式：

$$C_1 + \frac{C_2}{1+r} = (Y_1 - T_1 + dT_1) + \left[\frac{Y_2 - T_2 - dT_1(1+r)}{1+r}\right]$$

$$= \left[(Y_1 - T_1) + \frac{(Y_2 - T_2)}{1+r}\right] + \left[dT_1 - \frac{dT_1(1+r)}{1+r}\right]$$

$$=\left[(Y_1-T_1)+\frac{(Y_2-T_2)}{1+r}\right]$$
$$=W$$

消費者一生財富總值未改變（仍為 W），跨期預算線並未被移動，消費均衡點仍在 E 點，因此消費券之減稅政策並不會影響消費者跨期決策，唯一改變是稟賦點，由 A 點→B 點 $(Y_1-T_1+dT_1, Y_2-T_2-dT_2)$。

4.2008 年各國一窩蜂的降息，為什麼？並請以兩期跨期模型來解釋借貸市場的資金供需及消費受降息下的影響，並畫圖（橫軸是第一期消費，縱軸是第二期消費）來表示儲蓄者的所得效果及替代效果之變化。（假設消費是正常品）【98 台大商研所】(20%)

解：參閱講義

5.假設兩期模型，C1 為第一期的消費，C2 為第二期的消費，W1 為第一期的稟賦，W2 為第二期的稟賦，R 為市場借貸利率。請問：（以下題目請繪圖說明之）【提示：稟賦不得儲存】

(1) 請寫下跨期預算限制式。 (5%)

(2) 不存在借貸市場之下，個人的最佳消費組合為何？ (5%)

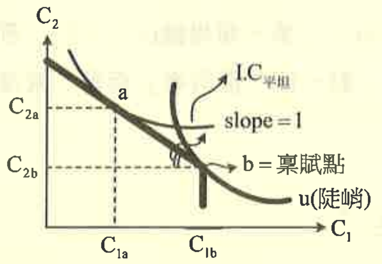
(3) 存在借貸市場之下，個人的最佳消費組合為何？ (5%)

(4) 請分析比較存在借貸市場是否較好？ (5%)

(5) 若借貸利率上升，對於借入者與貸放者的效用影響為何？ (10%)【98 暨南國企所】

解：(1) $C_1+\frac{C_2}{1+R}=W_1+\frac{W_2}{1+R}$

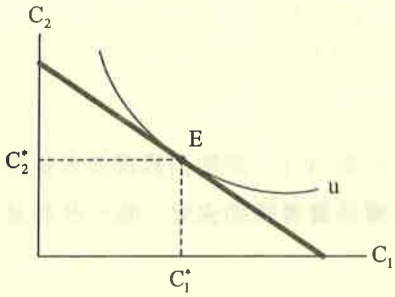
(2)不存在借貸市場下，即使第一期所得未用盡，留至第二期時使用，並不存在利息所得，故知： $C_1 \le W_1$ $\quad$ $C_2 \le W_2+(W_1-C_1)$



❶若無異曲線較平坦（偏好 C_2），均衡點為 a 點 $(W_1-C_1>0)$；

❷若無異曲線教陡峭（偏好 C_1），均衡點為 b 點 $(W_1=C_1)$。

(3)存在借貸市場下，預算線如(1)所示；均衡點滿足 $MRS=1+r \Rightarrow$ 無異曲線與預算線相切 $\Rightarrow$ 均衡點為 E。



(4)一般而言，由於存在借貸市場使消費者預算集合擴大，因此消費者福利較佳。

(5)面對實質利率上漲：

❶消費者原先為儲蓄者，因為實質利率上漲後，仍為儲蓄者，但是效用會上漲。

❷消費者原先為借取者，因為實質利率上漲後，仍為借取者，但是效用會下降。

❸◆狀況一：消費者原先為借取者，因為實質利率上漲後，變成儲蓄者，但是效用會上漲。

◆狀況二：消費者原先為借取者，因為實質利率上漲後，變成儲蓄者，但是效用會不變。

◆狀況三：消費者原先為儲蓄者，因為實質利率上漲後，變成借取者，是不理性決策。

6.請以經濟學的觀點解釋「寅吃卯糧」的意涵？當市場利率降低時，對於「寅吃卯糧」習慣的個人將有何種影響？ 【淡江企研所】

解：利用「跨期消費理論」說明：

(1)最適化模型：

$$\boxed{\begin{array}{l}\text{Max}\ \ U=U(C_1,C_2)\\ \text{s.t}\ \ C_1+\dfrac{C_2}{1+r}=\overline{Y}_1+\dfrac{\overline{Y}_2}{1+r}=W\end{array}}$$

利用$\dfrac{MU_{C_1}}{MU_{C_2}}=MRS=(1+r)$均衡條件，決定出最適兩期消費量$C^*_1$，$C^*_2$，若$C^*_1>\overline{Y}_1$，此時經濟個體在借貸市場變成「Borrower」，第一期借錢($C^*_1>\overline{Y}_1$)，到了第二期再償還，此時面對預算式為$C^*_2\le\overline{Y}_2-(C^*_1-\overline{Y}_1)(1+r)$，對一個「借取者」而言，其借貸行為如同「寅吃卯糧」。

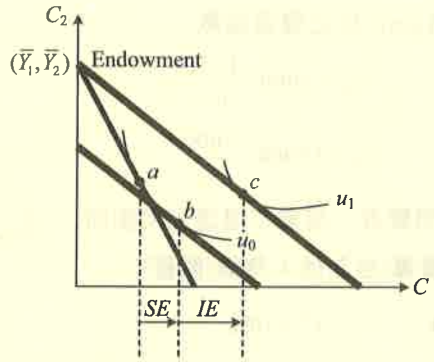
(2)比較靜態分析：

$ISE: r\downarrow\rightarrow C_1\uparrow, C_2\downarrow$

$IE: r\downarrow\rightarrow$ 實質所得增加，$C_1\uparrow, C_2\uparrow$

$PE: r\downarrow\rightarrow C_1\uparrow, C_2\,?$

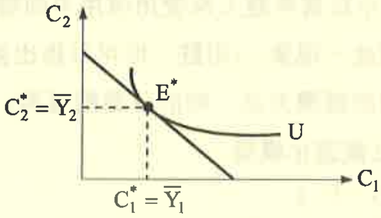
表$r\downarrow$，借取者第一期消費量必增加，資金需求量必增加，將使一個寅吃卯糧的人，更加「寅吃卯糧」，意味著「借更多錢」。



7.A consumer's consumption-utility function for a two-period horizon is $U = C_1 C_2^{\ 0.6}$; his income stream is $Y_1 = 1000$ and $Y_2 = 648$; and the market rate of interest is 8%. Determine values for C_1 and C_2 that maximize his utility. Is he a borrower or lender?

【96 北大國企所、中央財金所】

解：
$$\boxed{\begin{aligned} &Max \quad U = C_1 C_2^{0.6} \\ &s.t \quad 1000 + \frac{648}{1.08} = C_1 + \frac{C_2}{1.08} \end{aligned}}$$



$$L = C_1 C_2^{0.6} + \lambda \left[1600 - C_1 - \frac{C_2}{1.08} \right]$$

F.O.C $\dfrac{\partial L}{\partial C_1} = 0 \Rightarrow C_2^{0.6} - \lambda = 0$

$$\frac{\partial L}{\partial C_2} = 0 \Rightarrow 0.6 C_1 C_2^{-0.4} - \frac{\lambda}{1.08} = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow 1600 - C_1 - \frac{C_2}{1.08} = 0 \Rightarrow \frac{C_2}{0.6 C_1} = 1.08 \Rightarrow C_2 = (0.6)(1.08) C_1$$

代回$1600 = C_1 + 0.6 C_1$ $\therefore C_1^* = 1000$，$C_2^* = 648$，恰好為$\bar{Y}_1$，$\bar{Y}_2$

最適消費決策為$C_1^* = \bar{Y}_1$，$C_2^* = \bar{Y}_2$

此人為**借貸平衡者**，不借亦不貸

8.假設某一經濟係由兩種型態的消費者組成，即個體 A 和個體 B。

下列是 A 和 B 在本期和下期可分別收到的消費品。

	A	B	
本期	20,000	30,000	
下期	25,000	11,200	單位：個

已知 A、B 在各期之最佳消費方程式是 $C_1^A = 14000 + \dfrac{17500}{1+r}$, $C_2^{A*} = 7500 + 6000(1+r)$

$C_1^B = 15000 + \dfrac{5600}{1+r}$, $C_2^{B*} = 5600 + 15000(1+r)$；$C_1^*, C_2^*$分別代表本期和下期的最佳消費量，r 是實質利率。(1)請問該經濟體系的均衡實質利率是多少？

(2)請問 A、B 在各期的最佳消費量是多少？【台大財金所】

解：(1)先求出 Agent A 與 Agent B 之儲蓄函數

$$S_A = 20000 - \left(14000 + \frac{17500}{1+r}\right) \quad \therefore\ S_A = 6000 - \frac{17500}{1+r}$$

$$S_B = 30000 - \left(15000 + \frac{5600}{1+r}\right) \quad \therefore\ S_B = 15000 - \frac{5600}{1+r}$$

∵ 經濟體系只有 A , B 二個消費者，借貸市場達到均衡時， A , B 二人儲蓄總和為零

即 $S_A + S_B = 0$ [某人之儲蓄(債權)必為他人負債(債務)]

$$\therefore\ S_A + S_B = 21000 - \frac{23100}{1+r} = 0 \qquad \therefore\ r^* = 10\%$$

(2) $C_1^A = 14000 + \dfrac{17500}{1.1} = 29909.09 \qquad C_2^A = 7500 + 6000(1.1) = 14{,}100$

$C_1^B = 15000 + \dfrac{5600}{1.1} = 20090.09 \qquad C_2^B = 5600 + 15000(1.1) = 22{,}100$

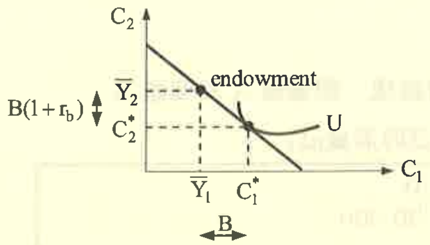
9.許多資料顯示目前年輕人常使用信用卡向銀行借錢，請使用經濟理論模型或圖形說明為何我們會觀察到此一現象。(附註：你可以提出多於一個以上的可能解釋，你越能運用經濟理論提出各種不同的解釋方法，你的分數就越高) 【台大財金所】

解：(1)先建立最適化模型

$$\boxed{\begin{array}{ll} \text{Max} & U = U(C_1, C_2) \\ \text{s.t} & Y_1 + \dfrac{Y_2}{1+r} = C_1 + \dfrac{C_2}{1+r} \end{array}}$$

消費者最適均衡條件為 $\dfrac{MU_{C_1}}{MU_{C_2}} = (1+r)$

理性經濟個體在市場利率 r 已知情況下，透過跨期消費決策，追求一生效用最大，求出最適消費為 C_1^*，若 $C_1^* > \overline{Y}_1$，則 Agent 為借取者(Borrower)，消費者會在第 1 期借款 $B = C_1^* - \overline{Y}_1$，然後在第 2 期進行償還，透過信用市場來進行借貸，使一生效用最大。



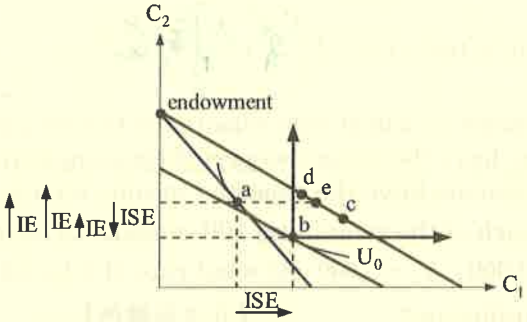
(2)比較靜態分析－

近幾年，金融創新後，隨著銀行業務開放自由競爭，造成信用卡日漸普及化，學生辦卡容易，再者，商業銀行間過度競爭下，常推出辦卡免年費、刷卡免手續費，循環利率下降，紅利點數兌換贈品…等優惠措施，造成使用信用卡之交易成本降低，產生 ISE 及 IE。

$\boxed{\text{ISE}}$ ⇒借款利率降低，當期消費 (C_1) 相對於未來消費 (C_2) 機會成本降低，使 $C_1\uparrow,\ C_2\downarrow$。

$\boxed{\text{IE}}$ ⇒利息成本減輕，實質所得增加，$C_1\uparrow,\ C_2\uparrow$。

$\boxed{\text{總效果}}$ ⇒ C_1 不斷增加，而未來消費不一定增加。



	ISE	IE	TE	C_1	C_2
ISE > IE	a→b	b→c	a→c	增加	減少
IE > ISE	a→b	b→d	a→d	增加	增加
ISE = IE	a→b	b→e	a→e	增加	不變

從圖中可知，隨著借款利率下降，年輕人消費金額不斷地增加。

(3)利用時間偏好率說明：

$$\boxed{\begin{array}{ll} \text{Max} & U = U(C_1) + \frac{1}{1+\rho} U(C_2) \\ \text{s.t} & Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r} \end{array}}$$

消費者均衡：$\frac{MU_{C_1}}{MU_{C_2}} = \frac{1+r}{1+\rho}$

若$\rho > r \Rightarrow \frac{MU_{C_1}}{MU_{C_2}} < 1 \Rightarrow MU_{C_1} < MU_{C_2}$

根據邊際效用遞減法則，則$C_1^* > C_2^*$，由於年輕人主觀上在維持一生效用水準不變情況下，減少一單位C_1所願意補償金額遠超於減少一單位C_1，即增加一單位儲蓄所獲得實際補償之利率。因此年輕人選擇現在消費較C_2（未來消費）多。

10.假定有一個人只活兩期，他在這兩期皆各有 110 元的所得，市場的利率是 10%。他在這兩期都要消費，他由本期消費C_1和下期消費C_2所得到的效用為$U = C_1^{(2/3)} C_2^{(1/3)}$。

(1)假定金融市場允許自由存款和借貸，則這個人在第一期和第二期分別會消費多少元？

(2)如果金融市場有流動性限制，則這個人在第一期和第二期分別會消費多少？他的效用有多高？ 【中央產經所】

解：(1) $Max \quad U = C_1^{\frac{2}{3}} C_2^{\frac{1}{3}} \qquad s.t \quad C_1 + \frac{C_2}{1.1} = 110 + \frac{110}{1.1} = 210$

$$L = C_1^{\frac{2}{3}} C_2^{\frac{1}{3}} + \lambda\left[210 - C_1 - \frac{C_2}{1.1}\right] \qquad \therefore \; C_1^* = 140, \; C_2^* = 77$$

(2)存在流動性限制，則預算式為$C_1 = 110, \; C_2 = 110$

$Max \quad U = C_1^{\frac{2}{3}} C_2^{\frac{1}{3}}$

$s.t \quad C_1 = 110, \ C_2 = 110$

則 $C_1^* = 110, \ C_2^* = 110 \quad U^* = (110)^{\frac{2}{3}}(110)^{\frac{1}{3}} = 110$

11.Consider a Treasury-Bill market in which only consumers borrow and lend. Suppose that all 200 consumers have the same two-period consumption-utility function: $U = 2C_1C_1$. Let each of 100 consumers have the expected-income stream $Y_1 = 5{,}000$, $Y_2 = 4{,}000$, and let each of the remaining 100 consumers have the expected-income stream $Y_1 = 3{,}000$, $Y_2 = 5{,}000$. At what rate of interest will the Treasury-Bill market be in equilibrium? 【北大合經所】

key: 都求 r 時 直接抓 S 解較快

解

第 1 類消費者消費函數	第 2 類消費者消費函數
Max $U = 2C_1C_2$ s.t $5000 + \frac{4000}{1+r} = C_1 + \frac{C_2}{1+r}$	Max $U = 2C_1C_2$ s.t $3000 + \frac{5000}{1+r} = C_1 + \frac{C_2}{1+r}$
$C_1 = 2500 + \frac{2000}{1+r}$ $C_2 = 2500(1+r) + 2000$	$C_1 = 1500 + \frac{2500}{1+r}$ $C_2 = 1500(1+r) + 2500$
$S_1 = 5000 - C_1$ $\therefore S_1^* = 2500 - \frac{2000}{1+r}$	$S_1 = 3000 - C_1$ $\therefore S_1^{**} = 1500 - \frac{2500}{1+r}$

社會總儲蓄為零 $\Rightarrow 100S_1^* + 100S_1^{**} = 0$

$$\left(250000 - \frac{200000}{1+r}\right) + \left(150000 - \frac{250000}{1+r}\right) = 0$$

$$400000 = \frac{450000}{1+r} \Rightarrow r^* = 0.125 \quad = 12.5\%$$

$\therefore r = \frac{4500}{4000} = \frac{1}{8} = 12.5\%$

11.Suppose you divide your life in to two periods-working age and retirement age. When you work, you earn labor income Y; when retired, you earn no labor income, but must live off your savings and the interest it earns. You have no initial assets. You save the amount S while working, earning interest at rate r, so you have (1 + r). S to live on when retired. Because you don't need to consume as much when retired, you want to set consumption when working twice as high as consumption when retired.

(1)Suppose you earn \$2 million over your working life, and the real interest rate for retirement saving is 50%. How much will you save, and how much will you consume in each part of your life?

(2)Suppose a social security system will pay you 25% of your working income when you are retired. Now (Y = \$2 million as in part (1)) how much will you save and how much will you consume each period? 【100 暨南財金所】

解：

(1) $C_1 + \frac{C_2}{1+r} = Y \Rightarrow C_1 + \frac{C_2}{1.5} = 2{,}000{,}000$

$if \ C_1 = 2C_2 \Rightarrow C_1^* = 1{,}500{,}000 \quad C_2^* = 750{,}000 \quad S^* = 500{,}000$

(2)

$$C_1+\frac{C_2}{1+r}=Y+\frac{25\%Y}{1+r}\Rightarrow C_1+\frac{C_2}{1.5}=2{,}000{,}000+\frac{500{,}000}{1.5}$$

$$if\ C_1=2C_2\Rightarrow C_1^{**}=1{,}750{,}000 \quad C_2^{**}=875{,}000 \quad S^{**}=250{,}000$$

12.John will live for only two periods. In the first period he will earn \$100,000. In the second period he will retire and live on his savings. John has a utility function $U(c_1,c_2)=c_1^2c_2$, where c_1 is his period 1 consumption expenditure and c_2 is his period 2 consumption expenditure. The real interest rate is r. If John saves \$1 in period 1, he can get \$1 + r in period 2.

(1) How much will John save in period 1?

(2) Suppose the government wants to encourage people to save more. To this end, the government raises the interest rate to 2r. What will be John's saving in period 1 when the interest rate becomes $2r$?【成大工管所】

解：$c_1(1+r)+c_2=100{,}000(1+r)$

(1) Max $u=c_1^2c_2$

s.t $c_1+\dfrac{c_2}{1+r}=100{,}000$

$c_1{}^*=\dfrac{200{,}000}{3}$； $c_2{}^*=\dfrac{100{,}000(1+r)}{3}$

$s_1{}^*=100{,}000-\dfrac{200{,}000}{3}=\dfrac{100{,}000}{3}$

(2) Max $u=c_1^2c_2$

s.t $c_1+\dfrac{c_2}{1+2r}=100{,}000$

$c_1'=\dfrac{200{,}000}{3}$； $c_2'=\dfrac{100{,}000(1+2r)}{3}$

$s_1'=100{,}000-\dfrac{200{,}000}{3}=\dfrac{100{,}000}{3}$

13.考慮某人的兩期消費模型。令兩期商品價格均為 1（即 $P_1=P_2=1$），所得原賦為 $(Y_1,Y_2)=(1000,540)$，名目利率為 8%。假設兩期消費的邊際效用分別為 $MU_1=1/C_1$ 與 $MU_2=0.7/C_2$。請問：

(1)該消費者的終身預算式為何？並畫出對應的預算線。

(2)此人在第一期與第二期最多可以消費多少？

(3)此人在第一期與第二期的最適消費各為多少？(15 分)【98 輔大管研所】

解：

(1) $C_1+\dfrac{C_2}{1+8\%}=1000+\dfrac{540}{1+8\%}\Rightarrow 1.08C_1+C_2=1620$

(2)第一期最多可消費量 $C_1=\dfrac{1620}{1.08}=1500$

第二期最多可消費量 $C_2=1620$

(3)消費者均衡條件：$\dfrac{MU_1}{MU_2}=(1+r)\Rightarrow\dfrac{1/C_1}{0.7/C_2}=1+8\%\Rightarrow C_2=0.756C_1$

$\therefore 1.08C_1+0.756C_1=1620\Rightarrow C_1^*=882.3529$，$C_2^*=667.0588$

14.一島上有二類人，其中 A 類人有 80 人，B 類人有 40 人。二類人都只活二期，其所得如下：

	A	B
第一期	100	50
第二期	80	110

二類人的效用函數均為$U=C_1C_2$，C_1與C_2分別是第一期與第二期的消費。

(1)令 r 為借貸利率，請分別求出 A、B 二類人第一期的消費函數。(2)小島的金融市場均衡利率為何？

(3)均衡時，A、B 二類人每人的儲蓄各為若干？(以整數表示)

(4)假設 B 類人的效用函數如上述，A 類人的效用函數為$U=C_1C_2^{0.6}$，則均衡利率與 A 類人的儲蓄將作何變動？請簡述理由。【淡江經研所】

解：(1)A 類消費者：
$$\boxed{\begin{array}{ll} Max & U=C_1C_2 \\ s.t & C_1+\dfrac{C_2}{1+r}=100+\dfrac{80}{1+r}\end{array}}$$

$C_1(1+r)+C_2=100(1+r)+80$

$$L=C_1C_2+\lambda\left[100+\frac{80}{1+r}-C_1-\frac{C_2}{1+r}\right]$$

F.O.C $\dfrac{\partial L}{\partial C_1}=0 \Rightarrow C_2-\lambda=0$

$\dfrac{\partial L}{\partial C_2}=0 \Rightarrow C_1-\dfrac{\lambda}{1+r}=0$

$\dfrac{\partial L}{\partial \lambda}=0 \Rightarrow 100+\dfrac{80}{1+r}-C_1-\dfrac{C_2}{1+r}=0\ldots$

$\Rightarrow \dfrac{C_2}{C_1}=(1+r)$ 代入 $100+\dfrac{80}{1+r}=C_1+C_1$ $\quad \therefore C_1^A=50+\dfrac{40}{1+r}$ $\quad C_2^A=50(1+r)+40$

同理，可得 B 類消費者消費函數：$\begin{cases} C_1^B=25+\dfrac{55}{1+r} \\ C_2^B=25(1+r)+55\end{cases}$

(2) $S_1^A=100-\left(50+\dfrac{40}{1+r}\right)=50-\dfrac{40}{1+r}$ $\quad S_1^B=50-\left(25+\dfrac{55}{1+r}\right)=25-\dfrac{55}{1+r}$

A 類消費者總儲蓄 $80\times S_1^A=80\left[50-\dfrac{40}{1+r}\right]=4000-\dfrac{3200}{1+r}$

B 類消費者總儲蓄 $40\times S_1^B=40\left[25-\dfrac{55}{1+r}\right]=1000-\dfrac{2200}{1+r}$

∵債權、債務總和為零 ⇒ 社會總儲蓄為零 $\quad 5000-\dfrac{5400}{1+r}=0 \quad \therefore r=8\%$

(3) $S_1^A=50-\dfrac{40}{1.08}=12.963\fallingdotseq 13$ $\quad S_1^B=25-\dfrac{55}{1.08}=-25.926\fallingdotseq -26$

(4)Max $\quad U=C_1C_2^{0.6}$

s.t $\quad C_1+\dfrac{C_2}{1+r}=100+\dfrac{80}{1+r}$ $\quad \Rightarrow C_1(1+r)+C_2=100(1+r)+80$

$\therefore C_1^A=62.5+\dfrac{50}{1+r}$ $\quad$ 同理可得 $C_1^B=25+\dfrac{55}{1+r}$

$80S_1^A=80\left(37.5-\dfrac{50}{1+r}\right)=3000-\dfrac{4000}{1+r}$ $\quad 40S_1^B=40\left(25-\dfrac{55}{1+r}\right)=1000-\dfrac{2200}{1+r}$

$80S_1^A+40S_1^B=0 \quad \Rightarrow 4000-\dfrac{6200}{1+r}=0 \quad \therefore r=0.55 \quad S_1^A=37.5-\dfrac{50}{1.55}=5.242$

∴經濟直覺為 $r\uparrow, S_1^A\downarrow$

15.Consider a bond market in which only consumers borrow and lend. Assume that all

200 consumers have the same two-period consumption-utility function: $U = 0.5C_1C_2$. Let each of 160 consumers have the expected-income stream $Y_1 = 5000$, $Y_2 = 4000$, and let each of the remaining 40 consumers have the expected-income stream $Y_1 = 4000$, $Y_2 = 8000$, At what rate of interest will the bond market be in equilibrium? Please show the calculation. 【95 北大國企所】

解：

<table>
<tr><td>A 類人</td><td>B 類人</td></tr>
<tr><td>$\begin{cases}\max U = 0.5C_1C_2 \\ s.t.\ \ C_1+\dfrac{C_2}{1+r}=5000+\dfrac{4000}{1+r}\end{cases}$</td><td>$\begin{cases}\max U = 0.5C_1C_2 \\ s.t.\ \ C_1+\dfrac{C_2}{1+r}=4000+\dfrac{8000}{1+r}\end{cases}$</td></tr>
<tr><td>$C_1^*=2500+\dfrac{2000}{1+r}$
$S_A^*=5000-C_1^*=2500-\dfrac{2000}{1+r}$</td><td>$C_1^*=2000+\dfrac{4000}{1+r}$
$S_B^*=4000-C_1^*=2000-\dfrac{4000}{1+r}$</td></tr>
<tr><td colspan="2">借貸市場均衡時，市場總儲蓄額為 0：
$160S_A^*+40S_B^*=0$
$\Rightarrow 160\left(2500-\dfrac{2000}{1+r}\right)+40\left(2000-\dfrac{4000}{1+r}\right)=0$
$\Rightarrow r^*=0$</td></tr>
</table>

16.考慮某消費者的跨期（兩期）消費模型。假設兩期商品價格均為 $P_1 = P_2 = 1$（沒有通貨膨脹），兩期所得各為 $Y_1 = 2{,}000$、$Y_2 = 2{,}750$，名目利率為 10%，假設兩期消費的邊際效用分別為 $MU_1 = 10/C_1$、$MU_2 = 8/C_2$，其中 C_1、C_2 為兩期的消費支出。

(1)請列出此消費者的終身預算限制式。

(2)此消費者於第一期最多可消費多少？

(3)該消費在最適消費選擇下，兩期的消費各為多少？第一期是儲蓄者或是賒借者？【95 屏科大企研所】

解：(1)預算限制式：$C_1+\dfrac{C_2}{1+R}=P_1y_1+\dfrac{P_2y_2}{1+R}\Rightarrow C_1+\dfrac{C_2}{1.1}=2000+\dfrac{2750}{1.1}=4500$

(2)消費者第一期最可以消費：$P_1y_1+\dfrac{P_2y_2}{1+R}=2000+\dfrac{2750}{1.1}=4500$

(3)利用消費者均衡條件：$\begin{cases}MRS=1+r\Rightarrow C_2=0.88C_1 \\ C_1+\dfrac{C_2}{1+R}=4500\end{cases}\Rightarrow C_1^*=2500, C_2^*=2200$

$\dfrac{MU_1}{MU_2}=\dfrac{\frac{10}{C_1}}{\frac{8}{C_2}}=\dfrac{10}{8}\dfrac{C_2}{C_1}=1.1$

$C_2=0.88C_1$

$C_1+\dfrac{0.88C_1}{1.1}=4500$

17.在兩期的 Fisher 消費模型中，某甲第一期為學生，靠在校園打工賺得$10000，第二期已畢業，賺取所得$76000。某甲選擇兩期各消費$40000。

(1)計算實質利率。

(2)若實際利率較(1)為高，將如何影響某甲的兩期消費？

(3)若某甲在第一期贏得樂透，樂透的獎金在第二期才會支付，這將如何影響某甲在第一期的

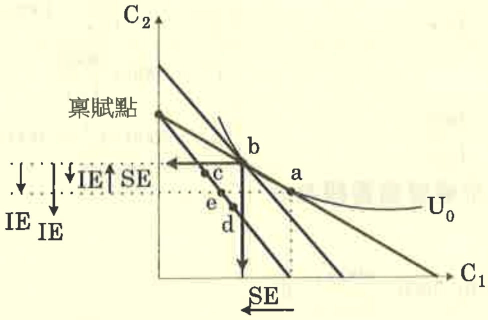
消費和儲蓄。【北大財政所】

解：(1)跨期消費預算限制式 $C_1+\frac{C_2}{1+r}=Y_1+\frac{Y_2}{1+r}$　$40000+\frac{40000}{1+r}=10000+\frac{76000}{1+r}$　$\therefore r^*=0.2$

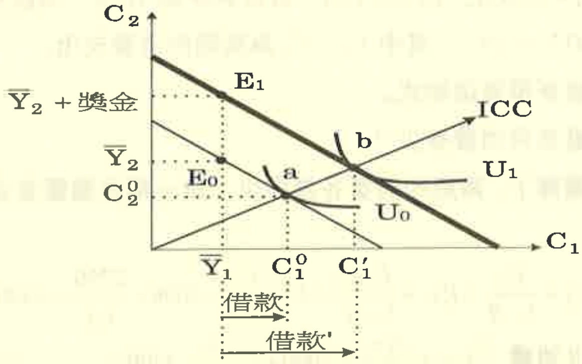
(2)由於 $C_1^*=40000>\overline{Y}_1=10000\Rightarrow$ 此人為借取者(Borrower)

$r_{借款}\uparrow\Rightarrow\begin{cases}|SE|\Rightarrow r上漲，第一期消費機會成本提高，C_1\downarrow,C_2\uparrow\\|IE|\Rightarrow 借款利率上漲，實質所得減少，若C_1與C_2為正常財\\下，C_1\downarrow,C_2\downarrow\end{cases}$

$\begin{cases}|SE|>|IE|\Rightarrow C_1\downarrow,C_2\uparrow & (a\to c)\\|SE|<|IE|\Rightarrow C_1\downarrow,C_2\downarrow & (a\to d)\\|SE|=|IE|\Rightarrow C_1\downarrow,\overline{C}_2 & (a\to e)\end{cases}$



(3)若甲在第一期贏得樂透，但樂透獎金在第二期才會支付，表示甲一生財富增加，跨期預算線整條平行右移，然而稟賦點從 E_0 垂直上移至 E_1（∵第一期所得不變，一生財富增加原因來自第二期所得增加所造成）。



原始狀態，甲消費 C_1^0, C_2^0，假設甲效用函數為 Homothetic，當甲一生財富增加時，C_1 與 C_2 會同比例增加，消費均衡點從 $a\to b$ 點，甲會增加第一期與第二期消費，因第一期所得未增加，因此甲在第一期借款金額會增加。

18.阿花的生存期間可分為兩期：第 0 期（青年）及第 1 期（老年）。她在兩期中的稟賦(endowments)分別為 I_0、I_1。她的效用函數為 $U(C_0,C_1)=\alpha\ln c_0+(1-\alpha)\ln c_1$，其中 C_0 為第 0 期的消費、C_1 為第 1 期的消費、$0<\alpha<1$。實質利率為 r。

(1)請計算並列出步驟：阿花的個人最適消費量及儲蓄量(S)。

(2)畫出標示清楚的圖形以表示阿花的跨時消費問題。

(3)由於就讀交通大學，阿花於年輕時申請助學貸款。請以本模型解釋阿花的儲蓄行為？【交大科管所】

(1) $\boxed{\begin{array}{l} \text{Max } \ U=\alpha_0 \ln C_0+(1-\alpha)\ln c_1 \\ s.t \quad C_0+\dfrac{C_1}{1+r}=I_0+\dfrac{I_1}{1+r} \end{array}}$

$$\mathrm{L}=\alpha\ln C_0+(1-\alpha)\ln C_1+\lambda\left[C_0+\frac{C_1}{1+r}-I_0-\frac{I_1}{1+r}\right]$$

$$F.O.C\ \frac{\partial \mathrm{L}}{\partial C_0}=0\Rightarrow\frac{\alpha}{C_0}+\lambda(1)=0 \qquad \frac{\partial \mathrm{L}}{\partial C_1}=0\Rightarrow\frac{1-\alpha}{C_1}+\lambda\left(\frac{1}{1+r}\right)=0$$

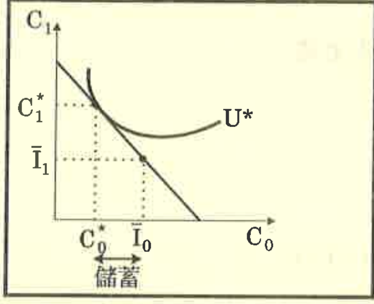
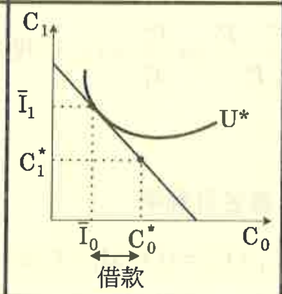
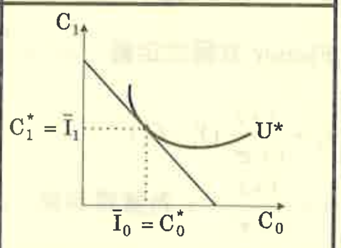
$$\frac{\partial \mathrm{L}}{\partial \lambda}=0\Rightarrow C_0+\frac{C_1}{1+r}-I_0-\frac{I_1}{1+r}=0 \qquad -\Rightarrow\frac{\alpha}{1-\alpha}\frac{C_1}{C_0}=1+r \text{ 代入}$$

$$C_0^*=\alpha\left(I_0+\frac{I_1}{1+r}\right)$$

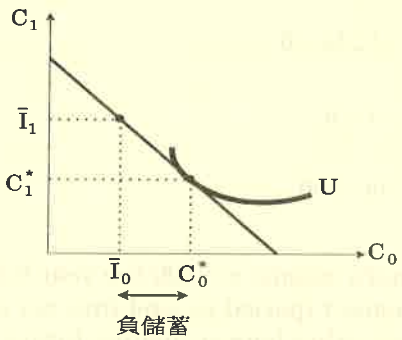
$$C_1^*=(1-\alpha)(1+r)\left[I_0+\frac{I_1}{1+r}\right]$$

$$S_0^*=I_0-C_0^*=(1-\alpha)I_0-\frac{\alpha I_1}{1+r}$$

(2)由於此題並未清楚標示此人之所得稟賦大小，故阿花跨期消費問題可分成三種狀況討論：

若 $C_0^*<\bar{I}_0\Rightarrow$ Lender	若 $C_0^*>\bar{I}_1\Rightarrow$ Borrower	若 $C_0^*=\bar{I}_1\Rightarrow$ 借貸平衡者
		

(3)因阿花年輕時(第 0 期)，所得稟賦低，必需申請就學貸款，表示阿花為借款者(Borrower)，在跨期消費決策下，最適消費決策為 (C_0^*, C_1^*)，因此在第 0 期必需申請就學貸款金額為 $(C_0^*-\bar{I}_0)$。為阿花負擔儲蓄金額，等到第一期（畢業後）再償還第 0 期之借款。



19.Consider a consumer, who is initially a lender.
(1)If the consumer remains a lender after a decline in interest rates, is he better off or worse off after the change in interest rates? Why? Graphically show your answer.
(2)If the consumer becomes a borrower after the change, is he better off or worse off? Why? Graphically show your answer. 【95 成大經研所】

解：(1)消費者原為儲蓄者，面對實質利率下降，仍為儲蓄者，效用會下降。

(2)消費者原為儲蓄者，面對實質利率下降，變成借取者，但是效用變化未確定，可能增加、下降、維持不變。

20.A person will live for only two periods. In the first period, his income is Y_1 and his consumption expenditure is C_1. In the second period, his income is Y_2 and his consumption is C_2. He can borrow and lend at an interest rate of r. There is a π rate of inflation. His utility function is $U(C_1, C_2) = C_1^{0.6}C_2^{0.2}$. His budget line is :

$aC_1 + C_2 = aY_1 + Y_2$

(1)What is the value of a?
(2)Suppose $Y_1 = 300, Y_2 = 625, r = 0.25, and\ \pi = 0$. What is C_1?【96 中興財金所】

解：(1)令 P_1 為第一期物價；P_2^e 為預期第二期物價；Y_1, Y_2 分別為兩期所得稟賦量

$$P_2^eC_2 = P_2^eY_2 + (1+i)(P_1Y_1 - P_1C_1)$$

$$\Rightarrow C_2 = Y_2 + (1+i)\left(\frac{P_1}{P_2^e}\right)(Y_1 - C_1)$$

根據 Fisher 方程式定義，$\pi^e = \dfrac{P_2^e - P_1}{P_1} \Rightarrow \dfrac{P_2^e}{P_1} = 1 + \pi^e$ 代入上式

$$C_2 = Y_2 + \frac{1+i}{1+\pi^e}(Y_1 - C_1)$$

令 $1 + r = \dfrac{1+i}{1+\pi^e}$，r 為實質利率；i 為名目利率

$$\Rightarrow C_2 = Y_2 + (1+r)(Y_1 - C_1) \Rightarrow (1+r)C_1 + C_2 = (1+r)Y_1 + Y_2 \Rightarrow a = 1+r$$

(2)由 Fisher Equation: $r = i - \pi^e \Rightarrow i = r = 0.25$

$$\begin{cases} Max\,U = C_1^{0.6}C_2^{0.2} \\ s.t.\quad 1.25C_1 + C_2 = 1.25(300) + 625 = 1{,}000 \end{cases}$$

$$\Rightarrow L = C_1^{0.6}C_2^{0.2} + \lambda(1{,}000 - 1.25C_1 - C_2)$$

F.O.C：$\dfrac{\partial L}{\partial C_1} = 0$，$0.6C_1^{-0.4}C_2^{0.2} - 1.25\lambda = 0$

$\dfrac{\partial L}{\partial C_2} = 0$，$0.2C_1^{0.6}C_2^{-0.8} - \lambda = 0$

$C_2^* = 250 \quad C_1^* = \dfrac{3}{1.25}(250) = 600$

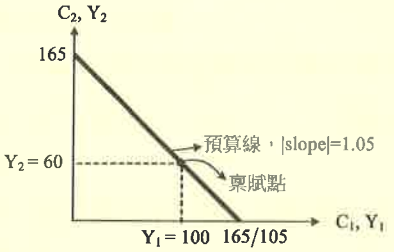
21.Suppose that an individual's income is \$100 the year before retirement (period 1) and \$60 the year after retirement (period 2), and that net savings over the two periods are zero. If the individual can either borrow against future income or lend from present income at a 5% interest rate(r),
(1)derive and plot the individual's budget constraint between period 1 and period 2.

(2)if the individual is in equilibrium by transferring \$20 of income from period 1 to period 2, draw a figure showing how this equilibrium was reached . How much does this individual consume in period 1 and period 2 at equilibrium

【96 中興經研所】

解：(1) $Y_1+\dfrac{Y_2}{1+r}=C_1+\dfrac{C_2}{1+r}\Rightarrow$

預算限制式為：$100+\dfrac{60}{1+0.05}=C_1+\dfrac{C_2}{1+0.05}\Rightarrow\dfrac{165}{1.05}=C_1+\dfrac{C_2}{1.05}$



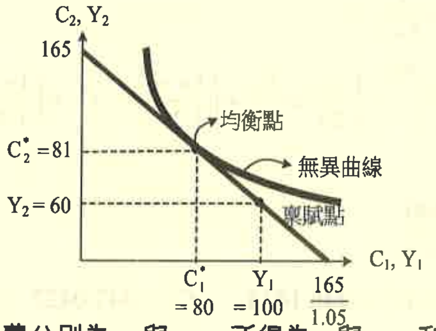
(2) $C_1^*=Y_1-20=100-20=80$; $C_2^*=Y_2+20(1+r)=60+21=81$

$S_1=100-80=20$

$S_2=60-81=-21$

$(1+r)S_1=21$

$(1+r)S_1-S_2=0$



22.在兩期的選擇模型中，消費分別為C_1與C_2，所得為I_1與I_2，利率為 r。若小吳有儲蓄的習慣，且其效用函數為：$U(C_1,C_2)=U(C_1)+U(C_2)/(1+\rho)$

(1)若利率下跌，C_1恆增加？

(2)ρ的經濟意義為何？若小吳認為年輕應多努力，貪圖現在的享受較無意義，老了的享受更重要，此將使得ρ變大？

(3)若$U(C_1,C_2)=C^{0.5}{}_1+0.8C^{0.5}{}_2$，兩期的所得為$I_1=I_2=400$，價格為$P_1=P_2=1$，利率$r=10\%$，求均衡解。 【96 中原國貿所】

解：(1)$\text{Max}\,U(C_1C_2)=U(C_1)+\dfrac{U(C_2)}{1+\rho}$ s.t. $C_1+\dfrac{C_2}{1+r}=I_1+\dfrac{I_2}{1+r}$(財富水準)

$$L=U(C_1)+\frac{U(C_2)}{1+\rho}+\lambda\left[I_1+\frac{I_2}{1+r}-C_1-\frac{C_2}{1+r}\right]$$

$$\text{F.O.C}\begin{cases}\dfrac{\partial L}{\partial C_1}=0,U'(C_1)=\lambda\\[2ex]\dfrac{\partial L}{\partial C_2}=0,\dfrac{1}{1+\rho}U'(C_2)=\lambda\dfrac{1}{1+r}\\[2ex]\dfrac{\partial L}{\partial \lambda}=0,C_1+\dfrac{C_2}{1+r}=I_1+\dfrac{I_2}{1+r}\end{cases}$$

消費者均衡條件$\Rightarrow (1+\rho)\dfrac{U'(C_1)}{U'(C_2)}=1+r$　若實質利率下降，$\therefore C_1$可能增加或減少

(2)　ρ為時間偏好率，一般而言：$\begin{cases}\rho\text{越大}\Rightarrow\text{越偏好現在消費}\\ \rho\text{越小}\Rightarrow\text{越偏好未來消費}\end{cases}$

消費者均衡下$\Rightarrow \dfrac{U'(C_1)}{U'(C_2)}(1+\rho)=(1+r)$

由題目可知，小吳較偏好未來消費(C_2)，∴ρ會變小，無異曲線變平坦。

(3)$\text{Max}\,U(C_1,C_2)=C_1^{0.5}+0.8C_2^{0.5}$　s.t.　$C_1+\dfrac{C_2}{1+r}=I_1+\dfrac{I_2}{1+r}$

$$L=C_1^{0.5}+0.8C_2^{0.5}+\lambda\left[I_1+\frac{I_2}{1+r}-C_1-\frac{C_2}{1+r}\right]$$

$$\text{F.O.C}\begin{cases}\dfrac{\partial L}{\partial C_1}=0\Rightarrow 0.5C_1^{-0.5}=\lambda\\ \dfrac{\partial L}{\partial C_2}=0\Rightarrow 0.4C_2^{-0.5}=\lambda\left(\dfrac{1}{1+r}\right)\\ \dfrac{\partial L}{\partial \lambda}=0\Rightarrow C_1+\dfrac{C_2}{1+r}=I_1+\dfrac{I_2}{1+r}\end{cases}$$

$$1.25\left(\frac{C_2}{C_1}\right)^{0.5}=1+r\qquad \therefore\left(\frac{C_2}{C_1}\right)^{0.5}=\frac{1+r}{1.25}\Rightarrow\frac{C_2}{C_1}=\left(\frac{1+r}{1.25}\right)^2\qquad C_2=\left(\frac{1+r}{1.25}\right)^2C_1$$

$$\Rightarrow C_1+\frac{\left(\dfrac{1+r}{1.25}\right)^2C_1}{1+r}=400+\frac{400}{1+r}$$

$$\Rightarrow C_1+0.704C_1=400+\frac{400}{1.1}\Rightarrow C_1^*=448.1434\qquad C_2^*=347.0422$$

$$\therefore\text{均衡下}\begin{cases}\text{第一期消費，}C_1^*=448.1434\\ \text{第二期消費，}C_2^*=347.0422\end{cases}$$

23.有一簡單跨期消費模型如下，家計單位的兩期效用函數為：$u(C_1,C_2)=\log C_1+\alpha\log C_2$；第一期原賦(endowment)為2，第二期為5，借貸市場實質利率為0.6，消費為正常財，請問：

(1)當$\alpha=0.5$時，求出第一期及第二期的最適消費水準?

(2)　若借貸實質利率上升，繪圖說明第一期及第二期的消費水準會如何變動?

(3)政府對每一期原賦所得均課稅，稅率為10%，繪圖說明課稅後，第一期及第二期的消費會如何變動?　　【97中興應經所】

解：

(1) $\boxed{\begin{array}{l}\text{Max}\ \ U=\log C_1+0.5\log C_2\\ s.t\quad C_1+\dfrac{C_2}{1+0.6}=2+\dfrac{5}{1+0.6}=\dfrac{41}{8}\end{array}}$　$L=\log C_1+0.5\log C_2+\lambda\left[\dfrac{41}{8}-C_1-\dfrac{C_2}{1.6}\right]$

$C_1^*=\dfrac{41}{12}$　　$C_2^*=\dfrac{41}{15}$　消費者為借取者

(2)消費者為借取者，面對實質利率上漲，第一期消費量一定會減少，第二期消費量不一定。

(3)由於對兩期所得皆課稅，因此並不會改變兩期相對價格，跨期預算線平行左移，一生財富減少，若兩期消費為正常財，課稅結果只會造成兩期消費皆下降。

<u>24.</u>約翰決定將所有財富在今明兩年內花完。約翰的跨期效用函數為$u=c_1c_2^{0.5}$，其中 ci 代表第 i 年的消費金額。約翰在這兩年內分別有$1700(今年，即第一年)和$440(明年，即第二年)的所得收入。除所得收入外，約翰並沒有其他存款或收入來源。另外，約翰可以到銀行以市場利率 r 進行借貸 。

(1)請問約翰在考慮跨期消費問題時，所面對的預算限制式為何?

(2) 若市場利率 r 為 10%，請問約翰第一年會借錢消費或進行儲蓄? 借錢(或儲蓄)的金額為多少?

(3) 若市場利率下跌，則約翰在第二年的消費金額會增加或減少? 此時約翰的效用會增加或減少? 【97 彰師商教所】

解：

(1) $c_1+\dfrac{c_2}{1+r}=1700+\dfrac{440}{1+r}$

消費者最適化問題：$\begin{cases}\max u=c_1c_2^{0.5}\\ s.t\quad c_1+\dfrac{c_2}{1.1}=1700+\dfrac{440}{1.1}=2100\end{cases}$

消費者均衡的必要條件：$\begin{cases}MRS=1+r=1.1\Rightarrow\dfrac{2c_2}{c_1}=1.1\Rightarrow 2c_2=1.12c_1\\ c_1+\dfrac{c_2}{1.1}=2100\end{cases}$

兩期最適消費量：$\begin{cases}c_1=\dfrac{1}{1.5}\times\dfrac{2100}{1}=1400\\ c_2=\dfrac{0.5}{1.5}\times\dfrac{2100}{\dfrac{1}{1.1}}=770\end{cases}\Rightarrow saving=1700-c_1=300$

25.Patience has the utility function $U(c_1,c_2)=c_1^{0.5}+2\cdot c_2^{0.5}$, where c_1 is her consumption in period 1 and c_2 is her consumption in period 2. She will earn 100 units of the consumption good in period 1 and 100 units of the consumption good in period 2. She can borrow or lend at an interest rate of 10%.

(1) Write an equation that describes Patience's budget.

(2) If Patience neither borrows nor lends, what will be her marginal rate of substitution between current and future consumption?

(3) If Patience does the optimal amount of borrowing or saving, what will be the ratio of her period 2 consumption to her period 1 consumption? 【97 成大經研所】

解：

(1) Patience 的跨期預算限制式： $c_1+\dfrac{c_2}{1.1}=100+\dfrac{100}{1.1}$

(2) 若 Patience <u>為借貸平衡者</u>，因此$c_1=100$、$c_2=100$，

邊際替代率為：$MRS=\dfrac{MU_1}{MU_2}=\dfrac{\frac{1}{2}c_1^{-\frac{1}{2}}}{c_2^{-\frac{1}{2}}}=\dfrac{1}{2}\left(\dfrac{c_2}{c_1}\right)^{\frac{1}{2}}=\dfrac{1}{2}\left(\dfrac{100}{100}\right)^{\frac{1}{2}}=\dfrac{1}{2}$

(3)當 Patience 從事最適的跨期消費決策下：

$$Max\ U(c_1,c_2)=c_1^{0.5}+2\cdot c_2^{0.5} \qquad c_1+\frac{c_2}{1.1}=100+\frac{100}{1.1}$$

$$MRS=\frac{1}{2}\left(\frac{c_2}{c_1}\right)^{\frac{1}{2}}=\frac{1}{\frac{1}{1+r}}=1+r=1.1\Rightarrow\left(\frac{c_2}{c_1}\right)^{\frac{1}{2}}=2.2\Rightarrow(\frac{c_2}{c_1})^*=4.84$$

26.【單選題】某信用卡使用者乙君的兩期預算限制式本為：$\$10{,}000+\dfrac{\$6{,}000}{1+r}=c_1+\dfrac{c_2}{1+r}$ 然而在成為卡債族後與銀行債務協商的結果，銀行讓他選擇在兩個還款方案中擇一；方案 A：每期所得扣 25%；或方案 B：每期從所得中扣$2,000，試問以下何者為真？(A)在成為卡債族前，實質利率上升會使乙君減少當期消費(B)若乙君仍可在借貸市場上任意借貸，則 A 方案對乙君較有利。(C)若聯合徵信中心將乙君列為債信不良人而乙君無法再借錢，則 A 方案仍對乙君較有利 。(D)乙君在欠債後之最適消費分別為($c_1=8{,}000, c_2=4{,}000$)。(E)以上皆非。 【97 中央財金所】

解：(A)正確；實質利率上升使本期消費的機會成本提高，因此本期消費減少。

(B)方案 A：$\dfrac{3}{4}\times\$10{,}000+\dfrac{\frac{3}{4}\times\$6{,}000}{1+r}=y_A=c_1+\dfrac{c_2}{1+r}$

方案 B：$\$8{,}000+\dfrac{\$4{,}000}{1+r}=y_B=c_1+\dfrac{c_2}{1+r}$

$y_a-y_B=\left(7{,}500+\dfrac{4{,}500}{1+r}\right)-\left(8{,}000+\dfrac{4{,}000}{1+r}\right)=\dfrac{500}{1+r}-500<0$；由上可知，A 方案較不利

(C)面對流動性限制下：方案 A：$\dfrac{3}{4}\times\$10{,}000+\dfrac{3}{4}\times\$6{,}000=12{,}000=y_A=c_1+\dfrac{c_2}{1+r}$

方案 B：$\$8{,}000+\$4{,}000=12{,}000=y_B=c_1+\dfrac{c_2}{1+r}$ $\qquad y_A=y_B=12{,}000\Rightarrow$兩案無差異

(D)題目並未給定此人之效用函數，無法求解最適消費量。

27.There is one nondurable consumption good, of which q_1 is consumed in period 1 and q_2 in period 2. The corresponding prices are p_1 and p_2, and incomes (paid at the beginning of each period) are y_1 and y_2. Let A_0 be the value of assets at the end of period 0 and r_1 and r_2 be the interest rates in the two periods, where interests are paid at the beginning of each period.
(1)Find a person's lifetime budget constraint.
(2)If the utility function is given by $u=\beta_1\ln(q_1-\bar{q}_1)+\beta_2\ln(q_2-\bar{q}_2)$ where $\bar{q}_1$ and $\bar{q}_2$ are two known constants, derive the Marshallian (ordinary) demands for q_1 and q_2.
【97 政大金融所】

解： (1) $\dfrac{q_1}{1+r_1}+\dfrac{q_2}{(1+r_1)(1+r_2)}=\dfrac{y_1}{1+r_1}+\dfrac{y_2}{(1+r_1)(1+r_2)}+A_0$

(2) 令消費者財富現值為：$y_1+\dfrac{y_2}{1+r}+A_0=\bar{y}$

$$\begin{cases} Max\ u=\beta_1\ln(q_1-\bar{q}_1)+\beta_2\ln(q_2-\bar{q}_2) \\ s.t.\quad q_1+\dfrac{q_2}{1+r_2}=\bar{y},\ \text{where}\ \bar{y}=y_1+\dfrac{y_2}{1+r_2}+A_0(1+r_1) \end{cases}$$

$$\Rightarrow \text{Larg\,range}\,(\ell)=\beta_1\ln(q_1-\bar{q}_1)+\beta_2\ln(q_2-\bar{q}_2)+\lambda\left(\bar{y}-q_1-\frac{q_2}{1+r_2}\right)$$

$$\frac{\partial\ell}{\partial q_1}=0,\quad \frac{\beta_1}{q_1-\bar{q}_1}=\lambda\ ① \qquad \frac{\partial\ell}{\partial q_2}=0,\quad \frac{\beta_2}{q_2-\bar{q}_2}=\frac{\lambda}{1+r}\ ② \qquad \frac{\partial\ell}{\partial\lambda}=0,\quad q_1+\frac{q_2}{1+r_2}=\bar{y}\ ③$$

由①/②可得：$\dfrac{\dfrac{\beta_1}{q_1-\bar{q}_1}}{\dfrac{\beta_2}{q_2-\bar{q}_2}}=1+r_2\Rightarrow\beta_2(1+r_2)q_1-\beta_1q_2=\beta_2(1+r_2)\bar{q}_1-\beta_1\bar{q}_2\ ....\ ④$

聯立求解③、④：$\begin{bmatrix}\beta_2(1+r_2) & -\beta_1 \\ 1 & \dfrac{1}{1+r_2}\end{bmatrix}\begin{bmatrix}q_1\\q_2\end{bmatrix}=\begin{bmatrix}\beta_2(1+r_2)\bar{q}_1-\beta_1\bar{q}_2 \\ \bar{y}\end{bmatrix}$

$$\begin{cases}\beta_2(1+r_2)=A \\ \dfrac{1}{1+r_2}=\delta \\ \beta_2(1+r_2)\bar{q}_1-\beta_1\bar{q}_2=D\end{cases}\Rightarrow\begin{bmatrix}A & -\beta_1\\ 1 & \delta\end{bmatrix}\begin{bmatrix}q_1\\q_2\end{bmatrix}=\begin{bmatrix}D\\ \bar{y}\end{bmatrix}$$

$$q_1^*=\frac{\begin{vmatrix}D & -\beta_1\\ \bar{y} & \delta\end{vmatrix}}{\begin{vmatrix}A & -\beta_1\\ 1 & \delta\end{vmatrix}}=\frac{D\delta+\beta_1\bar{y}}{A\delta+\beta_1}=\frac{\beta_2\bar{q}_1-\dfrac{\beta_1\bar{q}_2}{1+r}+\beta_1y_1+\beta_1\dfrac{y_2}{1+r_2}+\beta_1A_0(1+r_1)}{\beta_1+\beta_2}$$

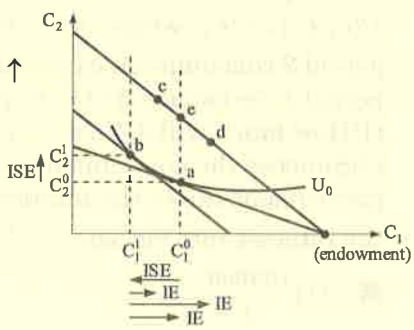
$$q_2^*=\frac{\begin{vmatrix}A & D\\ 1 & \bar{y}\end{vmatrix}}{\begin{vmatrix}A & -\beta_1\\ 1 & \delta\end{vmatrix}}=\frac{A\bar{y}-D}{A\delta+\beta_1}=\frac{\beta_2(1+r_2)y_1+\beta_2y_2+\beta_2(1+r_1)(1+r_2)A_0-\beta_2(1+r_2)\bar{q}_1+\beta_1\bar{q}_2}{\beta_1+\beta_2}$$

28.【是非題】 In a two–period inter-temporal decision model, a lender in the first period may remain a lender or become a borrower in the second period when the interest rate increases. However, if the lender remains a lender, the amount she/he lends to others must decrease.【清大經研所】

解：×；實質利率增加，比較靜態分析： <u>假設此人為 Lender</u>

$$r\uparrow\begin{cases} ISE：\text{現在消費相對於未來消費之機會成本增加，}C_1\downarrow,C_2\uparrow \\ \qquad\qquad\qquad(\because\text{假設消費為正常財}) \\ \text{IE：此人為儲蓄者，利息所得增加，實質所得增加，}C_1\uparrow,C_2\uparrow \end{cases}$$

$$\begin{cases} if\quad ISE>IE\Rightarrow r\uparrow,C_1\downarrow,C_2\uparrow \\ if\quad IE>ISE\Rightarrow r\uparrow,C_1\uparrow,C_2\uparrow \\ if\quad ISE=IE\Rightarrow r\uparrow,\bar{C}_1,C_2\uparrow \end{cases}$$



(2)對儲蓄者而言，面對實質利率上漲，有可能仍為儲蓄者，並且效用會增加；不過，儲蓄量增減未確定。若$|ISE|>|IE| \Rightarrow$實質利率上漲，儲蓄量會增加；若$|ISE|<|IE| \Rightarrow$實質利率上漲，儲蓄量會減少。不過此人原為儲蓄者，面對實質利率上漲，不可能變成借取者。

29.目前利率為 10%，目前麵包一個 20 元，小明本月零用錢收入 200 元，預期下月收入為 120 元，小明預期下月麵包漲價 4%，令小明本月買麵包X個，下月買麵包Y個，請寫出小明本月與下月的跨期預算限制式　　　　　　　　　　　　【中正財金所】

解：$P_1X+\dfrac{P_2Y}{1+R}=I_1+\dfrac{I_2}{1+R}$

$$20X+\frac{(1+4\%)20Y}{1+10\%}=200+\frac{120}{1+10\%}$$

$$20X+\frac{20Y}{\dfrac{1+10\%}{1+4\%}}=309.09 \therefore X+\frac{Y}{1.06}=15.45$$

30.若效用函數為$U=C_1^{\frac{1}{2}}+\dfrac{1}{1+\rho}C_2^{\frac{1}{2}}$，其中$\rho=0.25$。令兩期消費價格均為 1，兩期所得為$Y_1=200$，$Y_2=250$，則利率 6%時，此人在第一期借入或貸出金額多少？【94 暨南財金所】

解：Max $U=C_1^{\frac{1}{2}}+\dfrac{1}{1.25}C_2^{\frac{1}{2}}$

$$\text{s.t } C_1+\frac{C_2}{1+6\%}=200+\frac{250}{1+6\%}=435.8$$

$$L=C_1^{\frac{1}{2}}+\frac{C_2^{\frac{1}{2}}}{1.25}+\lambda\left(435.8-C_1-\frac{C_2}{1.06}\right)$$

F.O.C $\dfrac{\partial L}{\partial C_1}=0\Rightarrow\dfrac{1}{2}\cdot C_1^{\frac{-1}{2}}+\lambda\cdot(-1)=0$

$$\frac{\partial L}{\partial C_2}=0\Rightarrow\frac{1}{2}\cdot\frac{C_2^{\frac{-1}{2}}}{1.25}+\lambda\cdot\left(\frac{-1}{1.06}\right)=0$$

$\therefore C^*_1=259.7$　　$C^*_2=186.7$　　$C^*_1=259.7>\overline{Y}_1=200$

此人在第一期為借取者(Borrower)，借入金額為 59.7

31.She will live for only two periods. In the first period he will earn $100,000. in the second period he will retire and live on his saving. John has a utility function $U(C_1,C_2)=C_1^2C_2$, where C_1 is his period 1 consumption expenditure and C_2 is his period 2 consumption expenditure. The real interest rate is r. If John saves $1 in period 1, he can get $1+$r$ in period 2.

(1)How much will John save in period 1?

(2)Suppose the government wants to encourage people to save more. To this end, the government raises the interest rate to $2r$. What will be John's saving in period 1 when the interest rate becomes $2r$?　　　　　　　　　　　　【中興財金所】

解：(1)$\dfrac{100{,}000}{3}$

$$\boxed{\begin{array}{l}\text{Max } U=C_1^2C_2 \\ \text{s.t} \quad C_1+\dfrac{C_2}{1+r}=100{,}000\end{array}}$$

$C^*_1=\dfrac{200{,}000}{3}$ $\quad C^*_2=\dfrac{100{,}000}{3}(1+r)$ $\quad S^*_1=\dfrac{100{,}000}{3}$

(2) $\dfrac{100{,}000}{3}$

$$\boxed{\begin{array}{l}\text{Max } U=C_1^2C_2 \\ \text{s.t} \quad C_1+\dfrac{C_2}{1+2r}=100{,}000\end{array}}$$

$C_1'=\dfrac{200{,}000}{3}$ $\quad C_2'=\dfrac{100{,}000}{3}(1+2r)$ $\quad S_1'=\overline{Y}_1-C_1'=\dfrac{100{,}000}{3}$

32.假設消費者的消費選擇為： Max $U=f(C_1,C_2)=C_1^{\frac{1}{2}}C_2^{\frac{1}{2}}$

Subject to $C_1+\dfrac{C_2}{1+i}=1000+\dfrac{1320}{1+i}$

請求算出利率(i)從 10%上升至 20%情況下，Slutsky 定義下兩期消費的

(1)替代效果____；(2)所得效果____。【96-97 高科大風管所】

解：(1)

$$\boxed{\begin{array}{l}\text{Max } U=C_1^{\frac{1}{2}}C_2^{\frac{1}{2}} \\ \text{s.t } C_1+\dfrac{C_2}{1+i}=1000+\dfrac{1320}{1+i}\end{array}}$$

$C^*_1=500+\dfrac{660}{1+i}$

$C^*_2=500(1+i)+660$

原均衡 $i=10\%\Rightarrow C_1^0=1100,\quad C_2^0=1210$

新均衡 $i''=20\%\Rightarrow C_1''=1050,\quad C_2''=1260$

(2)依據 Slutsky 維持原來購買組合 $(C_1^0,C_2^0)=(1100,1210)$不變，就是維持消費者實質所得不變：

$$\boxed{\begin{array}{l}\text{Max } U=C_1^{\frac{1}{2}}C_2^{\frac{1}{2}} \\ \text{s.t } C_1+\dfrac{C_2}{1+0.2}=1100+\dfrac{1210}{1+0.2}\end{array}}$$

$C_1'=550+\dfrac{605}{1+0.2}\fallingdotseq 1054$

$C_2'=550\cdot(1+0.2)+605=1265$

(3) $SE\begin{cases}(C_1^0\to C_1'):1054-1100=-46\\(C_2^0\to C_2'):1265-1210=55\end{cases}$

$IE\begin{cases}(C_1'\to C_1''):1050-1054=-4\\(C_2'\to C_2''):1260-1265=-5\end{cases}$

32.John lives for two periods. In period one he earns Y_1 and in period two he earns Y_2. His intertemporal utility function is $(C_1, C_2) = u(C_1) + 0.8u(C_2)$ and the interest rate is 5 %. Assume that $u'>0$ and $u''<0$.

(1)Write down John's maximization problem.
(2)Does John consume more in period one or period two?
(3)If John gets all of his income in the first period, what happens to consumption in each period if the interest rate falls. 【中央企研所】

解：

(1)Max $U = u(C_1) + \dfrac{1}{1+\rho}u(C_2)$ s.t. $C_1 + \dfrac{C_2}{1+r} = Y_1 + \dfrac{Y_2}{1+r}$

消費者均衡條件：$\dfrac{(1+\rho)u_1(C_1)}{u_2(C_2)} = 1+r \Leftrightarrow \dfrac{u_1(C_1)}{u_2(C_2)} = \dfrac{1+r}{1+\rho}$

當ρ= r 時，$u_1(C_1) = u_2(C_2) \to C_1{}^* = C_2{}^*$

當ρ< r 時，$u_1(C_1) > u_2(C_2) \to C_1{}^* < C_2{}^*$

當ρ> r 時，$u_1(C_1) < u_2(C_2) \to C_1{}^* > C_2{}^*$

(2)Max $U = u(C_1) + 0.8u(C_2) = u(C_1) + \dfrac{1}{1+0.25}u(C_2)$

s.t. $C_1 + \dfrac{C_2}{1+0.05} = Y_1 + \dfrac{Y_2}{1+0.05}$

在已知ρ= 0.25 > r = 0.05 下，消費者最適為決策$C_1{}^* > C_2{}^*$。表示當消費者的時間偏好率高於利率時，其最適決策為第一期多消費，而第二期少消費。

(3)當利率下跌時，會使ρ更大於 r，則消費者必更增加第一期的消費，使第一期的消費更多而第二期消費會再減少，使第二期的消費數量更少。

33.Suppose the invention of Internet raises the productivity of capital by 10% permanently. Which of the following is (are) true? (A)The equilibrium interest rate must increases. (B)The demand for capital shifts upward. (C)The effect of this invention on the interest rate is larger than the effect of an invention which raises the productivity by 10% temporarily. (D)The quantity of current consumption must increases. (E)None of the above. 【96 台大經研所】

解：(A)(B)(C);(A)對。資本邊際報酬上升，使廠商對資本的需求增加，而使投資增加，故均衡利率必上升。 (B)對。資本需求曲線整條向上移動。(C)錯。資本邊際生產力上升對利率的影響，永久性影響將小於暫時性影響。(D)錯。利率上升將使當期的消費數量減少。

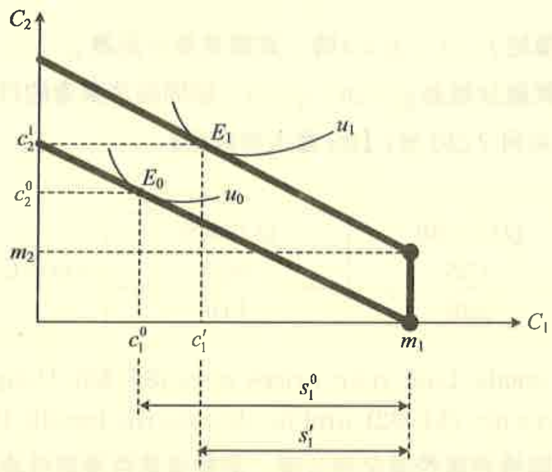
34.Assume that inflation rate is zero. All consumers can borrow and save money in the credit market. Initially, the interest rate to borrow money and the interest rate to save money are both 3%. Suppose the government imposes a tax on the banking sector such that the interest rate to borrow becomes 4% and the interest rate to save becomes 2%. Which of the following is (are) true? (A)All consumers must be strictly worth off. (B)A borrower may become a saver after the policy change. (C)A saver may become a borrower after the policy change. (D)The amount of transactions in the credit market must decline after the policy change. (E)None of the above. 【96 台大經研所】

解：<u>(A)(B)(D)</u>;原來借貸利率相等，皆為 3%，現今借款利率變為 4%，存款利率變為 2%

(A)對。課稅後整體消費者福利會下降。(B)對。對 borrower 而言，因為借款利率上升，消費

量必定減少，可能變為 saver。(C)錯誤。對 saver 而言，因為借款利率上升，存款利率下降，所以不可能變為 borrower。 (D)對。對 borrower 而言，因為借款利率上升，消費量必定減少，因此借貸市場之交易量減少。

35.(In this problem, you are required to answer by well-graphed diagrams with description and/or explanation. So be as specific as necessary to make your answer clear and complete.)
Suppose that the life of Lily can be divided into two periods：working and retirement and that a sensible decision has to be made on her consumption over these two periods c1 and c2, both of which are assumed to be normal. She is pretty sure that she is able to make an income of m_1 while working, but she probably needs to save a part of it because she will not have any income in her retirement. Denote the interest rate between two periods by r and assume that her preference is regular.
(1)Show her optimal choice (c_1, c_2 and the amount saved). Show how her choice changes in response to a change in r. (6%)
(2)She is glad to hear that the government is going to implement the National Annuity Program (國民年金制). Assume that under the new program, she is able to receive m_2 in her retirement without paying any cost (such as premium). Show her optimal choice now and determine whether she would save more or less compared to (1). (8%)
(3)Following (2), will she be better off or worse off if r goes down? (6%)【97 高雄應經所】
解：
(1) Lily 只有第 1 期存在所得稟賦m_1，若市場利率變動後，對c_1與S_1的影響未定。
(2) 當 Lily 面對國民年金制，可在第 2 期獲得m_2所得，預算線平行右移，沒有 SE，只有所得效果，假設兩期消費量均為正常財情況下，兩期消費量皆增加，但第 1 期的儲蓄量會減少。



(3) Lily 為儲蓄者面對r下降仍為儲蓄者，對c_1與S_1影響未定，但是效用水準必降低(Worse off)。

36.【是非題】From the microeconomic perspective, when the interest rate decreases, consumption will always increase. (10 分)【97 高雄第一科技金融】

解：錯誤，對於儲蓄者而言，實質利率下降，對消費量影響可能增加、減少、或是不變，必須考慮到 SE 與 IE；反之對於借取者而言，實質利率下降，當期消費量必定增加。

37.Please evaluate the statement: in an economy that includes current and future consumption, an increase in the interest rate cannot make lender become a borrower. (10 分)【97 暨南經研所】

解：正確，此人原為 Lender，面對實質利率上漲不可能變成 Borrower，仍為 Lender 且其效用水準會提高(Better off)。

主題六：顯示性偏好理論

1.假設消費者將其全部的所得用於購買 X 商品及 Y 商品。在上一期 X 商品的價格為\$4，Y 商品的價格為\$1，消費者購買了 50 單位的 X 及 100 單位的 Y。本期 X 商品的價格為\$2，Y 商品的價格為\$1，消費者購買了 75 單位的 X 及 75 單位的 Y。試比較消費者上一期與本期的福利何者為佳。【95 清大經研所】

解：

	Q_1 (50, 100)	Q_2 (75, 75)
P_1 (4, 1)	$\boxed{300}$	375
P_2 (2, 1)	200*	$\boxed{225}$

$\Rightarrow Q_2 > Q_1$

符合 $WARP$，Q_2 直接顯示偏好優於 Q_1，因此本期福利優於上期福利。

2.有某一消費者於商品價格 $P_1 = 3$，$P_2 = 4$ 時，其購買量分別為 $q_1 = 25$，$q_2 = 20$，當價格為 $P_1 = 4$，$P_2 = 6$ 時，其購買量分別為 $q_1 = 20$，$q_2 = 5$，試問該消費者的行為是否合乎顯示性偏好理論的公理，請問理由為何？(20 分)【98 嘉大應經所】

解：符合 WARP

	$Q_1(25, 20)$	$Q_2(20, 5)$
$P_1(3, 4)$	155	80*
$P_2(4, 6)$	220	110

$\Rightarrow Q_1 > Q_2$

3.Doug consumes two goods. Last year: prices were (\$6, \$3), Doug chose the bundle (9, 18). This year, the prices are (\$1, \$2), and he chooses the bundle (8, 14).【96 中興財金所】

解：利用顯示性偏好理論檢查是否滿足弱公理，消費者是否為理性決策：

$$\begin{cases}\Sigma P_0 \times Q_0 = 6\times 9 + 3\times 18 = 108 \\ \Sigma P_0 \times Q_1 = 6\times 8 + 3\times 14 = 90\end{cases} \quad \Rightarrow \Sigma P_0 Q_0 > \Sigma P_0 Q_1 \Rightarrow Q_0 > Q_1$$

$$\begin{cases}\Sigma P_1 \times Q_1 = 1\times 8 + 2\times 14 = 36 \\ \Sigma P_1 \times Q_0 = 1\times 9 + 2\times 18 = 45\end{cases} \quad \Rightarrow \Sigma P_1 Q_1 < \Sigma P_1 Q_0 \Rightarrow Q_0 > Q_1$$

$\Rightarrow$滿足顯示性偏好弱公理，表示消費者從事理性決策，Q_0 優於 Q_1

4.The consumer buys bundle X^i at prices P^i, $i=0, 1$. State whether the following choices satices satisfy Weak Axiom of Revealed Preference (WARP).

$P^0=(1,3), X^0=(4,2); P^1=(3,5), X^1=(3,1);$ 【96 暨南財金所】

解：

$P^0X^0=1\times4+3\times2=10>P^0X^1=1\times3+3\times1=6$

$P^1X^0=3\times4+5\times2=22>P^1X^1=3\times3+5\times1=14$

⇒符合 WARP

5.某甲有錢 200 元，他買 4 斤蘋果、8 斤香蕉，其中蘋果每斤 30 元，香蕉每斤 10 元。現若蘋果因開放進口每斤跌為 20 元，香蕉也因輸日談判協議達成，香蕉庫存壓力減輕，每斤漲為 20 元，某甲乃改買蘋果、香蕉各 5 斤。請問：某甲的行為是否滿足 Weak Axiom of Revealed Preference？為什麼？ 【中山人管所】

M=200 $Q_0>Q_1$

	$Q_0=(4,8)$	$Q_1(5,5)$
P(30,10)	200	200
(10,20)	200	150

解：令 $P_0=(30,10)$ $Q_0=(4,8)$

$P_1=(20,20)$ $Q_1=(5,5)$

$P_0Q_0=200=P_0Q_1=200$

$P_1Q_0=240>P_1Q_1=200$

∴ 此人行為滿足 WARP，顯示出 $Q_0=(4,8)$ 優於 $Q_1(5,5)$

6.A consumer's budget is entirely spent on milk and pizza. Here are his consumption patterns for two months:

	April	May
Milk price	3	8
Pizza	4	6
Milk consumption	4	3
Pizza consumption	3	4

Is the consumer's behavior consistent with the utility maximization model?

解

	$Q_1(4, 3)$	$Q_2(3, 4)$
$P_1(3, 8)$	36	41
$P_2(4, 6)$	34^*	36

第 1 期所得只有 36 元，買 Q_2 支出需 41 元，根本買不起 Q_2，第 2 期所得 36 元，買 Q_1 支出只需 34 元，買得起 Q_1，則 Q_2 優於 Q_1，符合 WARP，因此，消費者行為合乎理性，與追求效用極大化行為一致。

7.令 $P=(P_A,P_B)$ 代表 Apple 及 Banana 的價格，$Q=(Q_A,Q_B)$ 代表 Apple 及 Banana 的數量。金先生在 $P=(10,5)$ 時，買了 $Q=(3,3)$；在 $P=(8,6)$ 時，買了 $Q=(2,5)$。請問金先生的消費行為與效用極大的行為相一致嗎？為什麼？【政大金融所】

解：

	$Q_1(3, 3)$	$Q_2(2, 5)$	
$P_1(10, 5)$	45	45^*	⇒金先生消費行為不滿足 WARP，其消費行為與效用極大化行為不一致。
$P_2(8, 6)$	42^*	46	

8.
$X^{80}=(2, 4, 5)$ $P^{80}=(5, 6, 7)$
$X^{81}=(3, 3, 6)$ $P^{81}=(4, 8, 5)$
分別代表某甲在民國 80 年與民國 81 年在三種產品價格下(P)，所消費購買的產品數量(X)，請依據上述資料求：(1)拉氏指數(Laspeyres index) (2)巴氏指數(Paasche index) (3)判斷某甲在 80 年與 81 年度間福利水準的變化。 【北大經研所】

解：(1)拉氏物價指數，以基期數量 X_{80} 為衡量基準

$$L_P=\frac{P_{81}X_{80}}{P_{80}X_{80}}=\frac{(4,8,5)(2,4,5)}{(5,6,7)(2,4,5)}\times 100=\frac{4\times 2+8\times 4+5\times 5}{5\times 2+6\times 4+7\times 5}\times 100=\frac{65}{69}\times 100=94.203$$

(2)畢氏物價指數，以當期數量 X_{81} 為衡量基準

$$P_P=\frac{P_{81}X_{81}}{P_{80}X_{81}}=\frac{(4,8,5)(3,3,6)}{(5,6,7)(3,3,6)}\times 100=\frac{66}{75}\times 100=88$$

(3)真實物價指數：$T=\dfrac{P_{81}X_{81}}{P_{80}X_{80}}=\dfrac{(4,8,5)(3,3,6)}{(5,6,7)(2,4,5)}\times 100=\dfrac{66}{69}\times 100=95.65$

$\because\ L_P<T\ \Rightarrow\ P_{81}X_{80}<P_{81}X_{81}$ $P_P<T\ \Rightarrow\ P_{80}X_{81}>P_{80}X_{80}$

根據顯示性偏好弱公理，$X_{81}>X_{80}\therefore$ 80 年到 81 年，福利水準有改善。

9.We say that a consumer has well-behaved preferences if his preferences are complete, reflexive, transitive, monotonic, and weakly convex. We observe the consumption behaviors of Robert, Sam, and Tom in January and February. They only consume two goods, X and Y. Their tastes are unchanged in these two months. The prices of X and Y differ over time. The observed data are shown in the following table.

		Month	
		January	February
Price	(p_x, p_y)	(4 , 10)	(3 , 12)
consumption Quantities			
Robert	(x_r, y_r)	(12 , 2)	(10 , 3)
Sam	(x_s, y_s)	(12 , 2)	(9 , 3)
Tom	(x_t, y_t)	(12 , 2)	(8 , 2.5)

According to the available data, which of the following conclusion(s) is (are) true? (A)Robert may have well-behaved preferences. (B)Sam may have well-behaved preferences. (C)Tom must have well-behaved preferences. (D)Robert and Tom may have the same well-behaved preferences. (E)None of the above. 【96 台大經研所】

解：(A)

Robert:

	(x_r^J, y_r^J)	(x_r^F, y_r^F)
(p_x^J, p_y^J)	68	70
(p_x^F, p_y^F)	60*	66

$4\times12+10\times2=48+20=68$　　$3\times12+12\times2=36+24=60$

$4\times10+10\times3=40+30=70$　$3\times10+12\times3=30+36=66$　　$\therefore$ Robert 符合 WARP

Sam:

	(x_s^J, y_s^J)	(x_s^F, y_s^F)
(p_x^J, p_y^J)	68	66*
(p_x^F, p_y^F)	60*	63

$4\times12+10\times2=48+20=68$　　$3\times12+12\times2=36+24=60$

$4\times9+10\times3=36+30=66$　　$3\times9+12\times3=27+36=63$　　$\therefore$ Sam 違反 WARP

Tom:

	(x_t^J, y_t^J)	(x_t^F, y_t^F)
(p_x^J, p_y^J)	68	57*
(p_x^F, p_y^F)	60	54

$4\times12+10\times2=48+20=68$　　$3\times12+12\times2=36+24=60$

$4\times8+10\times2.5=32+25=57$　$3\times8+12\times2.5=24+30=54$　　$\therefore$ Tom 符合 WARP

(A)對。Robert 有良好的行為偏好。 (B)錯。Sam 違反 WARP,表示 Sam 並無良好的行為偏好。(C)錯。不一定；Tom 雖符合 WARP,但卻不一定符合 SARP。 (D)錯。Robert 與 Tom 並沒有一樣的行為偏好。

10.You are given the following partial information about a consumer's purchases. He consumes only two goods.

	Year 1 Quantity	Year 1 Price	Year 2 Quantity	Year 2 Price
Good 1	100	100	120	100
Good 2	100	100	Y	80

Over what range of quantities of good 2 consumed in Year 2 would you conclude: (a)That his behavior is inconsistent (i.e in contradiction with the weak axiom)? From now on, assume that the weak axiom is satisfied. (b)That the consumer's consumption bundle in Year 1 is revealed preferred to that in Year 2? (c)That the consumer's consumption bundle in Year 2 is revealed preferred to that in Year 1? (d)That there is insufficient information to justify (A), (B) and/or (C)? 【97 中山經濟所】

解：(a)

	$x_1(100,100)$	$x_2(120,Y)$
$p_1(100,100)$	20,000	12,000+100Y
$p_2(100,80)$	18,000	12,000+80Y

WARP 成立要件：$\begin{cases} p_1x_1 \ge p_1x_2 \\ p_2x_1 > p_2x_2 \Rightarrow p_2x_1 \le p_2x_2 \text{不可成立} \end{cases}$

由上可知：違反 WARP 的條件為：$\begin{cases} p_1x_1 \ge p_1x_2 \\ p_2x_1 > p_2x_2 \end{cases}$

$$\Rightarrow \begin{cases} 20{,}000 \geq 12{,}000 + 100Y \\ 18{,}000 \leq 12{,}000 + 80Y \end{cases} \Rightarrow 75 \leq Y \leq 80$$

由於題目為未定 Y 的值，因此無法判定其行為是否不一致，但若 $75 \leq Y \leq 80$，則其行為不具一致性。

(b) 要使此消費者比較喜歡 Year 1 的商品組合，則表示

$$\left.\begin{cases} p_1x_2 \leq p_1x_1 \Rightarrow 12{,}000+100Y \leq 20{,}000 \Rightarrow Y \leq 80 \\ p_2x_1 > p_2x_2 \Rightarrow 18{,}000 > 12{,}000+80Y \Rightarrow Y < 75 \end{cases}\right\} \Rightarrow Y < 75$$

(c) 要使此消費者比較喜歡 Year 2 的商品組合，則表示

$$\left.\begin{cases} p_1x_2 > p_1x_1 \Rightarrow 12{,}000+100Y > 20{,}000 \Rightarrow Y > 80 \\ p_2x_1 \leq p_2x_2 \Rightarrow 18{,}000 \leq 12{,}000+80Y \Rightarrow Y \geq 75 \end{cases}\right\} \Rightarrow Y > 80$$

(d) 由於上述(a)(b)(c)各小題的 Y 值無重疊，因此具有充分的資訊可以作判定。

11.【複選題】Let (x_1,x_2) be the chosen bundle when prices are (p_1,p_2) and (y_1,y_2) the chosen bundle when prices are (q_1,q_2). There are only two goods in the universe, x and y. Mr. E live on earth, Mr. M live on moon and Mr. J lives on Jupiter. We say that two persons have the same preference if whenever one weakly prefers a bundle to another, so does the other. E, M, J all have the same preference. In the following, let (a, b) denote the basket where x = a and y = b. The price on earth, moon and Jupiter are as follows.

	Earth	Moon	Jupiter
P_x	2	1	3
P_y	1	2	1

E's unique optimal consumption is (2, 4), M's unique optimal consumption is (4, 2), J's unique optimal consumption is (3, 2) (A)If you only observe E's and M's choices, you conclude that their behaviors are consistent with the weak axiom of revealed preference. (B)If you only observe M's and J's choices, you conclude that their behaviors are consistent with the weak axiom of revealed preference. (C)If you observe E's, M's and J's choices, you conclude that their behaviors are consistent with the strong axiom of revealed preference. (D)If you observe E's, M's and J's choices, you conclude that their behaviors are not consistent with the strong axiom of revealed preference.【台大經濟所】

解: <u>**(A) (B) (D)**</u>

	$Q_E = (2, 4)$	$Q_M = (4, 2)$	$Q_J = (3, 2)$
$P_E = (2,1)$	<u>8</u>	10	8
$P_M = (1, 2)$	10	<u>8</u>	7
$P_J = (3, 1)$	10	14	<u>11</u>

(A)不違反顯示性偏好弱公理，故正確　(B)亦不違反顯示性偏好弱公理

(D)不違反顯示性偏好強公理　(C)(E)皆產生矛盾的現象，違反顯示性偏好之公理

<u>12.</u>Please answer the following question:

(1) Mr. Lee is endowed with R^ = 24 hours of leisure per day and I^ = 40 units of income (dollars). His Marginal Rate of Substitution in Resources supply is MRS_R = I/R and the

market wage rate is $W_L = 10$. How many hours of labor will Mr. Lee, and what will be his income from labor?

(2) Please comment the statement that"in general there is a necessary relation between the importance of a commodity X (i.e. the share of the consumer budget spent on X) and its price elasticity."

(3)When price are $(P_1, P_2) = (1,2)$ a consumer demands $(X_1, X_2) = (1,2)$, and when prices are $(P_1, P_2) = (2, 1)$ a consumer demands $(X_1, X_2) = (2,1)$. Is this behavior consistent with the model of maximizing behavior? 【96 台科大企研所】

解：

(1) $MaxU = U(R,I) \quad st \quad I = W(24-R)+40 \Rightarrow MRS = W \Rightarrow \frac{I}{R} = 10$

$R^* = 14 \quad N^* = 24-14 = 10 \quad I^* = 140$

(2)利用 Slutsky Equation 說明：

若是財貨為正常財，當財貨支出占所得比重愈大，需求彈性愈大；當財貨支出占所得比重愈小，需求彈性愈小。若是財貨為劣等財，當財貨支出占所得比重愈大，需求彈性反而愈小；當財貨支出占所得比重愈小，需求彈性反而愈大。

(3)

	$Q_1 = (1,2)$	$Q_2 = (2,1)$
$p_1 = (1,2)$	5	<u>4</u>
$p_2 = (2,1)$	<u>4</u>	5

違反顯示性偏好弱公理，消費者違反理性決策。

13.【複選題】Let (x_1,x_2) be the chosen bundle when prices are (p_1,p_2) and (y_1,y_2) the chosen bundle when prices are (q_1,q_2). Which of the following statements is/are incorrect?

(A)If $p_1x_1 + p_2x_2 \le p_1y_1 + p_2y_2$ and $q_1y_1 + q_2y_2 > q_1x_1 + q_2x_2$, then the behavior is consistent with the model of maximizing utility.

(B)The principle of revealed preference says that if $q_1x_1 + q_2x_2 \ge q_1y_1 + q_2y_2$, we must have $(x_1, x_2) \succ (y_1, y_2)$. (C)If $p_1x_1 + p_2x_2 \ge p_1y_1 + p_2y_2$ and $q_1y_1 + q_2y_2 \ge q_1z_1 + q_2z_2$, then (x_1,x_2) is indirectly revealed preferred to (z_1,z_2)

(D)If $p_1x_1 + p_2x_2 \ge p_1y_1 + p_2y_2$ and $(x_1, x_2) \ne (y_1, y_2)$, then (x_1,x_2) is directly revealed preferred to (y_1,y_2)

(E)If $p_1x_1 + p_2x_2 \ge p_1y_1 + p_2y_2$ and $q_1y_1 + q_2y_2 > q_1x_1 + q_2x_2$, then the behavior is consistent with the model of maximizing utility. 【台大經濟所】

解: <u>(A) (B) (E)</u>

(A)若達成 $p_1x_1 + p_2x_2 \ge p_1y_1 + p_2y_2$ 且 $q_1y_1 + q_2y_2 \ge q_1z_1 + q_2z_2$ 的條件，表示滿足 WARP，此人為追求效用極大化的消費者

(B)若 $q_1x_1 + q_2x_2 \ge q_1y_1 + q_2y_2$ 且表示買 X 財比買 Z 財好，會較偏好 X 財

(E)若達成 $p_1x_1 + p_2x_2 \ge p_1y_1 + p_2y_2$ 且 $q_1y_1 + q_2y_2 > q_1x_1 + q_2x_2$ 的條件，表示滿足 WARP，此

人為追求效用極大化的消費者

14.假設有兩個產品：產品 1 與產品 2，其在上一期（稱為第 0 期）的價格分別是：$p_1^0=3$ 與 $p_2^0=4$。張三在上一期對兩個產品的消費量分別是：$q_1^0=20$ 與 $q_2^0=15$，因此其總支出乃是 $y^0=120$。兩個產品在本期（稱為第 1 期）的價格分別為：$p_1^1=5$ 與 $p_2^1=6$。張三在本期可用的總支出乃是 $y^1=191$。請問張三在本期的效用會提高或降低或不一定？(10 分)【98 中央產經所】

解：

	$Q_0=(20,15)$	Q_1
$p_0=(3,4)$	120	TE_0
$p_1=(5,6)$	190	191

消費者在商品價格為(5, 6)時選擇 Q_1 組合，亦買得起 Q_0 組合，表示 Q_1 優於 Q_0 組合，亦即本期效用水準會上升。然而若 $TE_0>120$，表示在價格為(3, 4)下買不起 Q_1 組合，消費者才會購買 Q_0 組合，此時符合 WARP；若 $TE_0\le120$ 時，表示在價格(3, 4)下，商品 Q_0 組合優於 Q_1，表示此人違反 WARP。

15.When prices are $(p_1,p_2)=(2,1)$ a consumer demands $(x_1,x_2)=(1,2)$, and when prices are $(q_1,q_2)=(1,2)$ the consumer demands $(y_1,y_2)=(2,1)$. Is this behavior consistent with the model of maximizing behavior? (10 分)【98 暨南財金所】

解：

	$Q_0=(1,2)$	$Q_1=(2,1)$
$p_0=(2,1)$	$\boxed{4}$	5
$p_1=(1,2)$	5	$\boxed{4}$

價格 p_0 情況下，買 Q_0 組合支小於買 Q_1 組合支出，因此此人買 Q_0 組合

價格 p_1 情況下，買 Q_1 組合支小於買 Q_0 組合支出，因此此人買 Q_1 組合

無法判斷此人是否違反 WARP，因此此人仍是理性決策。

16.有某一消費者於商品價格 $P_1=3$，$P_2=4$ 時，其購買量分別為 $q_1=25$，$q_2=20$，當價格為 $P_1=4$，$P_2=6$ 時，其購買量分別為 $q_1=20$，$q_2=5$，試問該消費者的行為是否合乎顯示性偏好理論的公理，請問理由為何？(20 分)【98 嘉義應經所】

解：

	$Q_0=(25,20)$	$Q_1=(20,5)$
$p_0=(3,4)$	$\boxed{155}$	80
$p_1=(4,6)$	220	$\boxed{110}$

滿足 WARP，並且 Q_0 優於 Q_1，表示此人為理性決策。

17.假定消費者只購買 X_1 與 X_2 兩種財貨(價格為 P_1，P_2)，若原來的價格為 $P_1=\$4.0$，$P_2=\$6.0$，而他的消費組合選擇為(20, 30)。

(1)當 P_1 漲價到\$5.0 時，他選擇購買 18, 34)的組合，試問 P_2 之漲跌。

(2)假定 X_1 與 X_2 的價格都漲成為(\$6.0, \$9.0)，而他的所得增加為\$364，試問消費者「比過去更快樂還是不快樂」？

(3)假定 P_1 與 P_2 都改變了，而之後「他買得起(25, 24)的組合」並且「在新的價格下(25, 24)比他原來的選擇(20, 30)還貴」。試以圖說明這個變化，並且分析在此變化後他會買更多 X_1？更多 X_2？「比過去更快樂還是不快樂」？(25 分)【97 台大農經所】

解：

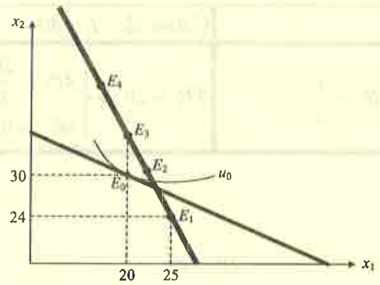
(1) 在 $P_1=4$，$P_2=6$ 時，購買(20, 30)組合，可得 $M=4(20)+6(30)=260$

當 P_1 漲價到 5 元時，購買(18, 34)組合可得 $5(18)+P_2(34)=260$ 表示 P_2 下跌到 $P_2'=5$ 元。

(2) 原來預算限制式：$4X_1+6X_2=260$　　新預算限制式：$6X_1+9X_2=364$

表示當價格皆上漲 1.5 倍，但名目所得漲幅卻小於 1.5 倍，顯示消費者實質所得下降，預算線往內移，效用水準必下降。

(3)



新的選擇必會比過去更快樂(效用提高)，但是新選擇下，x_1 消費量可能增加(E_2)、可能減少(E_4)、可能維持不變(E_3)但是 x_2 消費量必定增加。

18.請問下述選項何者為錯誤？(A)「$\frac{MU_X}{P_X}=\frac{MU_Y}{P_Y}$ 為消費者得到最大效用的充要條件」。(B)「凡是違反顯示性偏好弱性公理(WARP)者，其消費行為就不適合顯示性偏好理論所述；同理，凡是符合顯示性偏好強性公理(SARP)者，其消費行為就適合顯示性偏好理論所述」。(C)「若您的所得效用函數為 $U(M)=\sqrt{M}$，則您不會接受公平性的賭博遊戲」(D)「只要管制價格低於市場均衡價格，必有黑市發生」。　(E)「趨避風險者會買保險，但他們又喜好到賭場賭博，顯示其為風險愛好者，表示其行為是不合理性的矛盾現象」。【96 北大經研所】

解：**(A)(E)；**(A) $\frac{MU_X}{P_X}=\frac{MU_Y}{P_Y}$ 為消費者得到最大效用之必要條件。

(B)正確，違反 WARP 者，其選擇行為為不理性，符合 SARP 者，保證必滿足 WARP，其選擇行為則為理性。(C)正確，$U(M)=\sqrt{M}$ 為風險趨避者，不會參與公平賭局。(D)正確，管制價格小於市場價格，產生超額需求(ED)，因此會有黑市產生。

第五章　生產理論

1.令 L 代表勞動，K 代表資本。當 L=5、K=5，勞動的邊際產量是 3，資本的邊際產量是 4。請計算邊際技術替代率(marginal rate of technical substitution, MRTS)？並說明其意義。(10%)【98 靜宜企研所】

解：$\left|MRTS_{LK}\right| = \dfrac{\Delta K}{\Delta L} = \dfrac{MPP_L}{MPP_K} = \dfrac{3}{4} \Rightarrow$，

增加一單位勞動投入量.維持產量水準不變.資本投入量必須減少 $\dfrac{3}{4}$ 單位

2.若一傢俱工廠，每生產一張木桌(Q)，需 2 單位勞動(L)及 5 單位木材(K)：

(1)請寫出此工廠的生產函數。

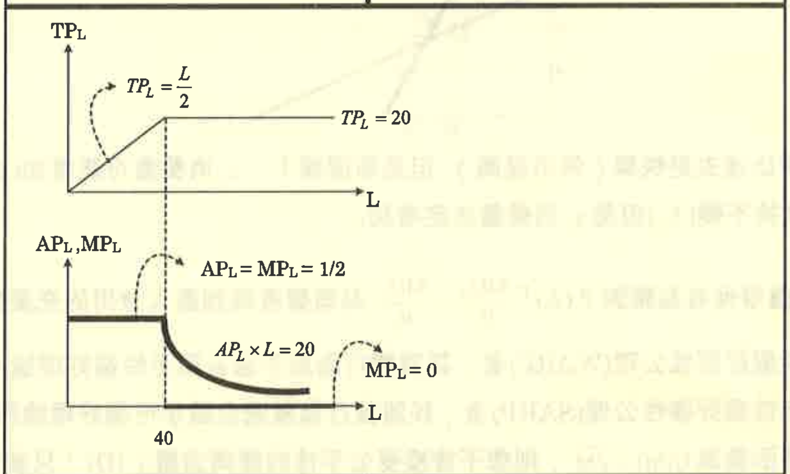
(2)在短期，若此工廠只擁有 100 單位木材，請畫此工廠之勞動的平均產量線(AP_L)及邊際產量線(MP_L)。　　【中興企研所】

/kəm'petətɪv/
競爭的
/mə'nɑpəlɪ/ monopoly
獨佔

解：(1) $Q = \min\left\{\dfrac{L}{2}, \dfrac{K}{5}\right\}$

(2)短期下，$TP_L = \min\left\{\dfrac{L}{2}, 20\right\}$

Case 1: $L \le 40$	Case 2: $L > 40$
$TP_L = \dfrac{L}{2} \Rightarrow AP_L = MP_L = \dfrac{1}{2}$	$TP_L = 20 \Rightarrow \begin{cases} AP_L = \dfrac{20}{L} \Rightarrow AP_L \times L = 20 \\ MP_L = 0 \end{cases}$



3.若勞動的平均產量：

$AP_L = 10$　　，當 $9 \ge L \ge 0$

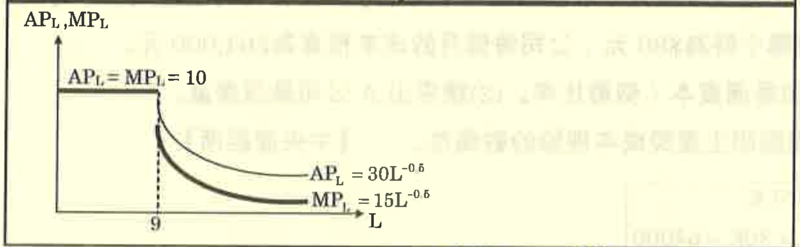
$30L^{-\frac{1}{2}}$　　，當 $L > 9$

試於同一座標平面上繪出勞動的平均產量曲線以及勞動的邊際產量曲線。【中興企研所】

解：

7.5k−3L=66−1.2k
8.7k=9L
3.9≤3L

Case 1: $0<L\le 9$	Case 2: $L>9$
$TP_L = AP_L \times L = 10L$ $MP_L = \dfrac{dTP_L}{dL} = 0$	$TP_L = AP_L \times L = 30L^{0.5}$ $MP_L = \dfrac{dTP_L}{dL} = 15L^{-0.5}$



4.True or False. Please explain your answers. 充足的 / sufficient (15%)

(1) Price equals marginal cost is a sufficient condition for firms' profit maximization in a competitive or monopoly industry.

(2) Suppose that a firm's production is determined by labors and capitals. In the long-run, the marginal product of labor is diminishing.

(3) In the short-run, a firm's producer surplus equals the firm's profits. 【98 高雄大學金融管理】

× PS=π

解：(1)錯誤；完全競爭廠商，利潤極大化的均衡條件為 P=MC 訂價法；而獨佔廠商利潤極大化訂價法為 MR=MC。

(2)正確；短期因為有固定生產要素存在，因此短期會滿足邊際生產力遞減法則；然而長期下所有生產要素均可變動，因此長期下生產要素不一定滿足邊際生產力遞減法則，長期下我們可以判斷生產函數是規模報酬遞增或是規模報酬遞減。

(3)錯誤；短期 $PS = TR - TVC = TR - (TC - TFC) = TR - TVC + TFC = \pi + TFC$。

題型：生產者均衡條件

5.IEM Corp. has estimated that it has the following production function：

$Q = 1.5LK - 0.3L^2 - 0.15K^2$, where L and K is the amount of labor and capital used with respectively.

Labor cost is \$60 and capital cost is \$75. IEM want to maximize output subject to the cost constraints of \$1,500.

(1)Explain your optimal condition in terms of ratio of the marginal product of each input to its own price. (5%)

(2)What amounts of labor and capital should be used? (5%)

(3)What is the total output from the above combination? 【98 清大工工、工管所】 (5%)

解：

(1)生產者均衡條件：$\text{MRTS}_{LK} = \dfrac{MPP_L}{MPP_K} = \dfrac{P_L}{P_K}$

(2) $\text{MRTS}_{LK} = \dfrac{MPP_L}{MPP_K} = \dfrac{P_L}{P_K} \Rightarrow \dfrac{1.5K - 0.6L}{1.5L - 0.3K} = \dfrac{60}{75}$ 帶回成本限制式：$1500 = 60L + 75K$

$\therefore L^* = 10.9 \quad K^* = 11.278$

$4L + 5K = 100$ ①
$37L - 30K = 0$ ②
$\therefore L = 9.8$

(3) $L^{*}=10.9 \quad K^{*}=11.278 \Rightarrow Q=129.67$

6.假設A公司的生產函數為$Q=5LK$，其中，Q表示生產數量，以公噸表示；L為勞動投入，以人工小時表示；K為資本投入，以機器運轉小時表示。A公司的生產成本，每人工小時為\$20元，每機器運轉小時為\$80元，公司每個月的成本預算為\$64,000元。
(1)請決定出A公司最適資本／勞動比率。(2)請求出A公司最適產量。
(3)請利用計算數據說明生產與成本理論的對偶性。　　【中央產經所】

解：(1) $\boxed{\begin{array}{ll} \text{Max} & Q=5LK \\ \text{s.t} & 20L+80K=64000 \end{array}}$

$$\mathcal{L}=5LK+\lambda[20L+80K-64000]$$

F.O.C $\dfrac{\partial \mathcal{L}}{\partial L}=0 \Rightarrow 5K+\lambda(20)=0 \qquad \dfrac{\partial \mathcal{L}}{\partial K}=0 \Rightarrow 5L+\lambda(80)=0$

$\dfrac{\partial \mathcal{L}}{\partial \lambda}=0 \Rightarrow 20L+80k-64000=0 \qquad \longrightarrow \boxed{\dfrac{K}{L}=\dfrac{1}{4}}$

故A公司最適資本／勞動比率為$\dfrac{1}{4}$

(2)將$\dfrac{K}{L}=\dfrac{1}{4}$代回　代

$20L+80\left(\dfrac{L}{4}\right)=64000 \Rightarrow L^{*}=1600$，$K^{*}=400$，

$Q^{*}=5(1600)(400)=3200{,}000$

$L=\dfrac{64000}{2\times 20}$　$\dfrac{64000}{2\times 20}=1600$　$\dfrac{64000}{2\times 80}=400$

(3) $\boxed{\begin{array}{ll} \text{Min} & C=20L+80K \\ \text{s.t} & 3200{,}000=5LK \end{array}}$

$$\mathcal{L}=20L+80K+\lambda[3200{,}000-5LK]$$

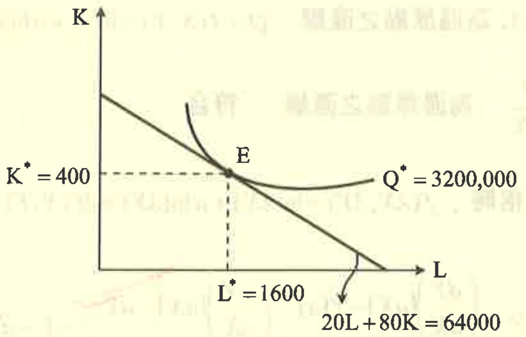
F.O.C $\dfrac{\partial \mathcal{L}}{\partial L}=0 \Rightarrow 20-5K=0$ ……………………………… ❶

$\dfrac{\partial \mathcal{L}}{\partial K}=0 \Rightarrow 80-5L=0$ ……………………………… ❷

$\dfrac{\partial \mathcal{L}}{\partial \lambda}=0 \Rightarrow 3200{,}000-5LK=0$ ……… ❸

$\longrightarrow \dfrac{1}{4}=\dfrac{K}{L}$ 代入❸式　　$3200{,}000=5(4K)(K)$　　$\therefore K^{*}=400$，$L^{*}=1600$

在成本預算為64,000元限制下，每小時工資為20元及每小時機器運轉費80元之下，可求出廠商最大產出為3200,000，利用對偶理論概念，若在產量限制3200,000之下，每小時工資為20元及每小時機器運轉費80元之下，廠商生產之最小成本為64,000元，而且此兩種方法所求出均衡L^{*}與K^{*}皆相同。



在產量限制$\overline{Q}=3200000$之下，最小成本之要素僱用組合為 E 點，
在成本限制$\overline{C}=64000$之下，最大產量之要素僱用組合為 E 點，皆為$L^*=1600$，$K^*=400$

題型：擴張線

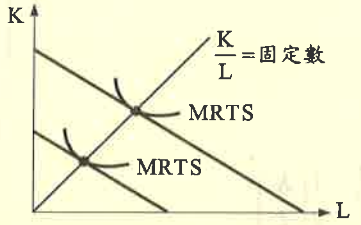
7.生產函數$X=AK^2L^2-BK^3L^3$為非齊次生產函數，P_K^0,P_L^0已知，試問其擴張曲線是否為一直線？　　【中興經研所】

解：擴張線方程式：$\dfrac{MPP_L}{MPP_K}=\dfrac{P_L}{P_K}\Rightarrow\dfrac{2ALK^2-3BL^2K^3}{2AL^2K-3BL^3K^2}=\dfrac{P_L}{P_K}\Rightarrow\dfrac{LK^2(2A-3BLK)}{L^2K(2A-3BLK)}=\dfrac{P_L}{P_K}$

$\dfrac{LK^2}{L^2K}=\dfrac{P_L}{P_K}\Rightarrow\dfrac{K}{L}=\dfrac{P_L}{P_K}=$固定數，因此擴張線是一條過原點直線。

8.如果生產函數$F(L,K)$當 homothetic，但並不是什麼齊次函數，設$MP_L(27,14)=5$，$MP_L(54,28)=3$，$MP_K(54,28)=9$，是否可決定$MP_K(27,14)$之值？如果是，則為多少？如果不是，解釋為什麼？【政大經研所】

解：若生產函數為 Homothetic，則擴張線為過原點直線；在既定$\left(\dfrac{K}{L}\right)$之下，其 Isoquant 之切線斜率會相等，即 MRTS 會相等。



$\dfrac{MP_L(27,14)}{MP_K(27,14)}=\dfrac{MP_L(54,28)}{MP_K(54,28)}$　　$\dfrac{5}{MP_K(27,14)}=\dfrac{3}{9}$　　$\therefore MP_K(27,14)=15$

9.【複選題】生產函數$Q=f(X,Y)=\ln X+a\ln Y$，$a>0$。(A)此生產函數為齊序(homothetic)生產函數 (B)此生產函數為固定規模報酬的生產函數 (C)此生產函數的邊際技術替代率遞減 (D)a 值愈大，要素之間的替代性愈小 (E)此生產函數的邊際技術替代率不受生產規模變動的影響　　【淡江經研所】

解：<u>(A)(C)(E)</u>

(A)齊序效用函數其 I.C.C 為過原點之直線：$Q=f(X,Y)=\ln X+a\ln Y \quad a>0$

$$MRTS=\frac{MPP_X}{MPP_Y}=\frac{\frac{1}{X}}{a\frac{1}{Y}}=\frac{Y}{aX}$$ 為過原點之直線 ∴符合

(B)當 X、Y 同時增加λ倍時，$f(\lambda X,\lambda Y)=\ln(\lambda X)+a\ln(\lambda Y)\neq\lambda f(X,Y)$

∴ 非固定規模報酬

(C) $MRTS=\frac{Y}{aX}$ $\quad \frac{\partial|MRTS|}{\partial X}=\frac{\left(\frac{dY}{dX}\right)(aX)-Y(a)}{(aX)^2}=\frac{\left(\frac{-Y}{aX}\right)(aX)-aY}{(aX)^2}=\frac{-Y-aY}{(aX)^2}<0$

∴邊際技術替代率遞減

(D)將生產函數還原：$Q=XY^a$（單調遞增轉換） $MRTS=\frac{MPP_X}{MPP_Y}=\frac{Y^a}{aXY^{a-1}}=\frac{Y}{aX}=\frac{1}{a}\times\frac{Y}{X}$

取 ln 再微分 $\quad \ln MRTS=\ln\left(\frac{Y}{X}\right)+\ln\left(\frac{1}{a}\right) \quad d\ln MRTS=d\ln\left(\frac{Y}{X}\right)$

∴替代彈性 $=\dfrac{d\ln\left(\frac{Y}{X}\right)}{d\ln MRTS}=1$ ∴和 a 無關

(E)當 X、Y 同時增加λ倍時 $\quad MRTS'=\frac{\lambda Y}{a\cdot\lambda X}=\frac{Y}{aX}=MRT$ ∴不變

題型：Cobb-Douglas 生產函數特性

10.設生產函數為 $Q=\sqrt{KL}$，P_K^0，P_L^0 已知，試證明：

(1) K、L 成比例增加，MP_L 和 AP_L 都沒有影響。

(2)此函數表示規模報酬不變，但合乎報酬遞減法則。

(3) 此函數的等產量曲線凸向原點。

(4)擴張路線(expaxsion path)為一直線。

(5)勞動和資本的產量彈性為何？

(6)生產彈性為一。

(7)替代彈性為何？【成大企研所】

解：(1) $APP_L=\left(\frac{K}{L}\right)^{\frac{1}{2}}$，$MPP_L=\frac{1}{2}L^{\frac{-1}{2}}K^{\frac{1}{2}}=\frac{1}{2}\left(\frac{K}{L}\right)^{\frac{1}{2}}$

當 (L,K) 同比例增加，APP_L 和 MPP_L 皆不變

(2) $f_{(\lambda L,\lambda K)}=\sqrt{\lambda^2LK}=\lambda\sqrt{LK}=\lambda\cdot f(L,K)$

∴CRS，一階齊次生產函數

$MPP_L=\frac{1}{2}L^{\frac{-1}{2}}K^{\frac{1}{2}}$，$\frac{dMPP_L}{dL}=\frac{-1}{4}L^{\frac{-3}{2}}K^{\frac{1}{2}}<0\rightarrow$ 邊際報酬遞減法則

(3) $MRTS_{LK}=\frac{dK}{dL}=\frac{-MPP_L}{MPP_K}=\frac{-K}{L}$

CD函數 $\sigma=\frac{1}{1-e}$

$\sigma=1$

完全互補 $\sigma=0$

$e=1$ $\quad \sigma=\frac{1}{1-1}=\infty$ 完全替代

$MP_L=\alpha L^{\alpha-1}K^{1-\alpha}$

$MP_K=(1-\alpha)L^{\alpha}K^{-\alpha}$

$MRTS_{xy}=\frac{\alpha}{1-\alpha}\left(\frac{K}{L}\right)^1$

$$\frac{d|MRTS_{LK}|}{dL}=\frac{\left(\frac{dK}{dL}\right)L-K\left(\frac{dL}{dL}\right)}{L^2}=\frac{\left(\frac{-K}{L}\right)L-K}{L^2}=\frac{-2K}{L^2}<0$$

(4) 擴張線：$\frac{MPP_L}{MPP_K}=\frac{P_L^0}{P_K^0}\Rightarrow\frac{K}{L}=\left(\frac{P_L^0}{P_K^0}\right)$

∴擴張線是一條過原點直線

(5) $\varepsilon_L=\frac{1}{2}$　　$\varepsilon_K=\frac{1}{2}$　　(6) $\varepsilon_P=\varepsilon_L+\varepsilon_K=1$　　(7) $\sigma=1$

11.假設某廠商的生產函數如下：$Q=f(L,K)=(\alpha L^\rho+\beta K^\rho)^{\phi/\rho}$，$\alpha+\beta=1$，$\rho<1$，$\rho\neq0$，$\alpha,\beta,\rho>0$

其中，L為勞動投入，K為資本投入，Q為生產量，試問：

(1)生產函數的邊際技術替代率為？

(2)生產函數的替代彈性為？

(3)在$\phi=1$時，該生產函數為固定規模報酬，請分別說明：❶生產函數的形式為完全替代時，$\rho=?$

❷生產函數為完全互補時，$\rho=?$　　【96-97 高科大風管所】

解：(1) $MRTS_{LK}=\frac{MPP_L}{MPP_K}=\frac{\frac{\phi}{\rho}(\alpha L^\rho+\beta K^\rho)^{\frac{\phi}{\rho}-1}\cdot\alpha\rho L^{\rho-1}}{\frac{\phi}{\rho}(\alpha L^\rho+\beta K^\rho)^{\frac{\phi}{\rho}-1}\cdot\beta\rho K^{\rho-1}}=\left(\frac{\alpha}{\beta}\right)\left(\frac{K}{L}\right)^{1-\rho}$

(2) $\ln MRTS_{LK}=\ln\left(\frac{\alpha}{\beta}\right)+(1-\rho)\ln\left(\frac{K}{L}\right)$　　$d\ln MRTS_{LK}=(1-\rho)d\ln\left(\frac{K}{L}\right)$

$\therefore\sigma=\frac{d\ln\left(\frac{K}{L}\right)}{d\ln MRTS_{LK}}=\frac{1}{1-\rho}$

(3) 完全替代生產函數$\sigma=\infty\Rightarrow$此時$\rho=1$

完全互補生產函數$\sigma=0\Rightarrow$此時$\rho\to\infty$

12.Consider the production function $y=L^\alpha K^{1-\alpha}$,where $L=\text{labor}$, $K=\text{capital}$, $y=\text{output}$, and a is restricted to values between 0 and 1.

(1)find the marginal products of labor and capital, MP_L, MP_K.

(2)find the rates of change of these marginal products due to changes in both labor and capital. Verify that the rate of change of MP_L with respect to K is the same as that of MP_K with respect to L.

(3)Does the law of diminishing marginal productivity hold for this production?【中山財管所】

解：(1) $MP_L=\frac{dy}{dL}=\alpha L^{\alpha-1}K^{1-\alpha}$　　$MP_K=\frac{dy}{dK}=(1-\alpha)L^\alpha K^{-\alpha}$

(2) $\frac{\partial MP_L}{\partial k}=f_{LK}=\alpha(1-\alpha)L^{\alpha-1}K^{-\alpha}$　　$\frac{\partial MP_K}{\partial L}=f_{KL}=\alpha(1-\alpha)L^{\alpha-1}K^{-\alpha}$

$$\therefore \frac{\partial MP_L}{\partial K} = \frac{\partial MP_K}{\partial L}$$

(3) $\dfrac{\partial MP_L}{\partial L} = f_{LL} = \alpha(\alpha-1)L^{\alpha-2}K^{1-\alpha} < 0 \quad [\because\ \alpha < 1]$

$\dfrac{\partial MP_K}{\partial K} = f_{KK} = (1-\alpha)(-\alpha)L^{\alpha}K^{-\alpha-1} < 0$

∴ 短期，符合**邊際報酬遞減法則**

13.生產函數$Q = F(K,L) = 10K^{0.3}L^{0.7}$，試求：(1) AP_K，MP_K，AP_L，MP_L，F_{KK}，F_{LL}。
(2)資本及勞動產出彈性？(3) $MRTS_{LK} = ?$　(4)（生產要素之）替代彈性$\sigma = ?$
(5)資本及勞動份額？（如果生產要素依其邊際生產力給付）【中山財管所】

解：$Q = 10L^{0.7}K^{0.3}$

(1) $AP_K = \dfrac{Q}{K} = \dfrac{10L^{0.7}K^{0.3}}{K} = 10L^{0.7}K^{-0.7}$　$MP_K = \dfrac{dQ}{dK} = 3L^{0.7}K^{-0.7}$　$AP_L = \dfrac{Q}{L} = 10L^{-0.3}K^{0.3}$

$MP_L = \dfrac{dQ}{dL} = 7L^{-0.3}K^{0.3}$　$F_{KK} = \dfrac{dMP_K}{dK} = -2.1L^{0.7}K^{-1.7}$　$F_{LL} = \dfrac{dMP_L}{dL} = -2.1L^{-1.3}K^{0.3}$

(2)勞動產量彈性：$\varepsilon_L = \dfrac{MP_L}{AP_L} = \dfrac{7L^{-0.3}K^{0.3}}{10L^{-0.3}K^{0.3}} = 0.7$　資本產量彈性：$\varepsilon_K = \dfrac{MP_K}{AP_K} = \dfrac{3L^{0.7}K^{-0.7}}{10L^{0.7}K^{-0.7}} = 0.3$

(3) $MRTS_{LK} = \dfrac{MPP_L}{MPP_K} = \dfrac{7K}{3L}$

(4) $\sigma = 1$

$d\ln MRTS_{LK} = d\ln\left(\dfrac{7}{3}\right) + d\ln\left(\dfrac{K}{L}\right) = d\ln\left(\dfrac{K}{L}\right)$　$\sigma = \dfrac{d\ln\left(\dfrac{K}{L}\right)}{d\ln MRTS_{LK}} = 1$

(5) $S_L = \dfrac{MPP_L \cdot L}{Q} = \dfrac{7L^{-0.3}K^{0.3}(L)}{10L^{0.7}K^{0.3}} = 0.7 = 70\%$　$S_K = \dfrac{MPP_K \cdot K}{Q} = \dfrac{3L^{0.7}K^{-0.7}(K)}{10L^{0.7}K^{0.3}} = 0.3 = 30\%$

題型：完全互補生產函數

14.假設生產函數為$Q = [\min(2K,3L)]^{0.5}$，其中，K 為資本，L 為勞動，則：
(1)該生產函數的規模報酬特性為何？
(2)當 K、L 的價格分別為 r = 5 與 w = 10，且固定成本為 120 時，最適的 K、L 各為多少？此時的產出又為何？(3)擴張路線(expansion path)及替代彈性(elasticity of substitution)各具有何種特性？
7 (4)短期成本函數計算　(5)長期成本函數計算　(6)成本彈性。 Ex $E_p > 1$ ∴ $E_p = 2$
6 (7)規模經濟或不經濟。(8)齊序生產函數？【中原企管所】

$E_p < 5$

解：(1) $f(\lambda L, \lambda K) = [\min(2\lambda K, 3\lambda L)]^{0.5} = \lambda^{0.5}[\min(2K, 3D)]^{0.5} \Rightarrow DRS$。

(2) 均衡解符合 $2K = 3L \Rightarrow 120 = 10L + 5K \Rightarrow L^* = \dfrac{48}{7}, K^* = \dfrac{72}{7}$

(3) 均衡時，最適要素雇用條件滿足：2K = 3L → K = 1.5L…此即擴張線方程式；Leontief 生產函數表達的是投入「完全互補之生產要素」，亦即σ = 0。

(4)短期成本計算：均衡時最適要素雇用量滿足：$Q^2=2\overline{K}=3L \Rightarrow L^*=\frac{Q^2}{3} \Rightarrow STC=10\left(\frac{Q^2}{3}\right)+5\overline{K}$

(5)長期成本計算：均衡時最適要素雇用量滿足：

$Q^2=2K=3L \Rightarrow$ 條件要素需求函數：$L^*=\frac{Q^2}{3}$ $\quad K^*=\frac{Q^2}{2} \Rightarrow LTC=10\left(\frac{Q^2}{3}\right)+5\left(\frac{Q^2}{2}\right)=\frac{35}{6}Q^2$

(6)成本彈性：$LTC=\frac{35}{6}Q^2 \Rightarrow \ell nC=\ell n\left(\frac{35}{6}\right)+2\ell nQ \Rightarrow d\ell nC=2\cdot d\ell nQ \Rightarrow \varepsilon^C=\frac{d\ell nC}{d\ell nQ}=2$ 規模彈性 = $\frac{1}{2}$

規模不經濟

(7)因為成本彈性 = 2 > 1，因此成本函數具有「規模不經濟」特性

(8) $Q=[\min(2K,3L)]^{0.5}$ 為齊序生產函數，因為擴張線方程式為 2K = 3L，擴張線是一條過原點直線。

15.設 CES 生產函數為：$Q=f(L,K)=[aL^\rho+bK^\rho]^{\frac{1}{\rho}}$，試求替代彈性？ 【淡江財金所】

解：$$MRTS_{LK}=\frac{MPP_L}{MPP_K}=\frac{\frac{1}{\rho}[\delta L^\rho+(1-\delta)K^\rho]^{\frac{1}{\rho}-1}\cdot\delta\cdot\rho L^{\rho-1}}{\frac{1}{\rho}[\delta L^\rho+(1-\delta)K^\rho]^{\frac{1}{\rho}-1}\cdot(1-\delta)\rho K^{\rho-1}}$$

$$\therefore \mathrm{MRTS_{LK}}=\frac{\delta}{1-\delta}\left(\frac{\mathrm{K}}{\mathrm{L}}\right)^{1-\rho}$$

$$\ln \mathrm{MRTS_{LK}}=\ln\left(\frac{\delta}{1-\delta}\right)+(1-\rho)\ln\left(\frac{\mathrm{K}}{\mathrm{L}}\right)$$

$$\mathrm{d}\ln \mathrm{MRTS_{LK}}=\mathrm{d}\ln\left(\frac{\delta}{1-\delta}\right)+(1-\rho)\mathrm{d}\ln\left(\frac{\mathrm{K}}{\mathrm{L}}\right)$$

$$\therefore \mathrm{d}\ln \mathrm{MRTS_{LK}}=(1-\rho)\mathrm{d}\ln\left(\frac{\mathrm{K}}{\mathrm{L}}\right)$$

替代彈性：$\frac{\mathrm{d}\ln\left(\frac{\mathrm{K}}{\mathrm{L}}\right)}{\mathrm{d}\ln \mathrm{MRTS_{LK}}}=\frac{\mathrm{d}\ln\left(\frac{\mathrm{K}}{\mathrm{L}}\right)}{(1-\rho)\mathrm{d}\ln\left(\frac{\mathrm{K}}{\mathrm{L}}\right)}=\frac{1}{1-\rho}$ 若替代彈性 $\geq 0 \Rightarrow \rho \leq 1$

16.設某廠商面對之生產函數為 $Q=9(2L^{0.3}+3K^{0.3})^2$。試求此一廠商在要素相對價格(P_L/P_K)上升百分之一時，其最適要素雇用比率(K/L)會變動百分之幾呢？ 【中興企研所】

解：

$$MP_L=\frac{dQ}{dL}=18\left[2L^{0.3}+3K^{0.3}\right]\cdot 0.6L^{-0.7} \qquad MP_K=\frac{dQ}{dK}=18\left[2L^{0.3}+3K^{0.3}\right]\cdot 0.9K^{-0.7}$$

$$\therefore \mathrm{MRTS_{LK}}=\frac{MP_L}{MP_K}=\frac{0.6}{0.9}\left(\frac{K}{L}\right)^{0.7}$$

令 $\frac{K}{L}=k,\ \therefore \mathrm{MRTS_{LK}}=\frac{2}{3}k^{0.7}$ $\qquad \sigma=\frac{1}{dMRTS/dk}\cdot\frac{\mathrm{MRTS}}{k}=\frac{1}{\frac{2}{3}\times 0.7k^{-0.3}}\cdot\frac{\frac{2}{3}k^{0.7}}{k}=\frac{10}{7}$

$\therefore$ 當 $\left(\frac{P_L}{P_K}\right)$ 變動百分之一，$\left(\frac{K}{L}\right)$ 會變動 $\frac{10}{7}$ 百分點。

17.It is possible to have decreasing marginal products for all inputs, and yet have increasing returns to scale. (10 分)　　　　　　　　　　　　【96 暨南經濟所】

解：有可能。

(1)邊際報酬遞減之原因⇒短期下生產者存在固定要素，故知，邊際報酬遞減是一種短期現象。

(2)規模報酬遞增指的是，所有的要素投入量同時增加λ倍，而產出增加大於λ倍之現象。

⇒所有要素同時變動表示沒有固定要素

亦即，是處在生產過程的長期之下，規模報酬遞增是一種長期現象。

(3)由上可知，所有要素呈現邊際報酬遞減與生產技術呈現規模報酬遞增兩者之間並無必然的關係。

18.【是非題】Suppose that a plant's production function is $Q = L^{0.6}K^{0.4}$ (L：labor, K：capital). If it takes the product price and the input prices as given, the total wages paid by the firm will equal 40% of its revenue.　　　　　　【東吳經研所】

解：此題敘述錯誤。勞動分配份額＝60%，並非為 40%

$$S_L = \frac{MPP_L L}{Q} = \frac{0.6L^{-0.4}K^{0.4}L}{L^{0.6}K^{0.4}} = 0.6 = 60\%$$

題型：規模報酬

19.Four production functions are listed as follows. A is the quantity of production. L and K are the quantities of labor and capital. Prove that whether each production function has the property of constant returns to scale.

(1)$Q = 5L^{0.7}K^{0.3}$ (2)$Q = \min(3L^{0.7}, 3K^{0.3})$ (3)$Q = 5(L^{0.7} + K^{0.3})$ (4)$Q = 5(L + K)$　【97 暨南財金所】

CRS　DRS　DRS　CRS

解：

$$(1)Q = 5L^{0.7}K^{0.3} \Rightarrow f(\lambda L, \lambda K) = 5(\lambda L)^{0.7}(\lambda K)^{0.3} = \lambda \cdot 5L^{0.7}K^{0.3} = \lambda \cdot f(L,K) \Rightarrow CRS$$

$$(2)Q = \min(3L^{0.7}, 3K^{0.3}) \Rightarrow f(\lambda L, \lambda K) = \min\left[3(\lambda L)^{0.7}, 3(\lambda K)^{0.3}\right]$$

$$= \lambda^{0.3} \cdot \min\left[3\lambda^{0.4}L^{0.7}, 3K^{0.3}\right] < \min\left[3L^{0.7}, 3K^{0.3}\right] \Rightarrow DRS$$

$$(3)Q = 5(L^{0.7} + K^{0.3}) \Rightarrow f(\lambda L, \lambda K) = 5\left[(\lambda L)^{0.7} + (\lambda K)^{0.3}\right]$$

$$= \lambda^{0.3}\left[\lambda^{0.4}5L^{0.7} + 5K^{0.3}\right] < \lambda \cdot \left[5L^{0.7} + 5K^{0.3}\right] \Rightarrow DRS$$

$$(4)Q = 5(L + K) \Rightarrow f(\lambda L, \lambda K) = 5(\lambda L + \lambda K) = \lambda \cdot f(L,K) \Rightarrow CRS$$

20.Suppose the production function for good q is given by $q = 3 \times K + 2 \times L$ where K and L are capital and labor inputs. Consider the following statements about this function:

(1)the function exhibits constant returns to scale

(2)the function exhibits diminishing marginal productivities to all inputs

(3)the function has a constant rate of technical substitution. Which of the above statements is true?　　　　　　　　　　【清大經研所】

解：$q = 3K + 2L$　CRS

(1) $f(\lambda K, \lambda L) = 3(\lambda K) + 2(\lambda L) = \lambda[3K + 2L] = \lambda[f(L, K)]$

$\therefore$ 為 1 階齊次生產函數，存在規模報酬固定不變特性　(1)描述正確

(2) $MPP_L = \dfrac{\partial q}{\partial L} = 3$　$\dfrac{dMPP_L}{dL} = 0$，$MPP_K = \dfrac{\partial q}{\partial K} = 2$，$\dfrac{dMPP_K}{dK} = 0$

短期，邊際報酬固定不變，並非遞減，此題敘述錯誤。

(3) $MRTS_{LK} = \dfrac{MPP_L}{MPP_K} = \dfrac{3}{2}$，固定不變，本題敘述正確。

$\lambda \Rightarrow IRS$　　$\lambda \Rightarrow CRS$

21.Do each of the following production functions exhibit decreasing, constant, or increasing returns to scale. (1) $Q = 5LK$　(2) $Q = 2L + 3K$【成大企研所】

解：(1) $f(\lambda L, \lambda K) = 5(\lambda L)(\lambda K) = \lambda^2(5LK) = \lambda^2 f(L,K)$

$\therefore$ 生產函數為 2 階齊次　$\Rightarrow$ IRS

(2) $f(\lambda L, \lambda K) = 2(\lambda L) + 3(\lambda K) = \lambda[2L + 3K] = \lambda[f(L,K)]$

$\therefore$ 生產函數為 1 階齊次　$\Rightarrow$ CRS

22.【是非題】$Q = L + K^{0.5}$ 為「規模報酬遞減」。

解：題目敘述有誤，說明如下：

$f(\lambda L, \lambda K) = (\lambda L) + (\lambda K)^{0.5} = \lambda L + \lambda^{0.5} K^{0.5}$

$\lambda Q = \lambda L + \lambda K^{0.5}$

$\because \lambda K^{0.5} > \lambda^{0.5} \therefore \lambda Q > f(\lambda L, \lambda K) \Rightarrow DRS$

λ DRS　　DRS

23.試判斷下列兩個生產函數的規模報酬特性：$Q_1 = 3L + 10K + 500$　$Q_2 = 10L^{0.5}K^{0.3}$【成大企研所】

解：**(1)DRS；　(2) DRS**

$$\begin{cases} f_1(\lambda L, \lambda K) = 3\lambda L + 10\lambda K + 500 \\ \lambda Q_1 = 3\lambda L + 10\lambda K + 500\lambda \end{cases} \Rightarrow \because 500\lambda \ge 500 \quad \therefore \lambda Q_1 \ge f_1(\lambda L, \lambda K) \Rightarrow DRS$$

$f_2(\lambda L, \lambda K) = \lambda^{0.8} Q \Rightarrow DRS$

$\sigma =$

24.令 F(K, L) 代表某廠商之生產函數，其中 K 與 L 分別為資本與勞動，其價格分別為 r 與 w：請計算下列生產函數之替代彈性：(1) $F(K,L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$

(2) $F(K,L) = 2K + L$　　　　【政大國貿所】

$\sigma = \infty$

解：(1) $MRTS_{LK} = \dfrac{\frac{1}{2} L^{-\frac{1}{2}} K^{\frac{1}{2}}}{\frac{1}{2} L^{\frac{1}{2}} K^{-\frac{1}{2}}} = \dfrac{K}{L}$　　$d\ln MRTS_{LK} = d\ln\left(\dfrac{K}{L}\right)$　　替代彈性 $(\sigma) = \dfrac{d\ln\left(\frac{K}{L}\right)}{d\ln MRTS_{LK}} = 1$

(2) $MRTS_{LK} = \dfrac{MPP_L}{MPP_K} = \dfrac{1}{2}$　　$d\ln MRTS_{LK} = d\ln\left(\dfrac{1}{2}\right) = 0$

替代彈性 $(\sigma) = \dfrac{d\ln\left(\frac{K}{L}\right)}{d\ln MRTS_{LK}} = \dfrac{d\ln\left(\frac{K}{L}\right)}{0} = \infty$

第六章　成本函數

TFC

$max\{10, \frac{q^2}{1000}\}$

1.A TV factory costs \$2 million to construct and the marginal cost of the q^{th} TV is Max $[10,\ q^2/1{,}000]$. (25%)

(1) What are average total costs? (10%) AC

(2) What is short run supply? (5%)

(3) What is the long run competitive supply of TVs? 【98 中央產經所】 (10%)

解：已知建造電視工廠花費 200 萬元，因此 TFC 為 200 萬元

$$(1)10=\frac{q^2}{1000}\Rightarrow q^*=100$$

$$\because MC=10\Rightarrow TVC=10q$$

$$\because MC=\frac{q^2}{1000}\Rightarrow TVC=\int\frac{q^2}{1000}dq=\frac{q^3}{3000}$$

$$\begin{Bmatrix} q\le 100 & TVC=10q \\ q>100 & TVC=\dfrac{q^3}{3000}\end{Bmatrix}\Rightarrow\begin{Bmatrix} q\le 100 & STC=10q+2000{,}000 \\ q>100 & STC=\dfrac{q^3}{3000}+2000{,}000\end{Bmatrix}\Rightarrow\begin{Bmatrix} q\le 100 & SAC=10+\dfrac{2000{,}000}{q} \\ q>100 & SAC=\dfrac{q^2}{3000}+\dfrac{2000{,}000}{q}\end{Bmatrix}$$

SMC　微分

(2)廠商短期供給曲線：AVC最低點以上SMC曲線 $\Rightarrow\begin{Bmatrix} q\le 100 & P=10 \\ q>100 & P=\dfrac{q^2}{1000}\end{Bmatrix}$

(3)廠商長期供給曲線：LAC最低點以上LMC曲線：

$$\frac{dAC}{dq}=0\Rightarrow\frac{-2000{,}000}{q^2}+\frac{q}{1500}=0\Rightarrow q^*=1000(3)^{\frac{1}{3}}\qquad P^*=LAC(\min)=3^{\frac{2}{3}}(1000)$$

$$\therefore 廠商長期供給曲線：P=MC\Rightarrow P=\frac{q^2}{1000};P\ge 3^{\frac{2}{3}}(1000)$$

2.已知廠商的固定成本為 100，唯一的變動成本為支付勞動的薪水；當它生產 50 單位產量時，勞動的邊際產量為 5，平均產量為 10，且一單位勞動的價格為 20；請問：此時的邊際成本與平均成本分別是多少？ (10%)【98 淡江財金、保險、國貿】

$$MC=\frac{dTVC}{dq}=\frac{d(\overline{w}\cdot L)}{dq}=\overline{w}\cdot\frac{dL}{dq}=\frac{\overline{w}}{MP_L}=\frac{20}{5}=4$$

解：$\because AP_L=\frac{q}{L}\Rightarrow 10=\frac{50}{L}\Rightarrow L=5$

$$\therefore TVC=w\cdot L=20\cdot 5=100\quad TFC=100\quad STC=200\quad SAC=\frac{STC}{q}=\frac{200}{50}=4$$

3.Consider the graph of Figure B.2 and identify:

(1)the marginal-cost curve.

(2)the average-total-cost curve.

(3)the average-variable-cost curve.

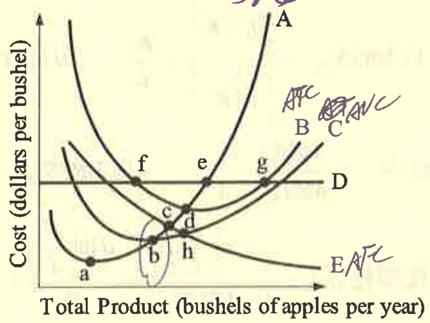
(4)the average-fix-cost curve.

(5)the short-run supply curve.

(6)the marginal-revenue curve.

(7)the average-revenue curve.

(8)the capacity output level.



(9)the break-even point on the short-run supply curve.

(10)the shutdown point on the short-run supply curve.【淡江企研所】

解：(1)MC：A 曲線

(2)ATC：B 曲線

(3)AVC：C 曲線

(4)AFC：E 曲線

(5)短期供給曲線：AVC(min)以上 MC 曲線

⇒ b 點以上 A 曲線

(6)MR：D 曲線

(7)AR：D 曲線

(8)MR = MC，決定 e 點，所對應產量水準

(9)損益平衡點：P = ATC ⇒ g 點

(10)短期歇業點：AVC(min)，即 b 點

4.A firm's total cost function is TC = 300 + 0.5Q

(1) Graph the firm's TC, MC and AC function.

(2) Find total fixed cost, total variable cost and average cost for this function. Break average cost into average fixed cost and average variable cost.

(3) What happens to average variable cost and average fixed cost as output rises?

(4) Will AC ever equal MC? If so, what quantity. If not, why not? 【97 暨南國企所】

解：(1) $TC = 300 + 0.5Q$　$AC = \dfrac{300}{Q} + 0.5$　$MC = 0.5$

(2) $TFC = 300$　$TVC = 0.5Q$　$AFC = \dfrac{300}{Q}$　$AVC = 0.5$　$SAC = \dfrac{300}{Q} + 0.5$

(3)AVC=0.5 ➔表示 AVC 是一條水平線；　$AFC = \dfrac{300}{Q}$ 是一條雙曲線，表示隨著產量增加，AFC 遞減。

(4) $AC = \dfrac{300}{Q} + 0.5$　$MC = 0.5 \Rightarrow$ 因此在任何產量水準下，AC 必大於 MC。

5.假設總成本函數為 TC = $\boxed{4000}$ + 30Q − 12Q² + 2Q³，請計算：

(1)產量為 1000 單位時，TFC 等於多少？AFC 等於多少？

(2)產量為 4000 單位時，TFC 等於多少？AFC 等於多少？

(3).產量為 40 單位時，AVC 等於多少？MC 等於多少？ATC 等於多少？ (4)在哪一個產量水準下，可變要素之邊際實物產量開始遞減？【中原企管所】

解：(1) $AFC = \dfrac{TFC}{Q} = \dfrac{4000}{1000} = 4$　(2) $AFC = \dfrac{TFC}{Q} = \dfrac{4000}{4000} = 1$

(3) $TVC = 30Q - 12Q^2 + 2Q^3$

$AVC = \frac{TVC}{Q} = 30 - 12Q + 2Q^2$ $\quad MC = 30 - 24Q + 6Q^2$ $\quad AC = \frac{4000}{Q} + 30 - 12Q + 2Q^2$

$$if\ Q = 40, \begin{cases} AVC = 2750 \\ AC = 2850 \\ MC = 8670 \end{cases}$$

(4) $SMC = \frac{P_L}{MP_L} = 30 - 24Q + 6Q^2$

$SMC = \frac{P_L}{MP_L} = 30 - 24Q + 6Q^2$ $\quad -\frac{b}{2a} = \frac{24}{12}$

$\frac{dSMC}{dQ} = 0 \Rightarrow 24 = 12Q, Q^* = 2$ $\quad \frac{d^2SMC}{dQ^2} = -12 < 0$

產量為2時，SMC為極大值，亦即MP_L開始遞減

6.假設某廠商之短期總成本函數為$Cs(Q) = \underbrace{a}_{TFC} + \underbrace{bQ - cQ^2 + dQ^3}_{TVC}$，Q 為其產量，且 a，b，c，d > 0，試分別求出(1)此廠商之變動成本函數和固定成本函數。

(2)此廠商之邊際成本曲線與平均變動成本曲線之交點所對應之產量。【台大財金所】

解：$C = a + bQ - cQ^2 + dQ^3$

(1) $TVC = bQ - cQ^2 + dQ^3$ $\quad TFC = a$

(2) $MC = b - 2cQ + 3dQ^2$ $\quad \mathrm{AVC = b - cQ + dQ^2}$

MC與AVC相交所對應產量，即MC = AVC

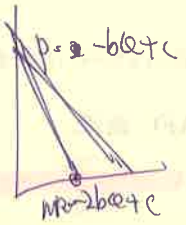
$b - 2cQ + 3dQ^2 = b - cQ + dQ^2$ $\quad 2dQ^2 - cQ = 0$ $\quad 2dQ - c = 0$ $\quad Q^* = \frac{c}{2d}$

7.Find the profit-maximizing quantity Q^* and price P^* for a monopolist with the linear demand curve $P = -bQ + c$ and constant AVC given by $AVC = a$ 【中央財管所】

解：獨佔廠商追求 Max π，採$MR = MC$訂價法

$TR = PQ = -bQ^2 + cQ$ $\quad \mathrm{MR = -2bQ + c}$ $\quad TVC = aQ$ $\quad MC = \frac{dTVC}{dQ} = a$

$\boxed{\mathrm{MR = MC}} \Rightarrow \underbrace{-2bQ + c}_{MR} = \underbrace{a}_{MC}$ $\quad \therefore Q^* = \frac{c - a}{2b}$ $\quad P^* = -b\left(\frac{c-a}{2b}\right) + c = \frac{a + c}{2}$



題型：短期生產結構與短期成本結構

8.【是非題】For a firm, if the marginal product increases, then its marginal cost will also increase. 【96 高第一科大金融營運】

解：此敘述有誤。$STC = \bar{w}L + \bar{r}K \Rightarrow SMC = \frac{d(\bar{w}L)}{dQ} = \bar{w}\frac{dL}{dQ} = \frac{\bar{w}}{MP_L} \Rightarrow MP_L$遞增$\Leftrightarrow SMC$遞減

由上可推知生產與成本結構的短期對偶關係：MP_L遞增$\Leftrightarrow SMC$遞減

9.【是非題】由於有沉沒成本的出現，廠商短期平均成本經常會低於長期平均成本。【台大國企所】

解：錯，因為長期平均成本曲線是短期平均成本曲線的下包絡曲線，所以短期平均成本必大

於或等於長期平均成本。

10.【是非題】The short-run total cost curves represent the minimal costs for producing various output levels. 【94 清大經濟】

解：×。長期總成本是所有短期總成本之包絡曲線。因此長期總成本才是代表各種產出水準下之最小成本組合的軌跡。

11.廠商甲的生產函數為 $Q = 0.25\sqrt{LK}$，其中 L 和 K 分別是勞動及資本的用量，Q 是產量；該廠商所面臨的勞動的單位價格 w = 5，資本的單位價格 r = 1；又該廠商所能採用的資本用量只有兩種：K = 400 (大廠) 與 K = 100 (小廠)。在此假設下，當 Q = 10 時，廠商甲的長期總成本為__(1)__；當 Q = 30 時，廠商甲的長期總成本為__(2)__。當 Q = 40 時，廠商甲的長期總成本為__(3)__。【94 台大財金所】

解：(1)180，(2)580，(3)720

短期成本函數$(\bar{K}_1 = 100)$：小廠	短期成本函數$(\bar{K}_2 = 400)$：大廠
$\boxed{\begin{array}{l} Min \quad STC_1 = 5L + 100 \\ s.t. Q = 2.5\sqrt{L} \end{array}}$	$\boxed{\begin{array}{l} Min \quad STC_2 = 5L + 400 \\ s.t. Q = 5\sqrt{L} \end{array}}$
$\because L = \dfrac{Q^2}{6.25}$	$\because L = \dfrac{Q^2}{25}$
$\therefore STC_1 = 5\left(\dfrac{Q^2}{6.25}\right) + 100 = 0.8Q^2 + 100$	$\therefore STC_2 = 5\left(\dfrac{Q^2}{25}\right) + 400 = 0.2Q^2 + 400$

(1) $Q = 10 \Rightarrow STC_1 = 0.8(10)^2 + 100 = 180 \qquad STC_2 = 0.2(10)^2 + 400 = 420$

$LTC = \min(STC_1 = 180, STC_2 = 420) = 180$

(2) $Q = 30 \Rightarrow STC_1 = 0.8(30)^2 + 100 = 820 \qquad STC_2 = 0.2(30)^2 + 400 = 580$

$LTC = \min(STC_1 = 820, STC_2 = 580) = 580$

(3) $Q = 40 \Rightarrow STC_1 = 0.8(40)^2 + 100 = 1380 \qquad STC_2 = 0.2(40)^2 + 400 = 720$

$LTC = \min(STC_1 = 1380, STC_2 = 720) = 720$

12.若某廠商短期總成本線 $STC = 0.04Q^3 - 0.9Q^2 + (11 - K)Q + 5K^2$，其中 K 為固定生產要素，試求出廠商之長期平均成本線(LAC)。【北大企研所】

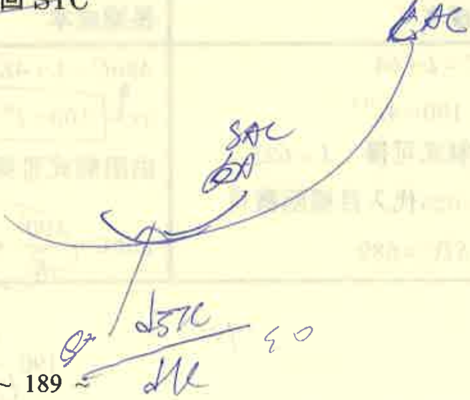
解：從 $STC \xrightarrow{\text{推導}} LTC$ $\boxed{\text{利用}\dfrac{dSTC}{dK} = 0}$ 求解

$\dfrac{dSTC}{dK} = 0 \quad \Rightarrow \; -Q + 10K = 0 \;\Rightarrow K^* = \dfrac{Q}{10}$ 代回 STC

$LTC = 0.04Q^3 - 0.9Q^2 + \dfrac{109}{10}Q^2 + 5\left(\dfrac{Q^2}{100}\right)$

$LTC = 0.04Q^3 + 10.05Q^2$

$LAC = 0.04Q^2 + 10.05Q$



13.(1)廠商甲的生產函數為：$y=(kl)^{1/4}$， y 為產出數量，k, l 為兩種要素的投入量。令 MP_k、MP_l 分別表示兩種要素的邊際產出(marginal product)，$|MRTS|\equiv|dk/dl|$ 為兩要素間的邊際技術替代率(marginal rate of technical substitution)。 (A) $MP_k=k^{3/4}\times l^{1/4}$ (B) $MP_k=k^{1/4}\times l^{3/4}$ (C) $MP_l=k^{3/4}\times l^{1/4}$ (D) $MP_l=k^{1/4}\times l^{3/4}$ (E) $|MRTS|=k/l$

(2)承續前題。要素 k 之價格為$16，要素 l 之價格為$1。兩種要素皆為變動要素。甲以最小的成本生產，當 y = 10，甲最適的要素使用量 k 與 l 為何？ (A) K = 100 (B) K = 400 (C) l = 100 (D) l = 400 (E)以上皆非 AC=8y $C=4y^2+4y^2=8y^2$ ∴ MC=16y

(3)承續前兩題。令 AC(y)與 MC(y)為甲全產 y 單位的平均成本與邊際成本。 (A) AC(y) = 4y (B) AC(y) = 8y (C) MC(y) = 4y (D) MC(y) = 8y (E) MC(y) = 16y

(4)承續前 3 題。產品 y 的市場為完全競爭，產品 y 之價格為$100。甲追求利潤極大，請問他會生產幾單位的 y？ (A) 20 (B) 25 (C) 30 (D) 35 (E) 以上皆非【97 台大經研所】 P=MR

解：<u>(1) (E) (2) (D) (3) (B)(E) (4) (E)</u>

14.Better Buys, Inc., is a leading discount retailer of wide-screen digital and cable-ready plasma HDTVs. Revenue and cost relations for a popular 55-inch model are:

TR = \$4,500Q - \$0.1Q² MR = \$4,500 - \$0.2Q

TC = \$2,000,000 + \$1,500Q + \$0.5Q² MC = \$1,500 + Q

(1)Calculate：output, marginal cost, average cost, price and profit at the average cost-minimizing activity level.

(2) Calculate：output, marginal cost, average cost, price and profit at the profit-maximizing activity level. 【97 彰師企研所】

解：$AC=\dfrac{2{,}000{,}000}{Q}+1500+0.5Q\Rightarrow\dfrac{dAC}{dQ}=\dfrac{-2000{,}000}{Q^2}+0.5=0\quad S.O.C\quad\dfrac{d^2AC}{dQ^2}=\dfrac{4000{,}000}{Q^3}>0$

$\therefore Q^*=2000\quad MC=3500\quad AC=3500\quad P=AR=4500-0.1(2000)=4300$

$\pi=8{,}600{,}000-7{,}000{,}000=1{,}600{,}000$

<u>題型：Cobb-Douglas 生產函數</u>

15.大有公司使用 L 與 K 兩種生產要素，生產函數為 $Q=L^{0.5}K^{0.5}$，其中，要素 L 的價格為$1，要素 K 的價格為$4。且要素 K 在短期下無法調整投入數量。目前要素 K 的使用量為 16。請問生產 100 單位的短期平均成本和長期平均成本各為多少？【96 彰師商教】

解：

短期成本	長期成本
$MinC=L+64$ $s.t.\quad 100=4L^{0.5}$ 由限制式可得：$L=625$ 將 L=625 代入目標函數可得：$STC=689$	$MinC=L+4K$ $s.t.\quad 100=L^{0.5}K^{0.5}$ 由限制式可得：$\dfrac{100^2}{K}=L$ 將上式代入目標函數可得： $\min C=\dfrac{100^2}{K}+4K$

$SAC = STC/Q = 6.89$	f.o.c $\dfrac{dC}{d\bar{K}} = 0 \Rightarrow -\dfrac{100^2}{\bar{K}^2} + 4 = 0 \Rightarrow \bar{K} = 50$ $LTC = \dfrac{100^2}{50} + 200 = 400$ LAC =400 /4=100

16.廠商的生產函數為$Q = 5K^{\frac{1}{2}}L^{\frac{1}{2}}$，$P_K = 20$，$P_L = 5$

(1)求出廠商短期總成本？ (2)求出廠商長期總成本？

(3)如果廠商設定產量$Q_0 = 400$，則最佳要素使用量 K，L 各為多少？總成本？平均成本？

(4)假設產品價格為 4，則利潤為何？ 【中山財管所、中山企研所】

解：(1)求廠商 STC $\boxed{\begin{array}{ll} Min & STC = 5L + 20\bar{K} \\ s.t & Q = 5K^{\frac{1}{2}}L^{\frac{1}{2}}, K = \bar{K} \end{array}}$

step 1：將生產函數變成$Q = 5\bar{K}^{\frac{1}{2}}L^{\frac{1}{2}}$

step 2：$Q^2 = 25\bar{K}L \rightarrow L = \dfrac{Q^2}{25\bar{K}}$

step 3：$STC = 5\left(\dfrac{Q^2}{25\bar{K}}\right) + 20\bar{K} = \dfrac{Q^2}{5\bar{K}} + 20\bar{K}$

(2)求廠商 LTC $\boxed{\begin{array}{ll} Min & LTC = 5L + 20K \\ s.t & Q = 5K^{\frac{1}{2}}L^{\frac{1}{2}} \end{array}}$

利用成本極小化條件：$\dfrac{MPP_L}{MPP_K} = \dfrac{P_L}{P_K}$求解

$L=4K$

即$\dfrac{\frac{5}{2}L^{\frac{-1}{2}}K^{\frac{1}{2}}}{\frac{5}{2}L^{\frac{1}{2}}K^{\frac{-1}{2}}} = \dfrac{5}{20} \Rightarrow \dfrac{K}{L} = \dfrac{1}{4} \Rightarrow L = 4K$代入生產函數 $Q=5\times 2K^{\frac{1}{2}}K^{\frac{1}{2}}$

$Q=10K$

$\therefore K=\frac{Q}{10}$

$Q = 5(4K)^{\frac{1}{2}}K^{\frac{1}{2}}$ $\therefore$ $Q = 10K$ $\rightarrow K = \dfrac{Q}{10}$，$L = \dfrac{2Q}{5}$

$\therefore$ $LTC = 5\left(\dfrac{2Q}{5}\right) + 20\left(\dfrac{Q}{10}\right) = 2Q + 2Q = 4Q$

$\therefore$ $LTC = 4Q$

(3) $\boxed{\begin{array}{ll} Min & LTC = 5L + 20K \\ s.t & 400 = 5K^{\frac{1}{2}}L^{\frac{1}{2}} \end{array}} \Rightarrow \begin{array}{l} L^* = \dfrac{2}{5}Q = \dfrac{2}{5}(400) = 160 \\ K^* = \dfrac{1}{10}Q = \dfrac{1}{10}(400) = 40 \end{array}$

$LTC = 4(400) = 1600$ $LAC = \dfrac{1600}{400} = 4$

(4) $\pi = P \cdot Q - LTC = 4(400) - 1600 = 0$

17.有一廠商的生產函數為$Y = f(X_1, X_2) = X_1^{\frac{1}{2}}X_2^{\frac{1}{2}}$，其中 Y 代表產出，$X_1$和$X_2$分別代表兩種要素投入的數量。此廠商不論市在產品市場或市要素市場都是價格接受者的角色。第一種要素的價格為\$1，第二種要素價格為\$4。目前廠商計量生產一單位的產出。請依據上述資訊回答

下列問題：

(1).請寫出廠商極小化成本的目標式。

(2)請依據(1)所要求列出的目標式計算出廠商最適的要素使用量，及廠商生產的總成本。

(3)若第一種要素的價格上漲為$2，其他條件維持不變，請問相較於(2)的結果，兩要素的使用量和總成本會有何變化？是增加、減少或不變。請提供符合直覺的解釋。(在此不需要計算出切確數值)　【95 彰師商教所】

解：

$$\begin{cases}\min C = w_1X_1 + w_2X_2 \\ s.t.\ X_1^{\frac{1}{2}}X_2^{\frac{1}{2}} = Y\end{cases} \Rightarrow L = w_1X_1 + w_2X_2 + \lambda\left(Y - X_1^{\frac{1}{2}}X_2^{\frac{1}{2}}\right) \Rightarrow f.o.c\begin{cases}MRTS = \dfrac{X_2}{X_1} = \dfrac{w_1}{w_2} \\ X_1^{\frac{1}{2}}X_2^{\frac{1}{2}} = Y\end{cases}$$

(1)
$$\Rightarrow \begin{cases}X_1^* = \left(\dfrac{w_2}{w_1}\right)^{\frac{1}{2}} Y \\ X_2^* = \left(\dfrac{w_1}{w_2}\right)^{\frac{1}{2}} Y\end{cases} \Rightarrow C = w_1X_1^* + w_2X_2^* = 2w_1^{\frac{1}{2}}w_2^{\frac{1}{2}}Y \Rightarrow w_1 = 1, w_2 = 4, Y = 1 : \begin{cases}X_1^* = 2 \\ X_2^* = \dfrac{1}{2} \\ LTC = 4\end{cases}$$

(2) $w_1 = 2, w_2 = 4, Y = 1 : \begin{cases}X_1^* = \sqrt{2} \\ X_2^* = \dfrac{1}{\sqrt{2}} \\ LTC = 4\sqrt{2}\end{cases}$　　(3) $\left(w_1 \uparrow, \overline{Y}\right) \Rightarrow X_1 \downarrow, X_2 \uparrow, LTC \uparrow$

18.某廠商的生產函數為 $Q = 5K^{0.5}L^{0.5}$，$P_L = 4$，$P_K = 1$　(1)短期資本數量固定 $K = 100$，計算廠商的短期總成本函數(成本為 Q 的函數)。(2)計算長期總成本函數。【94 成大企研所】

解：(1)求 STC：$\boxed{\begin{array}{ll} Min & C = 4L + K \\ s.t & Q = 5L^{0.5}K^{0.5}\end{array}}$

將 $\overline{K} = 100$ 代入生產函數，則　$Q = 5L^{0.5}(10)$　　∴ 將生產函數化簡為 $L^* = \dfrac{Q^2}{2{,}500}$

將　$L^* = \dfrac{Q^2}{2{,}500}$，$\overline{K} = 100$ 代入成本函數，即可求出 STC：　$STC = 4L^* + \overline{K} = \dfrac{Q^2}{625} + 100$

(2) $\boxed{\begin{array}{ll} Min & C = 4L + K \\ s.t & Q = 5L^{0.5}K^{0.5}\end{array}}$ 利用生產者均衡條件 $\dfrac{MPP_L}{MPP_K} = \dfrac{P_L}{P_K}$

$\dfrac{K}{L} = \dfrac{4}{1}$　$\Rightarrow$ $K = 4L$ 代入生產函數：　$Q = 5L^{0.5}(4L)^{0.5}$　$\Rightarrow$ $Q = 10L$　$\Rightarrow$ $L^* = \dfrac{Q}{10}$，$K^* = \dfrac{2}{5}Q$

再將條件要素需求函數 L^*，K^* 代回成本函數，即可求出：長期總成本：

$$LTC = 4L^* + K^* = 4\left(\frac{Q}{10}\right) + \frac{2}{5}Q$$

$$\therefore LTC = \frac{4}{5}Q$$

19.Consider the production function $Q = LK^2$. Let w, r be the factor prices of L, K.
(1)Find the constant-output(or conditional factor) factor demands.
(2)Find the cost function.

(3)Is the cost function derived from (2) homogeneous in w and r. 【成大國企所】

解：(1) $\boxed{\begin{array}{ll} Min & C = wL + rK \\ s.t & Q = LK^2 \end{array}}$ $\qquad \mathrm{L} = \mathrm{wL} + \mathrm{rK} + \lambda\left[\mathrm{Q} - \mathrm{LK}^2\right]$

F.O.C $\quad \dfrac{\partial \mathrm{L}}{\partial \mathrm{L}} = 0 \Rightarrow W - \lambda K^2 = 0 \qquad \dfrac{\partial \mathrm{L}}{\partial \mathrm{K}} = 0 \Rightarrow r - 2\lambda LK = 0 \qquad \dfrac{\partial \mathrm{L}}{\partial \lambda} = 0 \Rightarrow Q - LK^2 = 0$

$\Rightarrow \dfrac{K}{2L} = \dfrac{w}{r} \quad \Rightarrow \mathrm{K} = \dfrac{2\mathrm{w}}{\mathrm{r}}\mathrm{L}$ 代入 求解 $\quad Q = L\left(\dfrac{2w}{r}L\right)^2$

$\therefore Q = L^3\left(\dfrac{2w}{r}\right)^2 \quad \therefore L^* = Q^{\frac{1}{3}}\left(\dfrac{r}{2w}\right)^{\frac{2}{3}} \qquad K^* = Q^{\frac{1}{3}}\left(\dfrac{2w}{r}\right)^{\frac{1}{3}}$

(2)將條件要素需求函數L^*，K^*代入成本函數

$C = wL^* + rK^* = Q^{\frac{1}{3}}\left(\dfrac{1}{2}\right)^{\frac{2}{3}} w^{\frac{1}{3}} r^{\frac{2}{3}} + Q^{\frac{1}{3}}(2)^{\frac{1}{3}} w^{\frac{1}{3}} r^{\frac{2}{3}} \qquad \therefore$Cost function $\mathrm{C}^* = \mathrm{Q}^{\frac{1}{3}}\mathrm{w}^{\frac{1}{3}}\mathrm{r}^{\frac{2}{3}}\left[\left(\dfrac{1}{2}\right)^{\frac{2}{3}} + (2)^{\frac{1}{3}}\right]$

(3)令$C = C(w,r,Q)$：當 w, r 同時增加λ倍 $\Rightarrow C(\lambda w, \lambda r, Q) = Q^{\frac{1}{3}}(\lambda w)^{\frac{1}{3}}(\lambda r)^{\frac{2}{3}}\left[\left(\dfrac{1}{2}\right)^{\frac{2}{3}} + (2)^{\frac{1}{3}}\right]$

$\therefore C(\lambda w, \lambda r, Q) = \lambda^1 Q^{\frac{1}{3}} w^{\frac{1}{3}} r^{\frac{2}{3}}\left[\left(\dfrac{1}{2}\right)^{\frac{2}{3}} + (2)^{\frac{1}{3}}\right] \qquad C(\lambda w, \lambda r, Q) = \lambda^1 C(w,r,Q)$

∴ 成本函數為(w, r)之 Homogeneous of degree one

20.Consider the Cobb-Douglas production function $Q(L,K) = AL^{\alpha}K^{\beta}$, where A, α, β are all constants. Assume the price of input L is w, the price of input K is r.

(1)Calculate the elasticity of substitute σ.

(2)Does it exhibit increasing, decreasing, or constant returns to scale？ (hint：it may be depend on αand β)

(3)Find the Conditional labor demand curve and calculate its price elasticity.

(4)Given w = r =1, find the cost function C(Q), AC(Q), MC(Q), respectively.

(5)Given w = r =1. In the short run , k is fixed at 10, find AC, AVC, AFC, and MC. 【94 中山企研所】

解：

(1) $Q = AL^{\alpha}K^{\beta}$

$MP_L = \alpha AL^{\alpha-1}K^{\beta} \qquad MP_K = \beta AL^{\alpha}K^{1-\beta}$

將二式相除可得：$MRTS = \dfrac{\alpha}{\beta}\dfrac{K}{L} = \dfrac{\alpha}{\beta}k$

對上式取對數：$\ln MRTS = \ln\dfrac{\alpha}{\beta} + \ln k$

$\because \ln k = \ln MRTS - \ln\dfrac{\alpha}{\beta}$

$\therefore \sigma = \dfrac{d\ln k}{d\ln MRTS} = 1$

(2) $Q(\lambda L,\lambda K)=A(\lambda L)^{\alpha}(\lambda K)\beta$
$$=\lambda^{\alpha+\beta}AL^{\alpha}K^{\beta}$$
$$=\lambda^{\alpha+\beta}Q(L,K)$$

當α+β= 1 時，Q(L,K)為 constant return to scale。

當α+β< 1 時，Q(L,K)為 decreasing return to scale。

當α+β> 1 時，Q(L,K) is increasing return to scale。

(3) Min LTC = wL + wK

s.t. $Q = AL^{\alpha}K^{\beta}$

聯立求解上述一階條件可得 conditional factor demand function：

$$L=A^{\frac{-1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{-\beta}{\alpha+\beta}}\cdot Q^{\frac{1}{\alpha+\beta}}\qquad K=A^{\frac{-1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{\alpha}{\alpha+\beta}}\cdot Q^{\frac{1}{\alpha+\beta}}$$

勞動的需求彈性(E_L^d)

$$\ln L=\frac{-1}{\alpha+\beta}\ln A-\frac{\beta}{\alpha+\beta}\ln\frac{\beta}{\alpha}-\frac{\beta}{\alpha+\beta}\ln w+\frac{\beta}{\alpha+\beta}\ln r+\frac{1}{\alpha+\beta}\ln Q\qquad E_L^d=\frac{\partial\ln L}{\partial\ln w}=-\frac{\beta}{\alpha+\beta}$$

(4)成本函數：將勞動與資本的 conditional factor demand function 代入目標函數，可得如下成本函數：

$$LTC=\left[w\left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{-\beta}{\alpha+\beta}}+r\left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{\alpha}{\alpha+\beta}}\right]\cdot A^{\frac{-1}{\alpha+\beta}}Q^{\frac{1}{\alpha+\beta}}$$

令 $H=A^{\frac{-1}{\alpha+\beta}}\left[w\left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{-\beta}{\alpha+\beta}}+r\left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{\alpha}{\alpha+\beta}}\right]$

$$LTC=H\cdot Q^{\frac{1}{\alpha+\beta}}\qquad LAC=H\cdot Q^{\frac{1-\alpha-\beta}{\alpha+\beta}}\qquad LMC=\frac{H}{\alpha+\beta}Q^{\frac{1-\alpha-\beta}{\alpha+\beta}}$$

(5)Min STC = L + 10

s.t. $Q = A10^{\beta}L^{\alpha}$

$$\because L=A^{\frac{-1}{\alpha}}10^{\frac{-\beta}{\alpha}}\cdot Q^{\frac{1}{\alpha}}\qquad\therefore STC=A^{\frac{-1}{\alpha}}10^{\frac{-\beta}{\alpha}}\cdot Q^{\frac{1}{\alpha}}+10$$

$$TVC=A^{\frac{-1}{\alpha}}10^{\frac{-\beta}{\alpha}}Q^{\frac{1}{\alpha}}\to AVC=A^{\frac{-1}{\alpha}}10^{\frac{-\beta}{\alpha}}Q^{\frac{1-\alpha}{\alpha}}\qquad TFC=10\to AFC=10Q^{-1}$$

$$SAC=AVC+AFC=A^{\frac{-1}{\alpha}}10^{\frac{-\beta}{\alpha}}Q^{\frac{1-\alpha}{\alpha}}+\frac{10}{Q}\qquad SMC=\frac{1}{\alpha}A^{\frac{-1}{\alpha}}10^{\frac{-\beta}{\alpha}}Q^{\frac{1-\alpha}{\alpha}}$$

21.The production function of a coal-mine firm is given by $X=K^{0.5}\cdot L^{0.5}$, where x denotes million tons per year, and L and K denote the labor and capital inputs in millions of hours. The long run rental prices of L and K and $W_L=\$4$, and $W_K=\$9$.

(1)In the short-run the firm decides to maintain its capital input fixed at $K^0=4/3$ (million machine hours). Derive the firm's short-run total and marginal cost curves

(2)Derive the firm's long-run total and marginal cost curves

(3)What is the optimal output (sales) of coal in the short-run?

(4)If the firm were the only coal producer and the market demand for coal were given by $P=20-2X$, what would be the firm's maximum sales of coal in the long-run? Would this quantity be its optimal sales? 【成大財金所】

解：(1)設$K_0=\frac{4}{3}$ 代入生產函數 $\Rightarrow X=L^{0.5}\left(\frac{4}{3}\right)^{0.5}$ $\therefore L^*=\frac{3}{4}X^2$ 代入成本函數

因此$STC=4L^*+9K_0=4\left(\frac{3}{4}X^2\right)+9\left(\frac{4}{3}\right)$ $\therefore STC=3X^2+12$ $SMC=6X$

(2)求長期總成本，利用成本極小化模型求解

$\boxed{\begin{array}{ll}\underset{L^*,K^*}{Min} & C=4L+9K\\ s.t & X=L^{0.5}K^{0.5}\end{array}}$ $\Rightarrow$生產者均衡條件 $\frac{MPP_L}{MPP_K}=\frac{w}{r}$

$\frac{K}{L}=\frac{4}{9}\Rightarrow K=\frac{4}{9}L$ 代回生產函數 $X=L^{0.5}\left(\frac{4}{9}L\right)^{0.5}$ $\therefore X=\frac{2}{3}L$ $\Rightarrow$ $L^*=\frac{3}{2}X$, $K^*=\frac{2}{3}X$

再將條件要素需求函數L^*, K^* 代回成本函數，

$C^*=4L^*+9K^*$ $\Rightarrow$ $C=6X+6X$ $\Rightarrow C=12X$ $LMC=12$

(3)短期，令$K=\overline{K}$代回生產函數，化簡成$L^*=\frac{X^2}{\overline{K}}$代回成本函數，$STC=\frac{4X^2}{\overline{K}}+9\overline{K}$，

廠商在各個不同產量下，選擇最適生產規模，使生產總成本最小，

即$\frac{dSTC}{d\overline{K}}=0$ $\Rightarrow\frac{-4X^2}{\overline{K}^2}+9=0$，求出短期最適生產規模$\overline{K}=\frac{2}{3}X$

(4)若該產業為獨占，均衡訂價條件為$MR=MC$

$MR=20-4X$ $MC=12$ $\therefore 20-4X=12$ $\Rightarrow X^*=2$, $P^*=16$

利潤$=16\times2-12\times2=8$ $\therefore X^*=2$為最適銷售量

22.Given the following production function: $Y=\alpha\log K+(1-\alpha)\log L$ where $0<\alpha<1$, K and L are capital and labor inputs, the market return for capital and labor are r and w individually.

(1)Derive the demand curves for K and L for a competitive producer.

(2)Derive its cost function. (Hint: α, r, w are exogenous parameters. Solve: K^*‧L^* and C(Y).) 【中山經研所】

解：(1) $\boxed{\begin{array}{ll}Min & C=wL+rK\\ s.t & Y=\alpha\log K+(1-\alpha)\log L\end{array}}$ $L=wL+rK+\lambda[Y-\alpha\log K-(1-\alpha)\log L]$

F.O.C $\frac{\partial L}{\partial L}=0\Rightarrow w-\lambda\frac{1-\alpha}{L}=0$ $\frac{\partial L}{\partial K}=0\Rightarrow r-\lambda\frac{\alpha}{K}=0$ $\frac{\partial L}{\partial\lambda}=0$

$\Rightarrow Y-\alpha\log K-(1-\alpha)\log L=0$

$\Rightarrow$ $\frac{w}{r}=\frac{(1-\alpha)K}{\alpha L}$ $\Rightarrow K=\frac{\alpha w}{(1-\alpha)r}L$ 代入求解 $Y=\alpha\log\left(\frac{\alpha w}{(1-\alpha)r}L\right)+(1-\alpha)\log L$

$Y=\log\left[\frac{\alpha w}{(1-\alpha)r}L\right]^{\alpha}+\log L^{1-\alpha}$ $Y=\log\left[\frac{\alpha w}{(1-\alpha)r}L\right]^{\alpha}L^{1-\alpha}$

$\therefore 10^Y=L\left[\frac{\alpha w}{(1-\alpha)w}\right]^{\alpha}$ $\therefore$ $L^*=10^Y\left(\frac{(1-\alpha)r}{\alpha w}\right)^{\alpha}$ $K^*=10^Y\left(\frac{\alpha w}{(1-\alpha)r}\right)^{1-\alpha}$

(2)將L^*,K^*代回成本函數，$C^*=wL^*+rK^*$

$C^*=10^Y\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}w^{1-\alpha}r^{\alpha}+10^Y\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}w^{1-\alpha}r^{\alpha}$ $C^*=10^Y w^{1-\alpha}r^{\alpha}\left[\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}+\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right]$

23.有一廠商利用 x_1 和 x_2 兩種要素生產產品 y。兩種要素的價格分別為 w_1、w_2。生產函數為 $y = f(x_1,x_2) = \max(a_1x_1,a_2x_2)$。請問該生產函數對應的成本函數 $C(x_1,x_2,y)$為何？　【94 彰師商教所】

解：

$$\begin{cases}\min C = w_1X_1 + w_2X_2 \\ s.t.\ \ Y = \min(a_1X_1, a_2X_2)\end{cases} \Rightarrow a_1X_1 = a_2X_2$$

$$\Rightarrow \begin{cases} X_1^* = \dfrac{Y}{a_1} \\ X_2^* = \dfrac{Y}{a_2}\end{cases} \Rightarrow C = w_1X_1^* + w_2X_2^* = \left(\frac{w_1}{a_1} + \frac{w_2}{a_2}\right)Y$$

24.設廠商用 L 與 K 來生產 Q，其生產函數為 $Q = \min\{\sqrt{L}, K\}$。

(1)在短期中，假設廠商雇用「足夠多」資本 $\overline{K}$，故短期能影響產量(Q)的生產要素只有 L，亦即短期 $Q = \sqrt{L}$。設L的價格為P_L，K的價格為P_K，請證明：平均成本函數為 $C(Q,\overline{K}) = P_LQ + P_K\dfrac{\overline{K}}{Q}$。 (2)在短期，廠商的平均變動成本函數為何? 邊際成本函數為何? 【95 成大企研所】

解：

(1) $$\begin{cases}\min C = P_LL + P_KK \\ s.t.\ \ Q = \min\{\sqrt{L}, K\} \Rightarrow \text{將限制式代入目標函數：} STC = P_LQ^2 + P_K\overline{K} \\ K = \overline{K}\end{cases}$$

$$SAC = \frac{STC}{Q} = P_LQ + \frac{P_K\overline{K}}{Q}$$

(2) $$SAVC = \frac{TVC}{Q} = P_LQ \qquad SMC = \frac{dTC}{dQ} = 2P_LQ$$

25.假設廠商生產函數如下：$Q = \min\{5k, 10l\}$

短期下廠商的資本設備固定，$k = 1000$，資本的租賃價格(rental price) $r = 1$，勞動的工資率 $w = 1$，試問：(1)計算廠商長期總成本、平均成本及邊際成本

(2)計算廠商短期總成本、平均成本及邊際成本　【95 北大企研所】

解：(1) $$\begin{cases}\min C = l + k \\ s.t.\ \ Q = \min\{5k, 10l\}\end{cases} \Rightarrow Q = 5k = 10l \begin{cases} k^* = \dfrac{Q}{5} \\ l^* = \dfrac{Q}{10}\end{cases}$$

$$\Rightarrow LTC = l^* + k^* = 0.3Q \Rightarrow LAC = LMC = 0.3$$

$$\begin{cases}\min C = l + k \\ s.t.\ \ Q = \min\{5k, 10l\} \qquad \overline{k} = 1000\end{cases}$$

(2) $$\Rightarrow l = \frac{Q}{10} \Rightarrow STC = \frac{1}{10}Q + 1000$$

$$\therefore SMC = 0.1 \qquad SAC = \frac{1}{10} + \frac{1000}{Q}$$

26.已知生產函數為$Q=\min(4L,3K)$：(1) 若短期資本固定為$K=4$，且$P_K=4$，$P_L=1$，試求：AVC,SAC,SMC；(2) 若要素市場完全競爭，試求L,K的「條件要素需求函數」；(3) 試求「長期成本函數」；(4) 試求替代彈性；

解：

$$(1)\begin{cases}\min \quad C=L+16 \\ s.t. \quad Q=\min(4L,12)\end{cases} \Rightarrow 均衡時\ 4L=Q \Rightarrow L=\frac{Q}{4}$$

$$STC=\frac{Q}{4}+16 \qquad SAC=\frac{1}{4}+\frac{16}{Q} \qquad SMC=\frac{1}{4} \qquad TVC=\frac{Q}{4} \Rightarrow AVC=\frac{1}{4}$$

$$(2)\begin{cases}\min \quad C=L+4K \\ s.t. \quad Q=\min(4L,3K)\end{cases} \Rightarrow 均衡時\ Q=4L=3K \Rightarrow L=\frac{Q}{4}, K=\frac{Q}{3}$$

$$(3)\ LTC=P_L L+P_K K=\frac{Q}{4}+\frac{4}{3}Q=\frac{19}{12}Q$$

(4)生產函數完全互補$\Rightarrow \sigma=0$

題型：CES 生產函數計算

27.Suppose that a firm produces good Y with labor (X_1) and capital (X_2), whose price are $w_1=1$, and $w_2=1$. The technology of the firm is represented by a CES production function: $Y(X_1,X_2)=[X_1^\rho+X_2^\rho]^{1/\rho}$ (1) Assume that $\rho = 1/2$ and in the short-run the amount of capital is fixed at $X_2=100$. Find the short-run labor demand, and the short-run cost function.

(2) In a given market the firm a monopoly, facing the following constant elasticity demand for good Y: $Y(P)=100P^{-\theta}$ For what value of θ will the firm sell in this market?

(3)Assume the cost found in (1). If θ = 2, find the profit-maximizing price and quantity, and the monopoly profits. (4)承上題，假設$\rho=\frac{1}{2}$，請計算長期總成本函數與條件要素需求函數。

(5) 若生產函數仍為$Y=(X_1^\rho+X_2^\rho)^{1/\rho}$型式，請計算替代彈性與擴張線方程式。

(6)若生產函數仍為$Y=(X_1^\rho+X_2^\rho)^{1/\rho}$型式，此生產函數是齊序生產函數(Homothetic)嗎？

(7)若生產函數為$Y=(X_1^\rho+X_2^\rho)^{1/\rho}$型式，假設$w_1=w_2=1$，計算長期總成本函數與成本彈性。【政大國貿所】

解：

(1)短期總成本：

$$\boxed{\begin{array}{l}\text{Min STC}=w_1x_1+w_2x_2=x_1+x_2 \\ s.t \quad Y=(x_1^{\frac{1}{2}}+x_2^{\frac{1}{2}}),\ \bar{x}_2=100\end{array}}$$

短期$\bar{x}_2=100$代回生產函數，$Y=(x_1^{\frac{1}{2}}+100^{\frac{1}{2}})^2$

$x_1^{\frac{1}{2}}=\sqrt{Y}-10 \Rightarrow x_1=(\sqrt{Y}-10)^2$代回 STC 函數，

$STC=(\sqrt{Y}-10)^2+100$

(2) 市場需求曲線$Y^d=100P^{-\theta}$

$\ln Y^d=\ln 100-\theta\ln P$

$d\ln Y = -\theta \ln P \Rightarrow \varepsilon^d = \dfrac{-d\ln Y}{d\ln P} = \theta$

若獨占廠商採單一訂價法(MR = MC)，必在$|\varepsilon^d|>1$之處生產。$\therefore |\varepsilon^d| = \theta > 1$

(3) 市場需求曲線：$Y = 100P^{-2} \Rightarrow P^2 = \dfrac{100}{Y} \Rightarrow P = \dfrac{10}{\sqrt{Y}}$

獨占廠商短期最適決策：

$$\text{Max } \pi = \frac{10}{\sqrt{Y}}Y - [(\sqrt{Y}-10)^2 + 100]$$

$$F.O.C\ \frac{\partial \pi}{\partial Y} = 0 \Rightarrow 5Y^{\frac{-1}{2}} - 1 + 10Y^{\frac{-1}{2}} = 0 \Rightarrow Y^* = 225$$

$$P^* = \frac{10}{\sqrt{225}} = \frac{2}{3},\ \ \pi^* = \frac{2}{3}(225) - (15-10)^2 - 100 = 25$$

$S.O.C\ \dfrac{\partial^2 \pi}{\partial Y^2} = \dfrac{-5}{2}Y^{\frac{-3}{2}} - 5Y^{\frac{-3}{2}} < 0$（滿足利潤極大化充分條件）

(4) $\boxed{\begin{array}{l}\text{Min LTC} = X_1 + X_2 \\ s.t\ \ Y = (X_1^{\frac{1}{2}} + X_2^{\frac{1}{2}})^2\end{array}}$

$$L = X_1 + X_2 + \lambda[Y - (X_1^{\frac{1}{2}} + X_2^{\frac{1}{2}})^2]$$

利用生產者均衡條件 $MRTS = \dfrac{w_1}{w_2}$ 求解

$$\frac{2\left(X_1^{\frac{1}{2}} + X_2^{\frac{1}{2}}\right)\left(\frac{1}{2}X_1^{\frac{-1}{2}}\right)}{2\left(X_1^{\frac{1}{2}} + X_2^{\frac{1}{2}}\right)\left(\frac{1}{2}X_2^{\frac{-1}{2}}\right)} = 1 \Rightarrow X_1 = X_2 \text{ 代回生產函數}$$

$\therefore$ Conditional factor demand function 為 $X_1^* = \dfrac{Y}{4};\ X_2^* = \dfrac{Y}{4}\quad LTC = X_1 + X_2 = \dfrac{Y}{2}$

(5) $Y = (X_1^\rho + X_2^\rho)^{1/\rho}$

$$\text{MRTS} = \frac{MPP_{X_1}}{MPP_{X_2}} = \frac{\frac{1}{\rho}(X_1^\rho + X_2^\rho)^{\frac{1}{\rho}-1} \cdot \rho X_1^{\rho-1}}{\frac{1}{\rho}(X_1^\rho + X_2^\rho)^{\frac{1}{\rho}-1} \cdot \rho X_2^{\rho-1}} = \left(\frac{X_2}{X_1}\right)^{1-\rho}$$

$$\ln \text{MRTS} = (1-\rho)\ln\left(\frac{X_2}{X_1}\right)$$

$$d\ln \text{MRTS} = (1-\rho)d\ln\left(\frac{X_2}{X_1}\right) \qquad \therefore \text{替代彈性 } \sigma = \frac{d\ln\left(\frac{X_2}{X_1}\right)}{d\ln \text{MRTS}} = \frac{1}{1-\rho}$$

擴張線是滿足「$\dfrac{MPP_{X_1}}{MPP_{X_2}} = \dfrac{w_1}{w_2}$」均衡條件所形成軌跡：

$$\left(\frac{X_2}{X_1}\right)^{1-\rho} = \left(\frac{w_1}{w_2}\right) \Rightarrow \left(\frac{X_2}{X_1}\right) = \left(\frac{w_1}{w_2}\right)^{\frac{1}{1-\rho}} \text{......Expansion path}$$

(6)若生產函數可以化簡成 $MRTS = f\left(\frac{X_2}{X_1}\right)$，此為 Homothetic production function，

$MRTS = \frac{MPP_{X_1}}{MPP_{X_2}} = \left(\frac{X_2}{X_1}\right)^{1-\rho}$，若 $\rho \neq 0$ 時，無法將 MRTS 化簡成 $\left(\frac{X_2}{X_1}\right)$ 的比例函數，因此，

$Y = (X_1^{\rho} + X_2^{\rho})^{1/\rho}$ 並非為 Homothetic production function。

題型：拗折等產量曲線

28.A firm has a production function given by $f(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$. What is the condition demand function for factors x_1 and x_2 as a function of factor prices (w_1, w_2) and output y? What is the cost function for this technology?【96 台大農經所、朝陽企研財金】

解：生產者最適化模型 $\boxed{\begin{array}{l} \text{Min } TC = w_1x_1 + w_2x_2 \\ s.t \quad Y = \min(2x_1 + x_2, x_1 + 2x_2) \end{array}}$

(1) 若 $x_1 < x_2 \Rightarrow 2x_1 + x_2 < x_1 + 2x_2 \Rightarrow Y = 2x_1 + x_2 \Rightarrow MRTS = 2$

若 $MRTS = 2 < \frac{w_1}{w_2} \Rightarrow$ 全部使用 x_2 生產

條件要素需求函數 $\begin{cases} x_1^* = 0 \\ x_2^* = Y \end{cases}$ $TC = w_2Y$

(2) 若 $x_1 > x_2 \Rightarrow 2x_1 + x_2 > x_1 + 2x_2 \Rightarrow Y = x_1 + 2x_2 \Rightarrow MRTS = \frac{1}{2}$

若 $MRTS = \frac{1}{2} > \frac{w_1}{w_2} \Rightarrow$ 全部使用 x_1 生產：條件要素需求函數 $\begin{cases} x_1^* = Y \\ x_2^* = 0 \end{cases}$ $TC = w_1Y$

(3) 若 $x_1 = x_2 \Rightarrow 2x_1 + x_2 = x_1 + 2x_2 \Rightarrow Y = 3x_1$

若 $\frac{1}{2} < \frac{w_1}{w_2} < 2 \Rightarrow$ 條件要素需求函數 $\begin{cases} x_1^* = \frac{Y}{3} \\ x_2^* = \frac{Y}{3} \end{cases}$ $TC = w_1\left(\frac{Y}{3}\right) + w_2\left(\frac{Y}{3}\right) = \left(\frac{w_1 + w_2}{3}\right)Y$

(4) 成本函數：$\begin{cases} \frac{w_1}{w_2} > 2, x_1^* = 0, x_2^* = Y, TC = w_2Y \\ \frac{1}{2} < \frac{w_1}{w_2} < 2, x_1^* = \frac{Y}{3}, x_2^* = \frac{Y}{3}, TC = \left(\frac{w_1 + w_2}{3}\right)Y \\ \frac{w_1}{w_2} < \frac{1}{2}, x_1^* = Y, x_2^* = 0, TC = w_1Y \end{cases}$

29. 生產函數 $Q = \min(2L + 3K, 3L + 2K)$，請計算短期生產函數與長期成本函數。

解：

	生產函數 $Q = \min(2L + 3K, 3L + 2K)$
短期	$\begin{cases} 2L + 3K < 3L + 2K \Rightarrow L > \overline{K} \Rightarrow Q = TPP_L = 2L + 3\overline{K} \\ 2L + 3K > 3L + 2K \Rightarrow L < \overline{K} \Rightarrow Q = TPP_L = 3L + 2\overline{K} \end{cases}$

生產結構	$\begin{cases} L>\overline{K} \Rightarrow Q=TPP_L=2L+3\overline{K} \Rightarrow APP_L=2+\dfrac{3\overline{K}}{L} \quad MPP_L=\dfrac{\partial Q}{\partial L}=2 \\ L<\overline{K} \Rightarrow Q=TPP_L=3L+2\overline{K} \Rightarrow APP_L=3+\dfrac{2\overline{K}}{L} \quad MPP_L=\dfrac{\partial Q}{\partial L}=3 \end{cases}$
長期成本函數	由於題目為給定要素價格，因此成本極小要素組合有三種可能性，須視等成本線斜率值大小而定：

$MRTS=\dfrac{2}{3}<\dfrac{w}{r}<MRTS=\dfrac{3}{2}$	$\dfrac{w}{r}<MRTS=\dfrac{2}{3}$	$\dfrac{w}{r}>MRTS=\dfrac{3}{2}$
成本極小要素組合在「拗折點」 $\begin{Bmatrix} L=K \\ Q=\min(2L+3K,3L+2K)=5L \end{Bmatrix}$ 條件要素需求函數： $\begin{Bmatrix} L^*=\dfrac{Q}{5} \\ K^*=\dfrac{Q}{5} \end{Bmatrix}$	成本極小要素組合在「角隅解」 $\begin{Bmatrix} \text{全用}L\text{生產} \\ Q=\min(2L,3L)=2L \end{Bmatrix}$ 條件要素需求函數： $\begin{Bmatrix} L^*=\dfrac{Q}{2} \\ K^*=0 \end{Bmatrix}$	成本極小要素組合在「角隅解」 $\begin{Bmatrix} \text{全用}K\text{生產} \\ Q=\min(3K,2K)=2K \end{Bmatrix}$ 條件要素需求函數： $\begin{Bmatrix} L^*=0 \\ K^*=\dfrac{Q}{2} \end{Bmatrix}$
$C=w\cdot\dfrac{Q}{5}+r\cdot\dfrac{Q}{5}=\left(\dfrac{w}{5}+\dfrac{r}{5}\right)Q$	$C=w\cdot\dfrac{Q}{2}$	$C=r\cdot\dfrac{Q}{2}$
成本函數：$C=\min\left(\dfrac{w}{5}+\dfrac{r}{5},\dfrac{w}{2},\dfrac{r}{2}\right)Q$		

30.生產函數 $Q=\max(2L+3K,3L+2K)$，請計算短期生產函數與長期成本函數。

解：

	生產函數 $Q=\max(2L+3K,3L+2K)$
短期生產結構	$\begin{Bmatrix} 2L+3K<3L+2K \Rightarrow L>\overline{K} \Rightarrow Q=TPP_L=3L+2\overline{K} \\ 2L+3K>3L+2K \Rightarrow L<\overline{K} \Rightarrow Q=TPP_L=2L+3\overline{K} \end{Bmatrix}$ $\begin{cases} L>\overline{K} \Rightarrow Q=TPP_L=3L+2\overline{K} \Rightarrow APP_L=3+\dfrac{2\overline{K}}{L} \quad MPP_L=\dfrac{\partial Q}{\partial L}=3 \\ L<\overline{K} \Rightarrow Q=TPP_L=2L+3\overline{K} \Rightarrow APP_L=2+\dfrac{3\overline{K}}{L} \quad MPP_L=\dfrac{\partial Q}{\partial L}=2 \end{cases}$
長期	等產量曲線「凹向原點」，因此成本極小要素組合為「角隅解」：

成本極小要素組合在「角隅解」 $\begin{Bmatrix} \text{全用}L\text{生產} \\ Q=\max(2L,3L)=3L \end{Bmatrix}$	成本極小要素組合在「角隅解」 $\begin{Bmatrix} \text{全用}K\text{生產} \\ Q=\max(3K,2K)=3K \end{Bmatrix}$

成本函數	條件要素需求函數：$\begin{Bmatrix} L^*=\frac{Q}{3} \\ K^*=0 \end{Bmatrix}$	條件要素需求函數：$\begin{Bmatrix} L^*=0 \\ K^*=\frac{Q}{3} \end{Bmatrix}$
	$C=w\cdot\frac{Q}{3}$	$C=r\cdot\frac{Q}{3}$
	成本函數：$C=\min\left\{\frac{w}{3}+\frac{r}{3}\right\}\cdot Q$	

題型：成本極小化要素組合為角隅解

31.The production function for good y is $y=\max\{10X_1,4X_2\}$, where X_1 and X_2 are the amounts of factors 1 and 2, find the cost function for good y. 【96 台科大企研所】

解：$Min\ C=p_1x_1+p_2x_2$

由於生產函數並非凸向原點，因此均衡點必為角解(corner solution)：

情況 1：$\frac{p_1}{p_2}>\frac{10}{4}$	情況 2：$\frac{p_1}{p_2}<\frac{10}{4}$
條件要素需求函數：$x_1=0,x_2=0.25y$	條件要素需求函數：$x_1=0.1y,x_2=0$
$TC=0.25p_2y$	$TC=0.1p_1y$
結論：$TC=\min\{0.1p_1 0.25p_2\}y$	

32. Suppose the total cost functions for the two plants of a firm are $TC_a=38+5Q_a+Q_a^2$ and $TC_b=450+40Q_b+\frac{1}{2}Q_b^2$. If the total output is Q =15. How should the outputs be divided?【中正企研所】

解：由多工廠均衡條件：$\begin{cases} MC_a=MC_b\Rightarrow 5+2Q_a=40+Q_b \\ Q_a+Q_b=15 \end{cases}$

由上式可解得：$Q_a=\frac{50}{3},Q_b=-\frac{5}{3}<0,$不合

產量最適分配：$Q_a=Q=50,Q_b=0,$全由工廠a生產!

題型：規模報酬

34.Of the following production functions, which exhibit increasing, constant, or decreasing returns to scale?

(1) $F(L,K)=LK^2$ (2) $F(L,K)=5L+10\sqrt{K}$

(3) $F(L,K)=(LK)^{0.5}$ (4) $F(L,K)=\min\{2L,K\}$ 【96 海洋應經所】

解：

(1) $F(\lambda L,\lambda K)=(\lambda L)(\lambda K)^2=\lambda^3LK^2=\lambda^3\cdot F(L,K)\Rightarrow IRS$

(2) $F(\lambda L,\lambda K)=5(\lambda L)+10\sqrt{\lambda K}=\lambda^{0.5}(\lambda^{0.5}5L+10K^{0.5})<\lambda(5L+10\sqrt{K})\Rightarrow DRS$

(3) $F(\lambda L,\lambda K)=(\lambda L)^{0.5}(\lambda K)^{0.5}=\lambda(LK)^{0.5}=\lambda\cdot F(L,K)\Rightarrow CRS$

(4) $F(\lambda L,\lambda K)=\min\{2(\lambda L),(\lambda K)\}=\lambda\cdot\min\{2L,K\}=\lambda\cdot F(L,K)\Rightarrow CRS$

35.A firm has the production function $f(x_1,x_2)=(x_1^b+x_2^b)^c$, *where* $b>0$ *and* $c>0$

What is the condition for the firm to have constant returns to scale? 【95 淡江財金所】

並且計算成本彈性、符合規模經濟或是規模不經濟？

解： $f(\lambda x_1,\lambda x_2)=\left((\lambda x_1)^b+(\lambda x_2)^b\right)^c=\lambda^{bc}\cdot(x_1{}^b+x_2{}^b)^c \Rightarrow$ (bc)階齊次函數

成本彈性 = $\dfrac{1}{(bc)}$ **；生產力彈性 = (bc)；**

$(bc)=1\Rightarrow$ 生產力彈性=1 $\Leftrightarrow$ 成本彈性=1

$\Rightarrow$ 生產函數具有規模報酬固定不變,成本結構具有規模經濟固定不變

36.【是非題】Suppose the Red Cross Society of the ROC (中華民國紅十字會) is one of the suppliers to the perfectly competitive domestic blood market. Being a non-profit organization, its goal is to sell as much as possible without making profit. Consequently, the blood supply curve of the Red Cross is its AVC(average variable cost)curve. 【97 清大經研所】

解：錯誤。假設完全競爭廠商追求利潤極大化定價均衡條件為： $P=AR=MR=MC$

完全競爭廠商為價格接受者，利潤極大化產量在 MC 曲線，因此廠商短期供給曲線為 MC 曲線。然而中華民國紅十字會為非營利單位，在超額利潤等於零時，均衡條件為 $P=AC$ **，因此廠商供給曲線為 AC 曲線，並非 MC 曲線。**

37.【是非題】When a monopolist sells additional units, total revenues may rise, fall or remain unchanged. 【97 中央企研所】

解：錯誤。假設獨占廠商採取一般訂價法(MR=MC)，獨占廠商必在需求彈性大於一之處生產定價，廠商必須降價，銷售量才能增加，但是需求彈性大於一時，降價，總收入會增加。

38.【是非題】The production function f(L,K) exhibits constant returns to scale if and only if its corresponding cost function exhibits neither economies of scale nor diseconomies of scale. 【97 清大經研所】

解：若是生產函數呈現規模報酬固定不變，表示成本結構呈現「規模經濟固定不變」，此時總成本區線過原點直線，AC 與 MC 曲線為水平線。

39.【是非題】A firm's long run cost function is an increasing function of input prices. 【97 清大經研所】

解：正確。當要素價格上漲時，廠商長期總成本增加，長期平均成本亦增加，整條長期平均成本曲線上移，稱為「外部規模規模不經濟」，但是邊際成本曲線不一定上移，若為正常要素，當要素價格上漲時，MC 曲線上移；若為劣等要素，當要素價格上漲時，MC 曲線下移；若為中性要素，當要素價格上漲時，MC 曲線不受影響。

40.假定高大公司的生產函數為 $Q=LK$，其中 L 和 K 分別為勞動和資本使用量，Q 為最大產量，若勞動的價格固定為 w，資本的價格固定為 r。

(1)試以 Lagrangian 乘數法求取高大公司的長期成本函數 TC(Q)。

(2)繪出 TC–Q 關係圖，並討論該公司屬何種規模報酬？【98 高應大企研所】

解：(1) $\begin{cases}\min C = wL + rK \\ s.t.\ Q = LK\end{cases} \Rightarrow \text{Lagrange} = wL + rK + \lambda(Q - LK)$

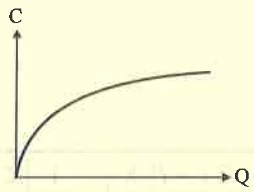
f.o.c $\begin{cases}\text{相切條件：} MRTS = \dfrac{w}{r} \Rightarrow K = \dfrac{w}{r}L \\ \text{限制條件：} Q = LK\end{cases}$

聯立求解上式可得「條件要素需求函數」: $L^* = \sqrt{Q\dfrac{r}{w}}$， $K^* = \sqrt{Q\dfrac{w}{r}}$

將上式代入目標函數，即為「成本函數」: $C^* = wL^* + rK^* = 2\sqrt{Q}\sqrt{wr}$

(2)令： $2\sqrt{wr} = A$， $A>0$， $C = A\sqrt{Q}$

$C(Q=0)=0$, $\dfrac{dC}{dQ} = \dfrac{1}{2}AQ^{-\frac{1}{2}} > 0$, $\dfrac{d^2C}{dQ^2} = -\dfrac{1}{4}AQ^{-\frac{3}{2}} < 0$



$f(tL, tK) = t^2LK > tQ = tLK \Rightarrow IRS$ ，規模報酬遞增

41.The cost function for an industry is

$C = bQ^a$, where C is the cost in thousands and Q is the output in hundreds.

(1)True or False. If $a>1$ the returns to scale in the industry is increasing. Note that you have to answer this question correctly to answer the rest of 4. So be careful. (5%)

(2)Estimate the average cost at m units. (5%)

(3)Estimate the marginal cost of the m^{th} unit. (5%)

(4)If the going price in the industry is p calculate the break even output. (5%)【98 清大工工、工管所】

解：(1) 錯誤。

$C = bQ^a \Rightarrow AC = bQ^{a-1} \Rightarrow \dfrac{dAC}{dQ} = b(a-1)Q^{a-2} > 0(\because a>1)$

⇒成本結構具有規模不經,因此生產函數呈現規模報酬遞減特色

(2) $Q = m \Rightarrow AC = bQ^{a-1} = bm^{a-1}$

(3) $Q = m \Rightarrow MC = abQ^{a-1} = abm^{a-1}$

(4)短期歇業點： $P = AVC(\min) \Rightarrow AVC = bQ^{a-1} \Rightarrow \dfrac{dAVC}{dQ} = (a-1)bQ^{a-2} = 0 \Rightarrow a = 1$

$\therefore P = AVC(\min) = bQ^{a-1} = bQ^0 = b$

因此短期歇業點： $\therefore P = AVC(\min) = b$ ，所對應產量等於零。

42.何謂「規模經濟(economies of scale)」？何謂「規模報酬(return to scale)」？請問兩者的關係為何？試證明之。(10%)【98 中山企管所】

解：在齊次生產函數下：規模報酬遞增與規模經濟具有若且唯若關係，規模報酬遞減與規模不經濟亦具有若且唯若關係。

成本彈性與生產力彈性關係：$\boxed{\varepsilon_P \times \varepsilon_C = 1}$

證明：$\varepsilon_P = \varepsilon_L + \varepsilon_K = \dfrac{MPP_L}{APP_L} + \dfrac{MPP_K}{APP_K} = MPP_L \times \dfrac{L}{Q} + MPP_K \times \dfrac{K}{Q}$

$$= \frac{MPP_L}{P_L} \times \frac{P_L \times L}{Q} + \frac{MPP_K}{P_K} \times \frac{P_K \times K}{Q}$$

❶利用生產者均衡條件，$\dfrac{MPP_L}{P_L} = \dfrac{MPP_K}{P_K}$，

❷ $MC = \dfrac{dC}{dQ} = \dfrac{P_L \cdot dL + P_K \cdot dK}{\dfrac{\partial Q}{\partial L} \cdot dL + \dfrac{\partial Q}{\partial K} \cdot dK} = \dfrac{P_L\left(dL + \dfrac{P_K}{P_L} \cdot dK\right)}{MPP_L \cdot dL + MPP_K \cdot dK} = \dfrac{P_L\left(dL + \dfrac{P_K}{P_L} \cdot dK\right)}{MPP_L\left(dL + \dfrac{MPP_K}{MPP_L} \cdot dK\right)} = \dfrac{P_L}{MPP_L}$

$\because MC = \dfrac{P_L}{MPP_L} = \dfrac{P_K}{MPP_K}$

$\therefore \dfrac{MPP_L}{P_L} = \dfrac{MPP_K}{P_K} = \dfrac{1}{MC}$

$\therefore \varepsilon_P = \dfrac{1}{MC}\dfrac{P_L \cdot L}{Q} + \dfrac{1}{MC}\dfrac{P_K \cdot K}{Q} = \dfrac{1}{MC}\left[\dfrac{P_L L + P_K K}{Q}\right] = \dfrac{1}{MC}\left[\dfrac{C}{Q}\right] = \dfrac{AC}{MC} = \dfrac{1}{\varepsilon_C}$

$\therefore \boxed{\varepsilon_P \times \varepsilon_C = 1}$

如果生產函數具有「規模報酬遞增」特色，表示「生產力彈性」> 1，生產力彈性與成本彈性具有倒數關係，表示「成本彈性」< 1，成本函數具有「規模經濟」特色。

43.An entrepreneur's short-run total cost function is $C = q^3 - 10q^2 + 17q + 66$.

(1)Determine the output level at which he maximize profit if p = 5

(2)Compute the output elasticity of cost at this output. 【97 交大財金所】

解：依題意 $STC = q^3 - 10q^2 + 17q + 66$

(1)當 $p = 5$ 下，廠商追求利潤極大化

$\text{Max } U = p \cdot q - STC = 5q - (q^3 - 10q^2 + 17q + 66)$

F.O.C $\dfrac{\partial \pi}{\partial q} = 0$ $5 - 3q^2 + 20q - 17 = 0$ $3q^2 - 20q + 12 = 0$ $(3q-2)(q-6) = 0$ $q = \dfrac{2}{3}$ 或 6

S.O.C $\dfrac{\partial^2 \pi}{\partial q^2} < 0$ $-6q + 20 < 0 \Rightarrow q > \dfrac{10}{3}$ 由 S.O.C 可知 $q^* = 6$ ($q = \dfrac{2}{3}$ 不合)

(2)令 The output elasticity of cost（成本的產出彈性）$= E_{Cq}$

$E_{Cq} = \dfrac{dC/C}{dq/q} = \dfrac{dC}{dq}\dfrac{q}{c} = (3q^2 - 20q + 17) \cdot \dfrac{q}{q^3 - 10q^2 - 17q + 55}$

將 $q^* = 6$ 代入 E_{Cq} 可知 $E_{Cq} = 5 \times \dfrac{6}{24} = 1.25$

題型：機會成本

1.呆呆今年考研究所，在就業與學業面臨抉擇。若唸研究所，每學期需繳學費 25,000 元，且每學期的書籍費用為 4,000 元，一年的生活費（食宿）為 120,000 元。若他選擇就業，可以去銀行當櫃員，月薪為 30,000 元。試問呆呆唸研究所一年的機會成本為何？若一年的生活費（食宿）漲為 200,000 元，則機會成本為何？【台大商研所】

解：機會成本：把資源投入某一個特定用途後，放棄其它選擇中最大利益者，即為機會成本。

(1)唸研究所機會成本 ＝ 外顯成本 ＋ 隱藏成本

$=(25{,}000\times2+4{,}000\times2+120{,}000)+(30{,}000\times12)=538{,}000$

(2)唸研究所機會本 $=(25{,}000\times2+4{,}000\times2+200{,}000)+(30{,}000\times12)=618{,}000$

∴生活費用提高，會使得唸研究所機會成本提高。

2.過去你一直為一家軟體開發公司工作，年薪為$35,000。如今決定自己開公司，計劃成為第二個比爾蓋茲。於是辭去工作，並以自己的銀行存款$10,000（利息 5%），購買最新的電腦軟體，用於發展業務。同時，又將住屋地下室改為新軟體公司的辦公室，該地下室租金每月為$250。此外，租用一些辦公設備，每年租金為$3,600，並雇用兩個兼職程式設計師，兩人年薪合計為$25,000，同時，年個月的暖氣與照明費用為$50。 (1) 新公司每年明示成本總額為何？ (2) 每年隱藏成本總額為何？ (3) 第一年底時，會計人員告知總銷售額為$55,000，並恭喜你賺得了利潤，試問她的道賀是否有理？理由？【95 銘傳管研所】

解：(1)外顯成本＝ $3{,}600+25{,}000+50\times12+10000=39{,}200$

(2)隱藏成本 ＝ $35{,}000+10{,}000\times0.05+250\times12=38{,}500$

(3)經濟利潤 ＝ $55{,}000-$外顯成本$-$隱藏成本＝ $-22{,}700$ 經濟利潤為負，會計人員並未考慮隱藏成本，其道賀無意義；長期下，若繼續虧損，決策者將會退出市場。

3.孫燕姿的演唱會票價為每張 400 元，你認為聽她的演唱值 500 元，所以你準備要去買票。在你買票之前，你因為參加電臺抽獎，得到一張周杰倫演唱會的免費入場券，這張入場券不能轉賣給任何人。不幸的是，周杰倫和孫燕姿的演唱會排在同一時間，所以你只能選一場聽。如果你選擇去聽周杰倫的演唱會，你的機會成本是多少？（假設除了門票外，聽演唱會沒有其它的花費。）【中興財金所】

解：500 元，即不聽孫燕姿演唱會而犧牲之效用

4.假設史奎謙成立的 True Yoga 的台灣分公司於第一年計有會員 5,000 人加入，史奎謙原來的律師年薪有 800 萬，經營 True Yoga 後兼任律師收入降為 400 萬，其他資料如下：

自有資金 4,000 萬（假設從事其他投資年報酬率為 5%）

貸款資金 4,000 萬（年利率 2.5%）

租金支出每月 200 萬

年初設備價格：3,600 萬

年末設備價格：3,200 萬

管銷費用 4,900 萬

會員一個月的上課費用：3000 元/一人

請根據上述資料分析此公司在這一年內的：

(1)會計成本；(2)會計利潤；(3)經濟折舊；(4)隱藏成本；(5)經濟利潤。【96 政大經濟】

解：(1) 會計成本（外顯成本）$=4000+4000\times 2.5\%+200\times 12+4900+(3600-3200)$
$=13{,}000$(萬)

(2) 會計利潤 = 總收入 − 會計成本 $=(3000\times 5000\times 12)-130{,}000{,}000$
$=5{,}000$(萬)

(3) 經濟折舊 $=3600-3200=400$萬

(4) 隱藏成本 $=400+4000(5\%)=600$萬

(5) 經濟利潤 = 會計利潤 − 隱藏成本 $=5000-600=4400$(萬)

5.【是非題】單身漢老王原本是計程車司機，一個月的淨收入約$40,000。老王決定改行開牛肉麵店，開張後每個月扣除所有的有形花費及成本後，淨賺$45,000。老王的牛肉麵店的經濟利潤為$45,000。

解：(×)：開牛肉麵店的經濟利潤 = 總收入 − 機會成本 $=45000-$ 無形成本 $=5000$。

6.老王在學校當老師，一個月的薪水為 50,000 元。上個月老王辭職，開了一家麵店賣麵，一個月下來麵店的總收入為 60,000 元，總支出為 33,000 元。則老王在這個月 (A)會計利潤為 10,000 元，經濟利潤為 60,000 元 (B)會計利潤為 27,000 元，經濟利潤為 27,000 元 (C)會計利潤為 27,000 元，經濟利潤為損失 23,000 元 (D)會計利潤為 10,000 元，經濟利潤為損失 17,000 元。

解：(C)

會計利潤 = 收入 − 有形成本 $=60{,}000-33{,}000=27{,}000$

經濟利潤 = 收入 − 有形成本 − 無形成本 $=60{,}000-33{,}000-50{,}000=-23{,}000$。

7.假設符號 A 為經濟利潤，符號 B 為正常利潤，符號 C 為會計利潤，那麼 ABC 三者之間的關係為 (A) $A=B+C$ (B) $A=B-C$ (C) $A=C-B$ (D) $A=B/C$ (E) $A=C/B$ 【95 北大企研所】

解：(C)

(1)經濟利潤 = 收入 − 外顯成本 − 內含成本

(2)正常利潤是指「經濟利潤為零」，正常利潤 = 內含成本

∴經濟利潤 = 會計利潤 − 內含成本 = 會計利潤 − 正常利潤。

8.若一家廠商總收益為$80000，勞動成本為$40000，原料成本為$20000，而該經營者到別處工作可賺到$1500，則設廠商的經濟利潤等於______，會計利潤等於______

(A)40000；5000 (B)5000；40000 (C)40000；15000 (D)20000；5000
(E)5000；20000。

解：(E)；會計利潤＝80,000−40,000−20,000＝20,000

經濟利潤＝會計利潤−無形成本＝20,000−15,000＝5,000。

9.小強想買一台價值 80000 元的發電機來補充電力，此時的儲蓄利率為 6%，投資股票的報酬率為 10%，假如小強為了增強實力而買了發電機，請問小強買發電機的機會成本為
(A)4800 元 (B)12800 元 (C)88000 元 (D)84800 元。

解：(C)；由於投資股票的報酬率 10%，大於儲蓄利率 6%，故買發電機的機會成本為
80000+80000×10%＝88000。

10.年假結束前，小華持有的消費券僅剩$500。他心中盤算著要把這僅有的$500 消費券用於購買一個新的隨身碟，或者邀請心愛的女友一同到爭＃迴轉壽司店共進晚餐。在 3C 的賣場中，有一個他鍾情的隨身碟原價$700，現在商家推出的消費券促銷專案中，則該商品只需支付$500，但該方案僅適用於使用消費券，是故不能以現金交易（現金交易必須支付$700）。若他選擇與女友到爭＃迴轉壽司用餐，該店目前同樣推出消費券的優惠活動，用餐原價$600 現在僅需支付$500，而該方案同樣只適用於消費券，即現金交易必須支付$600。請問：
(1)若小華決定使用消費券購買新的隨身碟，則該選擇的機會成本是多少？
(2)若以上兩家廠商的促銷活動同時適用於消費券與現金，即使用消費券與現金的價格是相同的，則上述問題(1)的機會成本又是多少？(10 分)【98 輔大管研所】

解： (1) 機會成本是指所有替代方案中價值最高的，因此機會成本 600 元。
(2) 同為 600 元，使手現金也可適用於促銷方案。

11.綜合您學習經濟學的心得，下列敘述何者為錯誤？ (A)經濟學的世界是面臨「取捨」的世界。 (B)經濟學強調「資源稀少性」，故須由「價格機能」達到有效率的資源配置。 (C)每一個「經濟體」必須在面對不同「誘因」時，做出「理性」的選擇。(D)選擇既有取捨，必然存在機會成本，所以「天下沒有白吃的午餐」。 (E)前述皆涉及一些社會、心理、與經濟體的體制等議題，故許多經濟理論係自由創造與思考而建立，不能稱為基本科學。(2 分)【99 台北公行所】

解：(E)

12.What impact will an increase in the marginal income tax rate from 50 percent to 70 percent have on the private opportunity cost (measured in terms of after-tax income foregone each year) of a $300,000 automobile purchased for business, assuming the interest rate is 10 percent? The after-tax income foregone per year will (A)increase from $15,000 to $30,000. (B) increase from $15,000 to $21,000. (C)decrease from $21,000 to $15,000. (D) decrease from $15,000 to $9,000. (3 分)【98 中正國經】

解：(D)；題意：企業以 300,000 買汽車，在原來所得稅率 50%時，稅後所得減少的機會成本為$300,000\times0.1\times(1-0.5)=15,000$

當所得稅率 70%時，稅後所得減少的機會成本為$300,000\times0.1\times(1-0.7)=9000$。

13.Sheila's Sports Shop is a very popular sporting goods store, which has a yearly revenue of $60,000. Sheila runs the business herself. Her alternative employment options are to be a college swimming coach for $50,000 per year or a construction worker for $40,000 per year. Sheila spends $230,000 purchasing goods for resale to her customer. She also has four employees, who each earn $25,000 per year. Sheila owns the building that her Sports Shop is housed in – she remodeled a house that she owns and that she could have rented out for $20,000 per year instead. Sheila's implicit costs equal (A)$70,000 per year. (B)$90,000 per year.(C)$110,000 per year.(D)$330,000 per year.【95 中山企研所】

解：(A)；隱藏成本包括放棄當游泳教練的 50,000 與放棄租房子的 20,000，總共 70,000。

14.過年後身材走樣，為了瘦身你想去上一堂「瘦身瑜珈」，但需要繳交 300 元的學費。同樣的時間若你不去上課，可以花 50 元搭捷運去內湖當「家教」賺到 400 元；身為影癡的你，也可以選擇一個人走路到東南亞戲院，看可能在韓國創下華語片賣座紀錄的「赤壁：決戰天下」，在付出 300 元電影票價之下獲得 600 元價值的「消費者剩餘」；更可以和新認識、和林志玲相似度 95%的女友，一起到台大文學院前的杜鵑花叢中共度浪漫情人節，獲得價值 500 元的滿足。請問你上這堂瑜珈課的「機會成本」是：(A)300 元(B)350 元(C)600 元(D)650 元(E)900 元。(3 分)【98 台大國發所】

解：(E)；上瑜珈課而放棄其他選擇中最高價值為 600，表示隱藏成本 600 元，而上瑜珈課學費 300 元為會計成本，因此機會成本$=600+300=900$。

15.在經濟學中的經濟成本也被稱為「機會成本」：(A)是指在做一項選擇時，以所放棄的其他選項中的最差選項來衡量的成本概念 (B)也就是一般在記帳時的實際支出 (C)上大學的經濟成本主要包括四年的學費以及所投注的時間成本 (D)以上皆是

解：(C)

(A)是指在做一項選擇時，以所放棄的其他選項中最高價值的選項來衡量的成本概念

(B)經濟成本 ＝ 記帳成本 ＋ 隱藏成本

16.大明每星期日擺五小時地攤，可以淨賺 3750 元。這天美華來訪，大明陪她逛街看電影，把擺地攤的五小時用掉，還花了計程車費 450 元、電影票 600 元，另外則吃了美華買的零食 350 元。請問對大明而言，陪女友出遊的機會成本是多少？【交大管科所】

解：陪女友出遊的機會成本$=450+600+3750=4800$。

17.如果你唸國發所，兩年的總支出 30 萬有全額獎學金。若你將到台大國發所讀兩年碩士的

時間挪做它用，你可以(1)當家教並獲得 80 萬元的淨收益；或(2)環遊世界名勝古蹟，雖然會花掉 40 萬元，且你覺得效用相當 140 萬元；或(3)談戀愛，雖然沒有收入，但完成你多年的希望，值得 200 萬元。請問你唸國發所碩士班的個人「機會成本」是： (A) 0 萬元 (B) 30 萬元 (C) 80 萬元 (D) 100 萬元 (E) 200 萬元 【94 台大國發所】

解：(E)

18.政府為獎勵投資，凡將個人所得或儲蓄轉作商業支出得予免稅，但利息所得則需課稅，今利率為 10%，有一人擬購$300,000 之汽車作為公司用車，若其所得稅由 50%增為 70%，則其購車之機會成本(opportunity costs)之變化為：(A) 由$15,000 增為$30,000 (B) 由$15,000 增為$21,000 (C) 由$21,000 減為$15,000 (D) 由$15,000 減為$9,000 (E) 以上皆非 【中正企研所】

解：(D)；題意：企業以 300,000 買汽車，在原來所得稅率 50%時，稅後所得減少的機會成本為 $300{,}000 \times 0.1 \times (1-0.5) = 15{,}000$

當所得稅率 70%時，稅後所得減少的機會成本為 $300{,}000 \times 0.1 \times (1-0.7) = 9000$。

19.王老師在台大與師大兼課，為了趕時間，所以王老師常坐計程車奔波兩校。台北市計程車很多，出兩校的校門都很容易招到車。王老師常跟科車的運將開玩笑：捧看他們的客人絡繹不絕，他們的錢真好賺。可是運將們卻多堅稱混口飯吃，沒賺沒賺。王老師覺得很奇怪，明明生意很好，卻又說沒賺。

(1)你能從經濟學的觀點為王老師解惑嗎？

(2)運將又說他剛花了 60 萬買新車，賺的錢還不夠繳汽車貸款，現在仍處於虧損中，請問在什麼情況下這位運將會繼續開計程車？而在什麼情況下，他會改行？【中原企研所】

解：經濟利潤＝收益－經濟成本

從經濟學的觀點，假如計程車每天營業收益為 1000 元，扣除汽油、汽車貸款、折舊等外顯成本（假設為 400 元）若計程車司機到公司上班的最高日薪為 600 元，這 600 元會計利潤對司機而言，並非其所賺，此時沒有經濟利潤，只有正常利潤而已。因此運將堅稱「混口飯吃，沒賺沒賺」是指他只賺到正常利潤（工錢），而沒有賺到超額利潤。

20.假設廠商面對的生產函數為 Q＝f(L,K)，在 P＝9 的價格下販賣，共賣了 10 單位的產品(Q＝10)。花在 L 與 K 上的成本分別為 25 與 40。這家廠商的老闆如果不做老闆，在別人的公司，最多可以賺到 20 報酬。請回答下列問題：

(1).廠商的經濟利潤是多少？ (2).正常利潤是多少？ (3).會計利潤是多少？

(4).該廠商的平均成本是多少？ 【淡江企研所】

(5).如果 K 在短期無法隨產量的變動而調整，則該廠商的平均變動成本？

解：

(1) $\pi = TR -$ 會計成本 $-$ 機會成本 $= 9\times10-(25+40)-20=5$

(2)當經濟利潤為零時，廠商所賺取的會計利潤恰足夠支付機會成本，此時稱廠商賺取「正常利潤」，其機會成本為 20

(3) $\pi_{會} = TR -$ 會計成本 $=90-25-40=25$

(4) 機會成本 $=25+40+20=85,\ AC=\frac{85}{10}=8.5$

(5) $STC = TVC + TFC, TVC = 25, AVC = 2.5$

21.College-age athletes who drop out of school to play professional sports (A)are making a bad economic decision, since they can't play forever. (B)are unaware of their opportunity cost of attending college. (C)underestimate the value of a college education. (D)are well aware that their opportunity cost of attending college is very high.【中山企管】

解: (D)；學生之所以會放棄學業而參與職業運動，乃是因為上學的機會成本太大，所以才會休學而去參加職業運到。

22.Assume that you have a piece of land which you could either rent for $8,000 per year, or sell of $75,000 and invest the proceeds of the sale in long-term bond yielding15% annually. The opportunity cost per year of retaining the land and not renting is (A) $3,250. (B) $8,000. (C) $11,250. (D) $75,000. 【成大工管所】

解: (C)機會成本是指做出選擇時所必須放棄的事項中最高的代價。

23.Suppose HUA-MAO earn $20,000 from a concert. If they choose to use the $20,000 to go to Kyoto, their opportunity cost of going to Kyoto is
(A)nothing, because opportunity cost is different from sunk cost.
(B)$20,000 (because HUA-MAO could have used the $20,000 to buy other things).
(C)$20,000 (because HUA-MAO could have used the $20,000 to buy other things) plus the value of the time spent in Kyoto.
(D)$20,000 (because HUA-MAO could have used the $20,000 to buy other things) plus the value of the time spent in Kyoto, plus the cost of the food and drink HUA-MAO consumed in Kyoto. (E)None of the above.【台大國企所】

解: (C)；機會成本 =外顯成本+隱藏成本 =旅行的費用+時間= 20,000 +時間。

24.John wins a ticket from a radio station to see a symphony orchestra perform at an outdoor concert. Mike has paid N.T.$490 for a ticket to the same concert. On the evening of the concert there is a tremendous thunderstorm. If John and Mike have the same tastes, which of them will be more likely to attend the concert, assuming that each decides whether to attend the concert on the basis of a standard cost-benefit comparison? 【政大風管所】

解：Mike 買一張音樂會門票花 490 元，表示音樂會門票市價值 490 元。而 John 雖然是從電台贏取一張門票，但機會成本亦是 490 元。假設兩人具有相同偏好，且觀賞音樂會的時間具有相同機會成本下，由於獲取音樂會門票的成本相同，因此兩人會有相同的選擇，即兩人是否參與音樂會會有相同的決策。

25.Suppose a country's workers can produce 4 watches per day or 12 rings per day. If there is no trade, (A)the opportunity cost of l watch is 3 rings. (B)the opportunity cost of l watch is 1/3 of a ring. (C)The opportunity cost of l watch is 4 rings. (D)The opportunity cost of l watch is 1/4 of a ring. (E)The opportunity of l watch is 12 rings. 【台大國企所】

解: (A)； Watch 的機會成本 = 12/4 = 3

即每增加一單位的 watch 就必須放棄 3 單位的 ring

26.The opportunity cost of holding money is the (A)nominal interest rate. (B)real interest rate. (C)rate of inflation. (D)prevailing Treasury bill rate.【中央人管所】

解: (A)，持有貨幣的機會成本把錢拿去投資的報酬率，也就是名目利率。

27.During the next hour John can play basketball, watch television, or read a book. The opportunity cost of reading a book (A)is how much the book cost when it was purchased.

(B)is the value of playing basketball if John prefers that to watching television. (C)is the value of playing basketball and the value of watching television. (D)equals how much John enjoys the book. 【94 成大國企所】

解: (B)；John 讀書的機會成本就是所放棄的玩樂當中，價值最高的；如果 John 喜歡打棒球勝於打籃球，則機會成本即是打棒球的價值。

28.A return on investment just sufficient to keep the owners of the business content to stay in that business is called (A) marginal product. (B) fair price. (C) total cost. (D) rate of return. (E) normal profit. 【交大科管所】

解: (E)，經濟利潤等於零時，表示無超額利潤，即為正常利潤。

29.Sunk costs are (A)costs that do not change when output changes. (B)the costs of starting the business. (C)no recoverable costs. (D)variable costs. (E)opportunity costs. 【中山企管、人管、資管】

解: (C)

30.The difference between fixed and sunk costs is that (A)fixed costs affect marginal costs while sunk costs do not. (B)fixed costs do not involve a cash outlay while sunk costs do. (C)some of the fixed costs might be recoverable while sunk costs cannot be recovered. (D)sunk costs do not involve a cash outlay while fixed costs do.【清大科管所】

解: (C)

31.Costs which are invariant across all alternatives are called (A) explicit costs. (B) incremental costs. (C) implicit costs. (D) sunk costs.【94 彰師企研所】

解: (D)，沉沒成本指一開始就投入且無法回收的成本。

32.Explain why sunk costs should not be included in a capital budgeting analysis, but opportunity costs and externalities should be included.【中山企研所】

解：廠商決定是否進入一產業與沉沒成本有關，若沉沒成本極高，換言之，廠商進出市場的成本極高時，將形成進入障礙；然而，一旦進入市場後，未來的經營決策則與沉沒成本無關。

33.Under what conditions does a general Cobb-Douglas production function, $q = AL^a K^b$, exhibit decreasing, constant, or increasing returns to scale? (10 分)【98 交大財金所】【98 東華經研所】

解：$\begin{cases} a+b<1 \Rightarrow DRS \\ a+B=1 \Rightarrow CRS \\ a+B>1 \Rightarrow IRS \end{cases}$

34.Under what conditions do the following production functions exhibit decreasing, constant, or increasing returns to scale? (10 分)【98 中山經研所】

(1) $Q = L + K$ (2) $Q = L^{\alpha} K^{\beta}$ (3) $Q = L^{\alpha} K^{\beta} + L + K$

解：

(1) $f(\lambda L, \lambda K) = \lambda(L+K) = \lambda \cdot f(L,K) \Rightarrow$ Constant Return to Scale

(2) $f(\lambda L, \lambda K) = \lambda^{\alpha+\beta} \cdot L^{\alpha} K^{\beta} = \lambda^{\alpha+\beta} \cdot f(L,K)$

若 $\alpha+\beta>1$ 呈現 IRS；$\alpha+\beta=1$ 呈現 CRS；$\alpha+\beta<1$ 呈現 DRS

(3) $f(\lambda L, \lambda K) = \lambda^{\alpha+\beta} \cdot L^{\alpha} K^{\beta} + \lambda(L+K)$

若 $\alpha+\beta>1$ 呈現 IRS；$\alpha+\beta=1$ 呈現 CRS；$\alpha+\beta<1$ 呈現 DRS

35.Use the information in the table to answer to questions below.

Labor (workers per day)	Total product (units per day)	Marginal Product	Average Product
0	0	0	0
1	2	2	2
2	(7)	(8)	4
3	12	(9)	(10)
4	15	(11)	

In the above table, the space (7) is ______ ; the space (8) is ______ ; the space (9) is ______ ; the space (10) is ______ ; the space (11) is ______ .(5 分)【98 中正國經所】

解：(7)8 (8)6 (9)4 (10)4 (11)3

36.

Techniques for making 100 automobiles		
Method	Labor (units)	Capital (units)
I	200	12
II	80	20
III	10	70
IV	85	30

The table above shows three production methods to produce 100 automobiles per day.

(1)Which method is not technologically efficient? Why? (5 分)

(2)Which method is economically efficient if the cost for capital is \$20 per unit and for labor is \$15 per unit? Show your work. (5 分)【97 高雄第一科技運籌管理】

解：

Techniques for making 100 automobiles					
Method	Labor (units)	Capital (units)	技術效率 technologically efficient	生產成本	經濟效率 economically efficient
I	200	12	符合	3240	不符合
II	80	20	符合	1600	不符合
III	10	70	符合	1550	符合
IV	85	30	不符合		

37.The above (incomplete) table provides information about the relationship between labor and various product measures.

Labor (units)	Total product (units)	marginal product	Average product
0	0		
1		3	
2			5
3	14		
4		2	
5	18		
6		1	

(1)What is the level of labor that maximizes the marginal product of labor? (5 分)

(2)What is the level of average product for the fourth unit of labor? (5 分)【97 高雄第一科技運籌管理】

解：

Labor (units)	Total product (units)	marginal product	Average product
0	0	----	------
1	3	3	3
2	10	$\boxed{7}$	5
3	14	4	14/3=4.67
4	16	2	$\boxed{4}$
5	18	2	3.6
6	19	1	19/6=3.17

(1)勞動邊際產量最大：L=2

(2)當 L=4 時，對應的平均產量為 4。

38.下列敘述何者為正確？(A)CES 生產函數的替代彈性恆為 1　(B)CES 生產函數的替代彈性恆為－1　(C)Cobb-Douglas 生產函數的替代彈性恆為－1　(D)生產者之成本函數彈性與生產函數彈性皆恆為 1　(E)生產者之成本函數彈性與生產函數彈性恆為倒數。【95 台北公行所】

解：(E)

39.當某一廠商的生產函數為$Q=9[0.5L^{0.5}+0.5K^{0.5}]^2$，則當廠商面對的因素價格比率$\left(\frac{P_L}{P_K}\right)$上漲1%時，其最適因素雇用比率$\left(\frac{K^*}{L^*}\right)$應變動的百分比為＿＿＿？(2 分)【95 台北公行所】

解：$MRTS_{LK}=\frac{MP_L}{MP_K}=\frac{0.25L^{-0.5}}{0.25K^{-0.5}}=\left(\frac{K}{L}\right)^{\frac{1}{2}}\Rightarrow \ln MRTS_{LK}=\frac{1}{2}\ln\left(\frac{K}{L}\right)$

$\therefore$替代彈性$\sigma=\frac{d\ln(K/L)}{d\ln MRTS_{LK}}=2$

40.Sue can do her accounting assignment by using: a personal computer; a pocket calculator, a pocket calculator and a pencil and paper; or a pencil and paper. With a PC, Sue completes the job in half an hour; with a pocket calculator, it takes 4 hour; with a pocket calculator and with a pencil and paper, it takes 5 hours ; and with a pencil and paper, it takes 14 hours. The PC and its software cost $2,000, the pocket calculator costs $15, and the pencil and paper cost $3.

a. Which, if any, of the methods is technologically efficient?

b. Which methods is economically efficient if Sue's wage rate is

(1)$10 an hour? (2) $20 an hour? (3) $50 an hour? (14%)【96 政大國貿所】

解：

方法	PC	calculator	P&P	Hour	$W=100$	$W=20$	$W=50$
A	1	0	0	0.5	2005	2010	2025
B	0	1	0	4	$\boxed{55}$	$\boxed{95}$	$\boxed{215}$
C	0	1	1	5			
D	0	0	1	14	143	283	703

(1)A、B、D 方法皆具有「技術效率」。

(2)不論每小時工資率為 10 元、20 元、30 元，方法 B 皆具有「經濟效率」。

41.

Production Information for Scully's Splendid spacecrafts		
Technique to produce 50 space crafts	Units of capital (thousands)	Hours of labor (thousands)
W	4	28
X	4	16
Y	8	4
Z	10	1

1. In the above table, if the price of labor is $10 per hour and the price of capital is $20 per unit, which method of producing 50 space crafts is economically efficient?
(A)Technique W (B)Technique X (C)Technique Y (D)Technique Z。【95 中山企研所】

解：(C)

技術	K	L	Cost	經濟效率
W	4	28	360	不符合
X	4	16	240	不符合

Y	8	4	200	符合
Z	10	1	210	不符合

42.假設某一廠商的生產函數為 $Q=f(L,K)=8L^{0.5}K^{0.5}$，且 $P_L=4$，$P_K=2$。請回答下列問題

(1)求 STC（短期總成本）(5 分)

(2)若 $K=10$，求 AVC（平均成本）與 SMC（短期邊際成本）(10 分)

(3)求 LTC（長期總成本）(10 分)【99 靜宜企研所】

解：

(1) 短期，$K=\bar{K}$ 代入生產函數，化簡成 $q=8L^{0.5}\bar{K}^{0.5}$

$\therefore L^*=\frac{q^2}{64\bar{K}}$，$STC=wL+r\bar{K}=4\left(\frac{q^2}{64\bar{K}}\right)+2\bar{K}=\frac{q^2}{16K}+2\bar{K}$

(2) $STC=\frac{q^2}{160}+20$；$AVC=\frac{TVC}{q}=\frac{q}{160}$；$SMC=\frac{dTVC}{dq}=\frac{q}{80}$

(3) $\boxed{\begin{array}{l} Min\ \ c=4L+2K \\ s.t\quad Q=8L^{0.5}K^{0.5}\end{array}}$ 生產者均衡條件：$\frac{MPP_L}{MPP_K}=\frac{P_L}{P_K}$

$\frac{K}{L}=\frac{4}{2}\Rightarrow K=2L$ 代入生產函數

$Q=8L^{0.5}(2L)^{0.5}\Rightarrow L^*=\frac{Q}{8\sqrt{2}}=\frac{\sqrt{2}Q}{16}$；

$C=4\left(\frac{\sqrt{2}Q}{16}\right)+2\left(\frac{\sqrt{2}Q}{8}\right)=\frac{\sqrt{2}Q}{2}$

(1) STC ⇒ $\bar{K}$

⇒ $\frac{Q^2}{64\bar{K}}=L$ 代入

$4L+2\bar{K}=C$

$\therefore STC(Q)=4\times\frac{Q^2}{64\bar{K}}+2\bar{K}=C$

$=\frac{Q^2}{16\bar{K}}+2\bar{K}$

43.Given the production function

(1)Assume $\alpha,\beta>0$. What restricti

(2)Input derived demand function

解：

(1) 生產函數為$(\alpha+\beta)$階齊次函數

(2) $\begin{cases} Min\ \ C=w_1x_1+w_2x_2 \\ s.t\quad Y=x_1^{\alpha}x_2^{\beta}\end{cases}$

$L=w_1x_1+w_2x_2+\lambda(Y-x_1^{\alpha}x_2^{\beta})$

F.O.C $\frac{\partial L}{\partial x_1}=0\Rightarrow w_1+\lambda(-\alpha x_1^{\alpha-1}x_2^{\beta})=$

$\frac{\partial L}{\partial x_2}=0\Rightarrow w_2+\lambda(-\beta x_1^{\alpha}x_2^{\beta-1})=$

$\frac{\partial L}{\partial \lambda}=0\Rightarrow Y-x_1^{\alpha}x_2^{\beta}=0$ ....

一：$\frac{w_1}{w_2}=\frac{\alpha x_2}{\beta x_1}\Rightarrow x_2=\frac{\beta w_1}{\alpha w_2}x_1$ 代回生產

$Y=x_1^{\alpha}x_2^{\beta}$

$\begin{cases} Min\ \ C=w_1x_1+w_2x_2 \\ s.t\quad Y=x_1^{\alpha}x_2^{\beta}\end{cases}$

$\mathcal{L}=w_1x_1+w_2x_1+\lambda(Y-x_1^{\alpha}x_2^{\beta})$

得 $\frac{\alpha x_2}{\beta x_1}=\frac{w_1}{w_2}$ $\therefore x_2=\left(\frac{\beta w_1}{\alpha w_2}\right)x_1$

$\Rightarrow Y=x_1^{\alpha}\cdot\left(\frac{\beta w_1}{\alpha w_2}\right)^{\beta}x_1^{\beta}$

$Y=\left(\frac{\beta w_1}{\alpha w_2}\right)x_1^{\alpha+\beta}$

$\therefore x_1=\sqrt[(\alpha+\beta)]{\left(\frac{\alpha w_2}{\beta w_1}\right)^{\beta}Y}$ #

整理後可得

可解出「條件要素需求函數」： $x_1^* = \left(\dfrac{\alpha w_2}{\beta w_1}\right)^{\frac{\beta}{\alpha+\beta}} Y^{\frac{1}{\alpha+\beta}}$

44.The next table shows the total product schedule of a bookstore.

Workers per day	Total Product (books sold per day)
0	0
1	10
2	24
3	40
4	58
5	73
6	83
7	87
8	80
9	90
10	90

(1)If the owner wants to have the highest average product of labor, how many workers should he hire?

(2)The wage for the each worker is \$6 per day. If labor cost is the only variable cost, then how many workers should the bookstore hire to have the minimum average variable cost (AVC)?

(3)If the bookstore's fixed cost is a rent of \$20 per day, then how many workers should the bookstore hire to have the minimum average total cost (ATC)?【95 中興財金所】

解：將生產函數表擴展列示於下。

L	TP	AP	$TVC = P_L \cdot L$	AVC	TFC	TC	ATC
0	0	0	0	0	20	20	∞
1	10	10	6	0.6	20	26	2.6
2	24	12	12	0.5	20	32	1.33
3	40	13.3	18	0.45	20	38	0.95
4	58	14.5	24	0.414	20	44	0.76
5	73	14.6	30	0.411	20	50	0.68
6	83	13.8	36	0.43	20	56	0.67
7	87	12.4	42	0.48	20	62	0.71
8	89	11.1	48	0.54	20	68	0.76
9	90	10	54	0.6	20	74	0.82
10	90	9	60	0.67	20	80	0.89

(1)由上表可知，$L=5$時，$AP=14.6$最大。

(2)由上表可知，$L=5$時，$AVC=0.411$最小。

(3)由上表可知，$L=6$時，$ATC=0.67$最小。可見ATC最低點產量比AVC最低點產量大。

45.

Quantity of output	Fixed costs	Variable costs
0	\$10	\$0
1	10	5
2	10	11
3	10	18
4	10	26

5	10	36

Table 3, please determine what the efficient scale of production is:
(A)zero unit (B)one unit (C)three units (D)four units.

解：(D)；將 Table 3 列出 TC 及 ATC，找出 ATC 最低點，即 $Q=4$ 時，$ATC=9$。

Q	TFC	TVC	TC	ATC
0	10	0	10	∞
1	10	5	15	15
2	10	11	21	10.5
3	10	18	28	9.3
4	10	26	36	9
5	10	36	46	9.2

46.Production and Cost

The production technology of representative firm can be expressed by $q=f(L)=\sqrt{L}$.

(1)Please derive the marginal product for L and plot it. (3 分) $MP_L = \frac{1}{2}L^{-\frac{1}{2}}$

(2)Is the return to scale constant, decreasing or increasing? Explain your answer. (3 分)

(3)Please derive the cost function mathematically or graphically. (3 分)

(4)Then according to your answer, derive the marginal cost and plot it. (3 分)【99 逢甲財稅；國貿所】

$C = wL + rK$

$C = wq^2$ $MC = 2wq = 2w\cdot(\sqrt{L}) = 2wL^{\frac{1}{2}}$

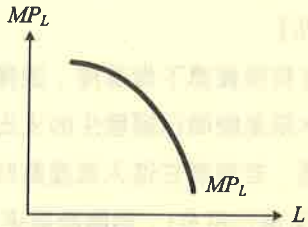
解：

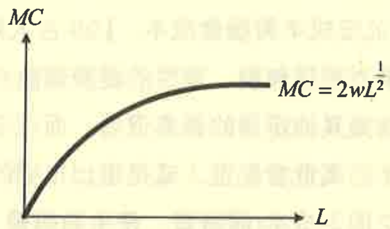
(1) $MP_L = \dfrac{dq}{dL} = \dfrac{1}{2}L^{\frac{-1}{2}}$

(2) $f(\lambda L) = \sqrt{\lambda L} = \lambda^{\frac{1}{2}}\cdot L^{\frac{1}{2}} = \lambda^{\frac{1}{2}}\cdot f(L)$，生產函數具有「DRS」特性

(3) $MC = \dfrac{dTVC}{dq} = \dfrac{d(\overline{w}\cdot L)}{dq} = w\dfrac{dL}{dq} = \dfrac{w}{MP_L} = \dfrac{w}{\frac{1}{2}L^{\frac{-1}{2}}} = 2wL^{\frac{1}{2}}$

(4)





47.The production function of firm 1 is $Q_1 = 20K_1^{0.5}L_1^{0.5}$, and the production function of firm 2 is $Q_2 = 20K_2^{0.8}L_2^{0.2}$, where Q is the total number of cell phones assembled per day, K (capital input) is machine hours per day, and L (labor input) is labor hours per day for each firm. (20%)

(1)If both firms use equal amounts of capital and labor $(K_i = L_i, i=1,2)$ in operation, then which firm would generate more output per day?

(2)Assuming the limitation of capital is 25 machine hours per day, but unlimited for labor. In which firm the marginal product of labor would be greater? 【99 中興行銷所】

解：

(1) 兩家廠商使用相同資本與勞動量，表示 $L_1 = K_1$ ，$Q_1 = 20L_1$ ，$L_2 = K_2$ ，$Q_2 = 20L_2$ ，表示兩家廠商在相同要素使用量下，會有相同產量水準。

(2) 廠商一：$MP_L^1 = \dfrac{\partial Q_1}{\partial L} = 10L_1^{-0.5}K_1^{0.5}$ ，廠商二：$MP_L^2 = \dfrac{\partial Q_2}{\partial L_2} = 4L_2^{-0.8}K_2^{0.8}$

若資本量限制在 $K_1 = K_2 = 25$ ，則 $MP_L^1 = 50L_1^{-0.5}$ ，$MP_L^2 = 4L_2^{-0.8}(25)^{0.8}$

$\dfrac{MP_L^1}{MP_L^2} = \dfrac{5L^{0.3}}{2\,(25)^{0.3}}$ ，當 $L > 1.1789$ 時，$MP_L^1 > MP_L^2$ ，當 $L < 1.1789$ 時，$MP_L^1 < MP_L^2$

48.在短期生產理論中，下列敘述，何項為正確？ (A)當邊際生產力等於平均生產力時，邊際生產力為最大。 (B) 邊際生產力的變化與平均生產力的變化，沒有什麼關係。 (C)當平均生產力遞增時，邊際生產力必然遞減。(D)當平均生產力到達最高時，邊際生產力等於平均生產力。(E)欲使平均生產力為最高時，所需之因素投入量，會低於使得邊際生產力為最高之因素投入量。(2 分)【99 台北公行所】

解：(D)； (A)若 APP_L 最高時，$APP_L = MPP_L$ 。(C)當 APP_L 遞增時，MPP_L 可能遞增或是遞減。

49.Give an example, not similar to the text material, where you erroneously took sunk costs into account where it was inappropriate to do so. (4%)【99 成大財金所】

解：沈沒成本是指已經發生支出而且無法回收的成本，廠商在短期決策分析中，不考慮沈沒成本，因為既然無法回收，對廠商決策沒有影響力。然而沈沒成本的高低又會影響廠商進入或退出市場的意願，若廠商在進入某產業時必須投入極高又無法回收的固定成本（店面預付租金，使用執照費；開辦費），此時廠商要退出市場的機會成本極高。例如廠商在短期決策分析中，常把每個月的租金費用或利息支出視為沈沒成本，這是不正確的，因為租金與利息是廠商營運成本視為可以回收的 TFC，並非沈沒成本。

50.名詞解釋並比較關聯性：沉沒成本與機會成本。【99 台大商研所】

解：機會成本是指在資源有限而慾望無窮，理性的經濟個體必須在有限資源下做選擇，選擇從事某一個經濟活動，必須放棄其他選擇的最高價值。而沉沒成本則是廠商已經發生的支出無法回收的成本；而沉沒成本的高低會影進入或是退出市場的意願，若廠商在進入某產業時必須投入相當大且無法回收之固定成本(開辦費、使用執照費，店面預付租金)，則廠商要退出市場的機會成本亦很高。

51.最近不少國內電子公司除了本業外，還想跨入新興領域例如太陽能光電，假設有一家電子公司只有本業的晶圓製造，其成本函數為 $c(y_1,0)$ 。若它只生產太陽能矽晶圓的成本函數為 $c(0,y_2)$ 。這家公司若同時進行本業的晶圓製造和太陽能矽晶圓製造，則其成本為 $c(y_1,y_2)$ 。在下列何種情況下，這家公司會決定同時生產兩種產品？

(A) $\dfrac{c(y_1,0)+c(0,y_2)-c(y_1,y_2)}{c(y_1,y_2)} > 0$ (B) $\dfrac{c(y_1,0)+c(0,y_2)-c(y_1,y_2)}{c(y_1,y_2)} < 0$

(C) $\dfrac{c(y_1,0)+c(0,y_2)+c(y_1,y_2)}{c(y_1,y_2)}=1$ (D) $\dfrac{c(y_1,0)+c(0,y_2)+c(y_1,y_2)}{c(y_1,y_2)}>1$ 【99 台大經研所】

解：選(A)；範疇經濟程度指標 $S.C=\dfrac{c(y_1,0)+c(0,y_2)-c(y_1,y_2)}{c(y_1,y_2)}$

52.The production function for a product is given by $Q=275KL$. If the price of capital is \$180 per day and the price of labor is \$60 per day, please calculate the minimum cost of producing 550 units of this product. (15%)【99 中興行銷所】

解：Min $TC=60L+180K$

s.t $550=275LK \Rightarrow 2=LK$

利用生產者均衡條件 $\dfrac{MP_L}{MP_K}=\dfrac{P_L}{P_K}$ 求解，$\dfrac{K}{L}=\dfrac{60}{180}$，$L=3K$ 代入生產函數可得 $2=3K^2$，條件要素

$K^2=\frac{2}{3}$

需求函數 $K^*=\left(\dfrac{2}{3}\right)^{\frac{1}{2}}$；$L^*=3\left(\dfrac{2}{3}\right)^{\frac{1}{2}}$

$K=\sqrt{\frac{2}{3}}$

成本函數 $C=180\left(\dfrac{2}{3}\right)^{\frac{1}{2}}+180\left(\dfrac{2}{3}\right)^{\frac{1}{2}}=360\left(\dfrac{2}{3}\right)^{\frac{1}{2}}$

53.最近媒體報導一家印刷工廠的經理因表現優異，老闆送給他的年終獎金是一輛賓士轎車和 100 萬元現金。他接受採訪表示，自己雖在傳統產業工作，但每天要花三小時學習新技術，他印刷書籍的主要原料投入為無毒碳粉(E)和再生紙(F)。假設每印一套書籍平均要消耗三單位的無毒碳粉和五單位的再生紙，且缺一不可。則下列何者為真？(A)該印刷工廠的生產函數為 $\max\left[\dfrac{E}{3},\dfrac{F}{5}\right]$ (B)該印刷工廠的生產函數為 $\min\left[\dfrac{E}{5},\dfrac{F}{3}\right]$ (C)若這個月決定印裝 100 份書籍，則無毒碳粉和再生紙的消耗量分別為 300 和 500 單位 (D)若這個月決定印裝 100 份書籍，則無毒碳粉和再生紙的消耗量分別為 500 和 300 單位【99 台大經研所】

解：(C)；假設每印一套書籍平均要消耗三單位無毒碳粉(E)和五單位再生紙(F)，表示生產函數為完全互補型式，則 $Q=\min\left(\dfrac{E}{3};\dfrac{F}{5}\right)$。滿足成本極小化條件下，$Q=\dfrac{E}{3}=\dfrac{F}{5}$，可計算「條件要素需求函數」：$E^*=3Q$；$F^*=5Q$，$Q=100$ 時，則 $E^*=3(100)=300$，$F^*=5(100)=500$。

54.A firm has two variable inputs and a production function $f(x_1,x_2)=\sqrt{2x_1+4x_2}$. Suppose that the price of the output is 4, the price of input 1 is 2, and the price of input 2 is 3. What are the profit-maximizing amount of factor 1, the profit-maximizing amount of factor 2, and the profit-maximizing output? 【97 高雄金融管理】

解：令 x_1,x_2 為二個可變的要素投入，x_1,x_2 要素價格分別為 $p_1=2, p_2=3$，Q 為產量：

$$Q=f(x_1,x_2)=(2x_1+4x_2)^{\frac{1}{2}}$$

$$MRTS=\frac{\frac{1}{2}(2x_1+4x_2)^{-\frac{1}{2}}\times 2}{\frac{1}{2}(2x_1+4x_2)^{-\frac{1}{2}}\times 4}=\frac{1}{2}<\frac{p_1}{p_2}=\frac{2}{3}$$

$\Rightarrow x_1, x_2$ 為完全替代要素，最適要素雇用方式為：$x_1^* = 0$，全部用 X_2 生產

$$\Rightarrow Q = 2(x_2)^{\frac{1}{2}} \Rightarrow x_2^* = \frac{Q^2}{4} \Rightarrow C^* = p_1x_1^* + p_2x_2^* = 2\cdot 0 + 3\cdot\frac{Q^2}{4} = \frac{3}{4}Q^2$$

$$Max\ \pi = 4Q - \frac{3}{4}Q^2 \quad \text{FOC} \quad \frac{d\pi}{dQ} = 0 \Rightarrow 4 - \frac{3}{2}Q = 0 \Rightarrow Q^* = \frac{8}{3}, x_2^* = \frac{1}{4}\times\frac{64}{9} = \frac{16}{9}, x_1^* = 0$$

SOC $\quad \dfrac{d^2\pi}{dQ^2} = -\dfrac{3}{2} < 0$，滿足利潤極大化充分條件

55.【是非題】If a firm maximizes its profit, then its marginal revenue must be equal to marginal cost. In contrast, if a firm's marginal revenue is equal to marginal cost, then this firm must be maximizing its profit. 【96 高第一科大金融營運】

解：此敘述有誤：

$MR = MC$ 只是廠商追求利潤極大化的必要條件，而非充分條件，說明如下：

$\max \pi = TR(Q) - TC(Q)$

F.O.C $\quad \dfrac{d\pi}{dQ} = 0$，$\dfrac{dTR}{dQ} - \dfrac{dTC}{dQ} = 0 \Rightarrow MR = MC$

S.O.C $\quad \dfrac{d^2\pi}{dQ^2} < 0$，$\dfrac{d^2TR}{dQ^2} - \dfrac{d^2TC}{dQ^2} < 0 \Rightarrow \dfrac{dMR}{dQ} - \dfrac{dMC}{dQ} < 0 \Rightarrow MR$斜率 $< MC$斜率

追求利潤極大化的廠商，其決策應同時滿足以上兩個條件，亦即：

$$\max \pi \Leftrightarrow \begin{cases} MR = MC \\ MR's\ slope < MC's\ slope \end{cases}$$

$MR = MC$ 不保證 $\max \pi$

56.有一生產函數：$Y = 100L^{\frac{1}{2}}K^{\frac{1}{4}}$，L 是勞動，K 是資本，Y 為產出； ℓ

(1)若資本之單位成本是 r，而工資為 w，試求最小成本之投入組合；

(2)若 K 已知且產品價格為 P，試求勞動之需求曲線；

(3)試求擴張方程式。【台大財金所】

解：(1)最小成本最適化模型 $\quad \boxed{\begin{array}{ll} \text{Min} & C = wL + rK \\ \text{s.t} & Y = 100L^{\frac{1}{2}}K^{\frac{1}{4}} \end{array}}$

$$L = wL + rK + \lambda\left[Y - 100L^{\frac{1}{2}}K^{\frac{1}{4}}\right]$$

F.O.C $\quad \dfrac{\partial L}{\partial L} = 0 \Rightarrow w - 50\lambda L^{-\frac{1}{2}}K^{\frac{1}{4}} = 0$

$\dfrac{\partial L}{\partial K} = 0 \Rightarrow r - 25\lambda L^{\frac{1}{2}}K^{-\frac{3}{4}} = 0$

$\dfrac{\partial L}{\partial \lambda} = 0 \Rightarrow r - 100L^{\frac{1}{2}}K^{\frac{1}{4}} = 0$ …………

$K = \frac{w}{2r}L$

$Y = 100$

$$\Rightarrow \frac{w}{r} = \frac{2K}{L} \Rightarrow K = \frac{w}{2r}L \text{代入} \quad Y = 100L^{\frac{1}{2}}\left(\frac{w}{2r}\right)^{\frac{1}{4}}L^{\frac{1}{4}} \quad \therefore Y = 100L^{\frac{3}{4}}\left(\frac{w}{2r}\right)^{\frac{1}{4}} \quad L^{\frac{3}{4}} = \left(\frac{Y}{100}\right)\left(\frac{2r}{w}\right)^{\frac{1}{4}}$$

$\therefore L^* = \left(\frac{Y}{100}\right)^{\frac{4}{3}}\left(\frac{2r}{w}\right)^{\frac{1}{3}}$　　$K^* = \left(\frac{Y}{100}\right)^{\frac{4}{3}}\left(\frac{w}{2r}\right)^{\frac{2}{3}}$　求出 L^*，K^* 稱為 **Conditional factor demand function**

(2)利用利潤極大化模型：

$$\underset{L^*}{Max}\ \pi = 100PL^{\frac{1}{2}}\overline{K}^{\frac{1}{4}} - wL - r\overline{K}$$

F.O.C　$\frac{\partial \pi}{\partial L} = 0 \Rightarrow 50PL^{-\frac{1}{2}}\overline{K}^{\frac{1}{4}} - w = 0$

S.O.C　$\frac{\partial^2 \pi}{\partial L^2} = -25PL^{-\frac{3}{2}}\overline{K}^{\frac{1}{4}} < 0$（滿足利潤極大 S.O.C）

$\frac{50P}{w}\overline{K}^{\frac{1}{4}} = L^{\frac{1}{2}}$　　$\therefore L^* = \frac{2500P^2}{w^2}\overline{K}^{\frac{1}{2}}$

(3)擴張線線上每一點皆為生產者均衡點：

Max	$Y = 100L^{\frac{1}{2}}K^{\frac{1}{4}}$
s.t	$C = wL + rK$

利用邊際生產力均等法則求解：$\frac{MPP_L}{P_L} = \frac{MPP_K}{P_K}$

即　$\frac{50L^{-\frac{1}{2}}K^{\frac{1}{4}}}{25L^{\frac{1}{2}}K^{-\frac{3}{4}}} = \frac{w}{r}$　$\Rightarrow \frac{2K}{L} = \frac{w}{r} \Rightarrow \frac{K}{L} = \frac{w}{2r}$

∴擴張線過原點直線，為 Homothetic production function

57.A firm has the following total cost and demand functions, respectively

$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$　　$Q = 100 - P$

P = 100 − Q

(1)Write out total-revenue function TR in terms of Q.
(2)Formulate the total-profit function π in the terms of Q.
(3)Find the profit-maximizing output Q. (4)What is the maximum profit? 【中興企研所】

解：(1) $TR = 100Q - Q^2$　(2) $\pi = -\frac{1}{3}Q^3 + 6Q^2 - 11Q - 50$

TR = P(Q) = P(100 − Q) = (100 − Q)Q

(3) $\frac{d\pi}{dQ} = 0 \Rightarrow -Q^2 + 12Q - 11 = 0$　$(Q-1)(Q-11) = 0$　$\therefore Q^* = 1$ 或 $Q^* = 11$

TR = 100Q − Q²

$\frac{d^2\pi}{dQ^2} < 0 \Rightarrow -2Q + 12 < 0$　$\therefore Q > 6$，故 $Q^* = 1$(不合)

$Q^* = 11$，$P^* = 89$

MR − MC = 0

(4) $\pi^* = -\frac{1}{3}(11)^3 + 6(11)^2 - 11(11) - 50$　$\therefore \pi^* = 111.34$

58.An electronic manufacturing company employs 100 workers and has two factories, one that produces computer (CO) and one that makes television (TV). With m workers, the computer factory can make m^2 COs per day. With n workers, the television factory can make $5n^2$ TVs per day. (15%)
(1) Show the form of production possibilities frontier. (5%)

(2) Assume computers sell for \$20,000 and TVs sell for \$25,000. What assignment of workers maximizes revenue? 【98 中央產經所】 (10%)

解：

$CO_S = m^2 \quad TV_S = 5n^2$

(1)$m = \sqrt{CO_S} \quad n = \sqrt{\dfrac{TV_S}{5}} \quad \Rightarrow m + n = 100 \Rightarrow \sqrt{CO_S} + \sqrt{\dfrac{TV_S}{5}} = 100$......生產可能曲線

(2)$Max \quad TR = 20{,}000CO_S + 25{,}000TV_S \Rightarrow TR = 20{,}000m^2 + 125{,}000n^2 \quad s.t. \quad m + n = 100$

$if \quad m = 100, n = 0 \Rightarrow TR = 20{,}000(100)^2$

$if \quad m = 0, n = 100 \Rightarrow TR = 125{,}000(100)^2$...較大

因此廠商在收入極大化目標下，會雇用 100 人生產 TV，0 人生產電腦。

59.K, L 為二生產要素，價格分別是 $P_K = 2$ ，$P_L = 3$。生產函數 $Q = \min(K, \sqrt{L})$。在短期，相對於個種可能生產數量而言，K 的數量「足夠」，每家皆為 K。L 為變動要素。市場需求函數 $Q = a - bP$。在短期有 N 家相同廠商，N 很大。 (a)邊際成本 $MC = 4Q$ (b)平均成本 $AC = 3Q + 3K/Q$ (c)市場均衡交易量為 $Na/(6-b)$(若 $6 > b$) (d)市場均衡價格為 $a/[(N/6)+b]$ (e)若橫軸為 Q，縱軸為 P，廠商短期供給函數在縱軸的截距，會隨 P_K 之增加而增加【台大經研所】

解：(d)

(a)(b)成本函數 $C = P_L \cdot L + P_K \cdot K = 3L + 2K$

$$\boxed{\begin{aligned} &Max \quad Q = Min(K, \sqrt{L}) \\ &S.T \quad C = P_L \cdot L + P_K \cdot K = 3L + 2K \end{aligned}}$$

均衡條件：$Q = \sqrt{L} \quad \Rightarrow \quad L = Q^2$ 代入限制式 $STC = 3Q^2 + 2K$

$\because MC = \dfrac{\partial STC}{\partial Q} = 6Q$.........(a)錯誤

$AC = \dfrac{STC}{Q} = 3Q + \dfrac{2K}{Q}$ ...(b)錯誤

(c)(d)市場需求函數 $Q = a - bP$

廠商供給函數 $P = 6q$，市場供給函數 $Q = \dfrac{P}{6} \cdot N$

均衡時 $\Rightarrow \quad a - bP = \dfrac{P}{6} \cdot N \quad \Rightarrow \quad 6a - 6bP = P \cdot N$

$\Rightarrow \quad (N + 6b)P = 6a \quad \Rightarrow \quad P = \dfrac{6a}{N + 6b} = \dfrac{a}{\dfrac{N}{6} + b}$ ...(d)正確

$Q = \dfrac{N}{6} \times \dfrac{6a}{N + 6b} = \dfrac{aN}{N + 6b}$ ....(c)錯誤

(e)廠商短期供給曲線為 SMC 高於 SAVC min 部份不受 P_K 的影響而改變與縱軸截距

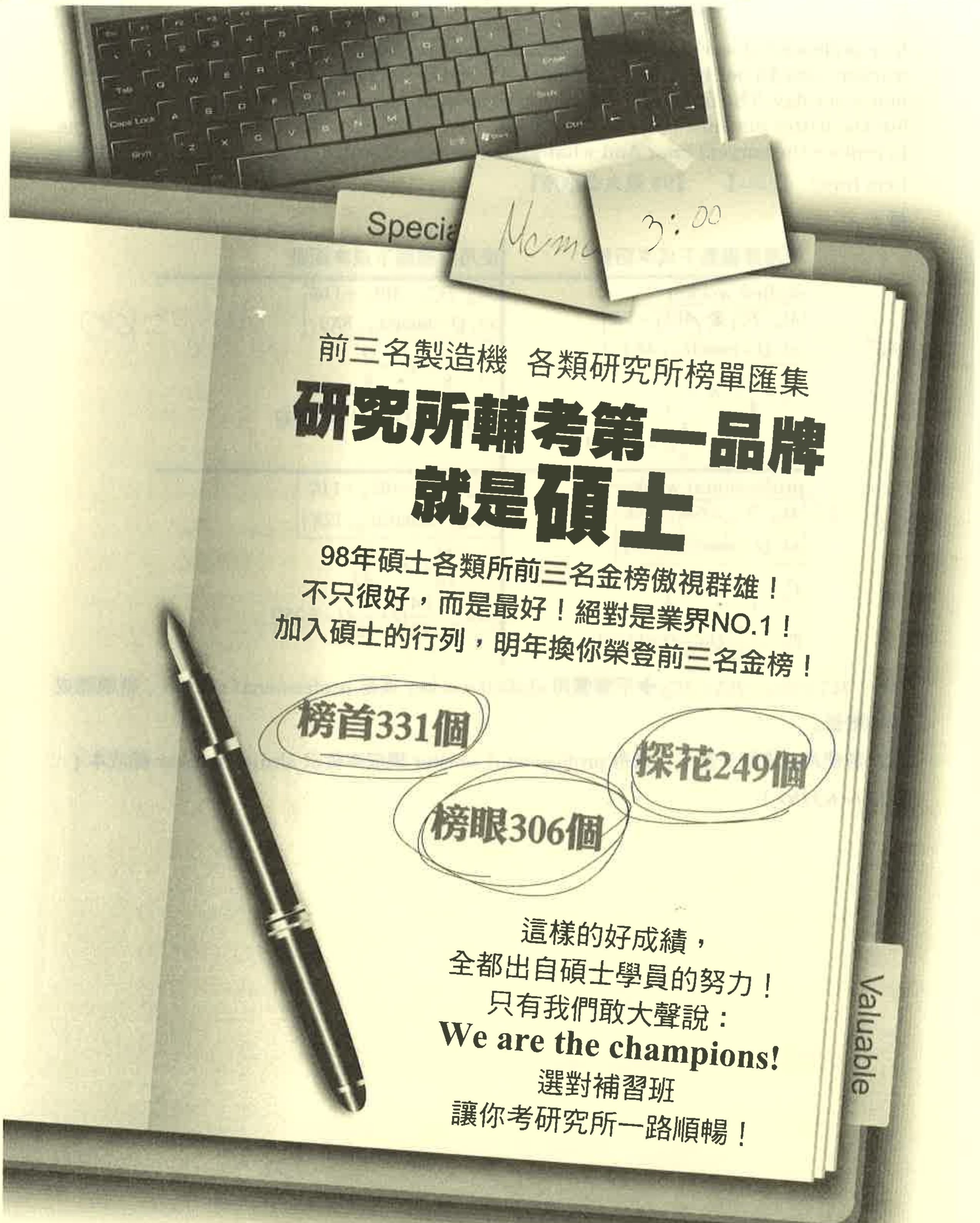
60.A firm is producing widgets with a combination of hired worked and rented machine. Each worker must work on a machine and the current machine costs \$5 per day to rent. The firm can hire skilled worker, which produces an average of 4 widgets per day, or hire professional worker, which produces an average of 6 widgets per day. Skilled worker costs \$5 per hour and professional worker costs \$8 per hour. Worker works 8 hours per day. The firm can rent a better machine that will double output per worker, but the better machine costs \$11 per day to rent. Should the firm rent the new machine to replace the current one? And what kind of worker (skilled or professional) should the firm hire? 【25%】 【95 政大金融所】

解：

使用原機器下成本函數	使用新機器下成本函數
skilled worker 5×8=40 $\boxed{\begin{array}{l} M_{in}\ TC_S = 40L_S + 5K \\ s.t.\ Q=\min(4L_S,\ 4K) \end{array}}$ $L_S^* = \dfrac{Q}{4}$; $K^* = \dfrac{Q}{4}$ $TC_S = \dfrac{40}{4}Q + \dfrac{5}{4}Q = 11.25Q$	$\boxed{\begin{array}{l} M_{in}\ TC'_S = 40L_S + 11K \\ s.t.\ Q=\min(8L_S,\ 8K) \end{array}}$ 8×5=40 $L'_S = \dfrac{Q}{8}$; $K' = \dfrac{Q}{8}$ $TC'_S = \dfrac{40}{8}Q + \dfrac{11}{8}Q = 6.375Q$
professional worker 8×8=64 $\boxed{\begin{array}{l} M_{in}\ TC_p = 64L_p + 5K \\ s.t.\ Q=\min(6L_p,\ 6K) \end{array}}$ $L_p^* = \dfrac{Q}{6}$; $K^* = \dfrac{Q}{6}$ $TC_p = \dfrac{64}{6}Q + \dfrac{5}{6}Q = 11.5Q$	$\boxed{\begin{array}{l} M_{in}\ TC'_p = 40L_p + 11K \\ s.t.\ Q=\min(12L_p,\ 12K) \end{array}}$ $L'_p = \dfrac{Q}{12}$; $K' = \dfrac{Q}{12}$ $\therefore TC'_p = \dfrac{64}{12}Q + \dfrac{11}{12}Q = 6.25Q$

(1) $\because TC'_S < TC_S$； $TC'_P < TC_P$ ➔不管僱用 skilled worker 或是 professional worker，新機器成本均較低。

(2) 若使用新機器生產下，僱用 professional worker 總成本低於 skilled worker 總成本（$\therefore 6.25Q < 6.375Q$）



Memo
3:00
前三名製造機 各類研究所榜單匯集
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